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Financial Derivatives

Third Edition

Robert W. Kolb
James A. Overdahl

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10 9 8 7 6 5 4 3 2 1

To my splendid Lori, an original who is anything but derivative.

R.W.K.

To Janis, who is consistently above fair value.

J.A.O.

preface

Financial Derivatives introduces the broad range of markets for financial derivatives. A *financial derivative* is a financial instrument based on another more elementary financial instrument. The value of the financial derivative depends on, or derives from, the more basic instrument. Usually, the base instrument is a cash market financial instrument, such as a bond or a share of stock.

Introductory in nature, this book is designed to supplement a wide range of college and university finance and economics classes. Every effort has been made to reduce the mathematical demands placed on the student, while still developing a broad understanding of trading, pricing, and risk management applications of financial derivatives.

The text has two principal goals. First, the book offers a broad overview of the different types of financial derivatives (futures, options, options on futures, and swaps), while focusing on the principles that determine market prices. These instruments are the basic building blocks of all more complicated risk management positions. Second, the text presents financial derivatives as tools for risk management, not as instruments of speculation. While financial derivatives are unsurpassed as tools for speculation, the book emphasizes the application of financial derivatives as risk management tools in a corporate setting. This approach is consistent with today's emergence of financial institutions and corporations as dominant forces in markets for financial derivatives.

This edition of *Financial Derivatives* includes three new chapters describing the applications of financial derivatives to risk management. These new chapters reflect an increased emphasis on exploring how financial derivatives are applied to managing financial risks. These new chapters—Chapter 3 (Risk Management with Futures Contracts), Chapter 5 (Risk Management with Options Contracts), and Chapter 7 (Risk Management with Swaps)—include several new applied examples. These application chapters follow the chapters describing futures (Chapter 2), options (Chapter 4), and the market swaps (Chapter 6). Chapter 1 (Introduction), surveys the major types of financial derivatives and their basic applications. The chapter discusses three types of financial derivatives—futures, options, and swaps. It then considers *financial engineering*—the application of financial derivatives to manage risk. The chapter concludes with a discussion of the markets for financial derivatives and brief comments on the social function of financial derivatives.

Chapter 2 (Futures) explores the futures markets in the United States and the contracts traded on them. Futures markets have a reputation for being incredibly risky. To a large extent, this reputation is justified, but futures contracts may also be used to manage many different kinds of risks. The chapter begins by explaining how a futures exchange is organized and how it helps to promote liquidity to attract greater trading volume. Chapter 2 focuses on the principles of futures pricing. Applications of futures contracts for risk management are explored in Chapter 3.

The second basic type of financial derivative, the option contract, is the subject of Chapter 4 (Options). Options markets are very diverse and have their own particular jargon. As a consequence, understanding options requires a grasp of the institutional details and terminology employed in the market. Chapter 4 begins with a discussion of the institutional background of options markets, including the kinds of contracts traded and the price quotations for various options. However, the chapter focuses principally on the valuation of options. For a potential speculator in options, these pricing relationships are of the greatest importance, as they are for a trader who wants to use options to manage risk.

Applications of options for risk management are explored in Chapter 5. In addition to showing how option contracts can be used in risk management, Chapter 5 shows how the option pricing model can be used to guide risk management decisions. The chapter emphasizes the role of option sensitivity measures (i.e., "The Greeks") in portfolio management.

Compared to futures or options, swap contracts are a recent innovation. A *swap* is an agreement between two parties, called *counterparties,* to exchange sets of cash flows over a period in the future. For example, Party A might agree to pay a fixed rate of interest on $1 million each year for five years to Party B. In return, Party B might pay a floating rate of interest on $1 million each year for five years. The cash flows that the counterparties make are can be tied to the value of debt instruments, to the value of foreign currencies, the value of equities or commodities, or the credit characteristics of a reference asset. This gives rise to five basic kinds of swaps: **interest rate swaps, currency swaps, equity swaps, commodity swaps, and credit swaps.** Chapter 6 (The Swaps Market) provides a basic introduction to the swaps market, a market that has grown incredibly over the last decade. Today, the swaps market has begun to dwarf other derivatives markets, as well as securities markets, including the stock and bond markets. New to this edition's treatment of swaps is a section on counterparty credit risk. Also, applied examples of swaps pricing have been added.

Applications of swaps for risk management are explored in Chapter 7. New to this edition are sections on duration gap management, uses of equity swaps, and swap portfolio management. This last section describes the concepts of value at risk (VaR) and stress testing and their role in managing the risk of a derivatives portfolio.

Chapter 8 (Financial Engineering and Structured Products) shows how forwards, futures, options, and swaps are building blocks that can be combined by the financial engineer to create new instruments that have highly specialized and desirable risk and return characteristics. While the financial engineer cannot create instruments that violate the well–established trade–offs between risk and return, it is possible to develop positions with risk and return profiles that fit a specific situation almost exactly. The chapter also examines some of the high-profile derivatives debacles of the past decade. New to this edition are descriptions of the Metallgesellschaft and Long-Term Capital Management debacles.

As always, in creating a book of this type, authors incur many debts. All of the material in the text has been tested in the classroom and revised in light of that teaching experience. For their patience with different versions of the text, we want to thank our students at the University of Miami and Johns Hopkins University. Shantaram Hegde of the University of Connecticut read the entire text of the first edition and made many useful suggestions. For their work on the previous edition, We would like to thank Kateri Davis, Andrea Coens, and Sandy Schroeder. We would also like to thank the many professors who made suggestions for improving this new edition.

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JAMES A. OVERDAHL

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CHAPTER 1
Introduction

By now the headlines are familiar: "Gibson Greetings Loses $19.7 Million in Derivatives" . . . "Procter and Gamble Takes $157 Million Hit on Derivatives" . . . "Metallgesellschaft Derivatives Losses Put at $1.3 billion" . . . "Derivatives Losses Bankrupt Barings." Such popular press accounts could easily lead us to conclude that derivatives were not only *involved* in these losses, but were *responsible* for them as well. Over the past few years, derivatives have become inviting targets for criticism. They have become demonized—the "D" word—the junk bonds of the New Millennium. But what are they?

Actually, there is not an easy definition. Economists, accountants, lawyers, and government regulators have all struggled to develop a precise definition. Imprecision in the use of the term, moreover, is more than just a semantic problem. It also is a real problem for firms that must operate in a regulatory environment where the meaning of the term often depends on which regulator is using it.

Although there are several competing definitions, we define a *derivative* as a *contract that derives most of its value from some underlying asset, reference rate, or index*. As our definition implies, a derivative must be based on at least one underlying. An *underlying* is the asset, reference rate, or index from which a derivative inherits its principal source of value. Falling within our definition are several different types of derivatives, including commodity derivatives and financial derivatives. A *commodity derivative* is a derivative contract specifying a commodity or commodity index as the underlying. For example, a crude oil forward contract specifies the price, quantity, and date of a future exchange of the grade of crude oil that underlies the forward contract. Because crude oil is a commodity, a crude oil forward contract would be a commodity derivative. A *financial derivative,* the focus of this book, is a derivative contract specifying a financial instrument, interest rate, foreign exchange rate, or financial index as the underlying. For example, a call option on IBM stock gives its owner the right to buy the IBM shares that underlie the option at a predetermined price. In this sense, an IBM call option derives its value from the value of the underlying shares of IBM stock. Because IBM stock is a financial instrument, the IBM call option is a financial derivative.

In practice, financial derivatives cover a diverse spectrum of underlyings, including stocks, bonds, exchange rates, interest rates, credit characteristics, or stock market indexes. Practically nothing limits the financial instruments, reference rates, or indexes that can serve as the underlying for a financial derivatives contract. Some derivatives, moreover, can be based on more than one underlying. For example, the value of a financial derivative may depend on the difference between a domestic interest rate and a foreign interest rate (i.e., two separate reference rates).

In this chapter, we briefly discuss the major types of financial derivatives and describe some of the ways in which they are used. In succeeding sections, we discuss four types of financial derivatives—forward contracts, futures, options, and swaps. We then turn to a brief consideration of *financial engineering*—the use of financial derivatives, perhaps in combination with standard financial instruments, to create more complex instruments, to solve complex risk management problems, and to exploit arbitrage opportunities. We conclude with a discussion of the markets for financial derivatives and brief comments on their social function.

FORWARD CONTRACTS

The most basic forward contract is a *forward delivery contract*. A forward delivery contract is a contract negotiated between two parties for the delivery of a physical asset (e.g., oil or gold) at a certain time in the future for a certain price fixed at the inception of the contract. The parties that agree to the forward delivery contract are known as *counterparties*. No actual transfer of ownership occurs in the underlying asset when the contract is initiated. Instead, there is simply an agreement to transfer ownership of the underlying asset at some future delivery date. A forward transaction from the perspective of the buyer establishes a *long position* in the underlying commodity. A forward transaction from the perspective of the seller establishes a *short position* in the underlying commodity.

A simple forward delivery contract might specify the exchange of 100 troy ounces of gold one year in the future for a price agreed on today, say \$400/oz. If the discounted expected future price of gold in the future is equal to \$400/oz. today, the forward contract has no value to either party *ex ante* and thus involves no cash payments at inception. If the *spot price* of gold (i.e., the price for immediate delivery) rises to \$450/oz. one year from now, the purchaser of this contract makes a profit equal to $5,000 ($450 minus $400, times 100 ounces), due entirely to the increase in the price of gold above its initial expected present value. Suppose instead the spot price of gold in a year happened to be $350/oz. Then the purchaser of the forward contract loses $5,000 ($350 minus $400, times 100 ounces), and she would prefer to have bought the gold at the lower spot price at the maturity date.

For the short, every dollar increase in the spot price of gold above the price at which the contract is negotiated causes a $1 per ounce loss on the contract at maturity. Every dollar decline in the spot price of gold yields a $1 per ounce increase in the contract's value at maturity. If the spot price of gold at maturity is exactly $400/oz., the forward seller is no better or worse off than if she had not entered into the contract.

From our example, we can see that the value of the forward contract depends not only on the value of the gold, but also on the creditworthiness of the contract's counterparties. Each counterparty must trust that the other will complete the contract as promised. A default by the losing counterparty means that the winning counterparty will not receive what she is owed under the terms of the contract. The possibility of default is known in advance to both counterparties. Consequently, this kind of forward contract can reasonably take place only between creditworthy counterparties or between counterparties who are willing to mitigate the credit risk they pose by posting collateral or other credit enhancements.

The most notable forward market is the foreign exchange forward market, in which current volume is in excess of one-third of a trillion dollars per day. Forward contracts on physical commodities are also commonly observed. Forward contracts on both foreign exchange and physical commodities involve *physical* settlement at maturity. A contract to purchase Japanese yen for British pounds three months hence, for example, involves a physical transfer of sterling from the buyer to the seller, in return for which the buyer receives yen from the seller at the negotiated exchange rate. Many forward contracts, however, are *cash-settled forward contracts*. At the maturity of such contracts, the long receives a cash payment if the spot price on the underlying prevailing at the contract's maturity date is above the purchase price specified in the contract. If the spot price on the underlying prevailing at the maturity date of the contract is below the purchase price specified in the contract, then the long makes a cash payment.

Forward contracts are important not only because they play an important role as financial instruments in their own right but also because many other financial instruments embodying complex features can be decomposed into various combinations of long and short forward positions.

FUTURES CONTRACTS

A *futures contract* is essentially a forward contract that is traded on an organized financial exchange such as the Chicago Mercantile Exchange (CME).[1] Organized futures markets as we know them arose in the mid-1800s in Chicago. Futures markets began with grains, such as corn, oats, and wheat, as the underlying asset. *Financial futures* are futures contracts based on a financial instrument or financial index. Today, financial futures based on currencies, debt instruments, and financial indexes trade actively. *Foreign currency futures* are futures contracts calling for the delivery of a specific amount of a foreign currency at a specified future date in return for a given payment of U.S. dollars. *Interest rate futures* take a debt instrument, such as a Treasury bill (T-bill) or Treasury bond (T-bond), as their underlying financial instrument. With these kinds of contracts, the trader must deliver a certain kind of debt instrument to fulfill the contract. In addition, some interest rate futures are settled with cash. A popular cash-settled interest rate futures contract is the CME's Eurodollar futures contract, which has a value at expiration based on the difference between 100 and the then-prevailing three-month London Interbank Offer Rate (LIBOR). Eurodollar futures are currently listed with quarterly expiration dates and up to 10 years to maturity. The 10-year deferred contract, for example, has an underlying of the three-month U.S. dollar LIBOR expected to prevail 10 years hence.

Financial futures also trade based on financial indexes. For these kinds of financial futures, there is no delivery, but traders complete their obligations by making cash payments based on changes in the value of the index. *Stock index futures* are futures contracts that are based on the value of an underlying stock index, such as the S&P 500 index. For these futures, movements in the index determine the gains and losses. Rather than attempt to deliver a basket of the 500 stocks in the index, traders settle their accounts by making cash payments that are consistent with movements in the index. Table 1.1 lists the world's major futures exchanges and the types of financial futures that they trade.[2] Financial futures were introduced only in the early 1970s. The first financial futures contracts were for foreign exchange, with interest rate futures beginning to trade in the mid-1970s, followed by stock index futures in the early 1980s.

Most futures transactions in the United States occur through the *open outcry* trading process, in which traders literally "cry out" their bids to go long and offers to go short in a physical trading "pit." This process helps ensure that all traders in a pit have access to the same information about the best available prices. In recent years, there have been several attempts to replicate the trading pit with online computer networks. Replicating the interactions of traders has proven to be a difficult task and computer-based

TABLE 1.1 World Futures Exchanges and the Financial Futures Contracts They Trade

Exchange	FX	IRF	Index
Chicago Board of Trade (USA)	♦	♦	
Chicago Mercantile Exchange (USA)	♦	♦	♦
EUREX (Germany and Switzerland)		♦	♦
London International Financial Futures Exchange (UK)		♦	♦
New York Board of Trade (USA)	♦		♦
Kansas City Board of Trade (USA)			♦
Mid–America Commodity Exchange (USA)	♦		
Bolsa de Mercadorios de Sao Paulo (Brazil)	♦	♦	♦
New York Mercantile Exchange (USA)			♦
London Securities and Derivatives Exchange (UK)			♦
Tokyo International Financial Futures Exchange (Japan)	♦	♦	
Osaka Securities Exchange (Japan)			♦
Tokyo Stock Echange (Japan)		♦	♦
Korea Stock Exchange (South Korea)			♦
Singapore Exchange (Singapore)	♦	♦	♦
Marche a Terme International de France (France)		♦	♦
Hong Kong Futures Exchange (China)	♦	♦	♦
New Zealand Futures Exchange (New Zealand)		♦	♦
Sydney Futures Exchange (Australia)		♦	♦
Montreal Exchange (Canada)		♦	♦
Toronto Futures Exchange (Canada)			♦
OM Stockholm AB (Sweden)		♦	♦
Cantor Financial Futures Exchange (USA)		♦	
BrokerTec Futures Exchange (USA)		♦	

Notes: FX indicates foreign exchange, IRF indicates interest rate futures, and Index indicates any of a variety of indexes, including stock indexes, interest rate indexes, and physical commodity indexes. The New York Board of Trade is the parent company of the Coffee, Sugar, and Cocoa Exchange, the New York Cotton Exchange, FINEX, and the New York Futures Exchange. In addition to the exchanges listed in the table, several other exchanges exist but are not operational.

Sources: Commodity Futures Trading Commission (CFTC), the *Wall Street Journal, Futures Magazine, Intermarket Magazine,* various issues, various exchange publications.

trading has not grown as fast as many industry professionals forecast a decade ago.

FORWARDS VERSUS FUTURES

To say that a futures contract is a forward contract traded on an organized exchange implies more than may be obvious. This is because trading on an organized exchange involves key institutional features aimed at overcoming the biggest problems traders face in using forward contracts: credit risk exposure, the difficulty of searching for trading partners, and the need for an economical means of exiting a position prior to contract termination.

To mitigate credit risk, futures exchanges require periodic recognition of gains and losses. At least daily, futures exchanges mark the value of all futures accounts to current market-determined futures prices. The winners can withdraw any gains in value from the previous mark-to-market period, and those gains are financed by the losses of the "losers" over that period.

Marking to market creates a difference in the way futures and forward contracts allow traders to lock in prices. With a forward contract, the price of the asset exchanged at delivery is simply the price specified in the contract. With a futures contract, the buyer pays and the seller receives the spot price prevailing at the delivery date. If this is so, then how is the price locked in? The answer is that gains and losses on a futures position are recognized daily so that over the life of the futures contract the accumulated profits or losses—coupled with the spot price at delivery—yield a net price corresponding with the futures price quoted at the time the futures position was established. The marking-to-market procedure requires that customers post a performance bond that, loosely speaking, covers the maximum daily loss on their futures position. Those who fail to meet their margin call have their positions liquidated by the exchange before trading resumes. But how does the exchange know what the maximum daily loss is? The answer is that the exchange imposes daily price limits on its contracts (both on the up side and the down side) to define the maximum loss. For example, the New York Mercantile Exchange limits price movements for its nearby crude oil contract to $7.50 per barrel from the previous day's settlement price. If the limit is hit, then trading halts for the day and can resume that day only at prices within the limit. The point is that marking-to-market—coupled with daily price limits—serve to reduce exposure to credit risk.

In addition to marking to market and price limits, futures exchanges use a clearinghouse to serve as the counterparty to all transactions. If two traders consummate a transaction at a particular price, the trade immediately becomes two legally enforceable contracts: a contract obligating the buyer to buy from the clearinghouse at the negotiated price, and a contract obligating the seller to sell to the clearinghouse at the negotiated price. Individual traders thus never have to engage in credit risk evaluation of other traders. All futures traders face the same credit risk—the risk of a clearinghouse default. To further mitigate credit risk, futures exchanges employ additional means, such as capital requirements, to reduce the probability of clearinghouse default.

A second problem with a forward contract is that the heterogeneity of contract terms makes it difficult to find a trading partner. The terms of forward contracts are customized to suit the individual needs of the counterparties. To agree to a contract, the unique needs of contract counterparties must correspond. For example, a counterparty who wishes to sell gold for delivery in one year, may find it difficult to find someone willing to contract now for the delivery of gold one year from now. Not only must the timing coincide for the two parties, but both parties must want to exchange the same amount of gold. Searching for trading partners under these constraints can be costly and time consuming, leaving many potential traders unable to consummate their desired trades. Organized exchanges, by offering standardized contracts and centralized trading, economize on the cost of searching for trading partners.

A third and related problem with a forward contract is the difficulty in exiting a position, short of actually completing delivery. In the example of the gold forward contract, imagine that one party to the transaction decides after six months that it is undesirable to complete the contract through the delivery process. This trader has only two ways to fulfill his or her obligation. The first way is to make delivery as originally agreed, despite its undesirability. The second is to negotiate with the counterparty, who may in fact be perfectly happy with the original contract terms, to terminate the contract early, a process that typically requires an inducement in the form of a cash payment. As explained in Chapter 2, the existence of organized exchanges makes it easy for traders to complete their obligations without actually making or taking delivery.

Because of credit risk exposure, the cost and difficulty of searching for trading partners, and the need for an economical means of exiting a position early, forward markets have always been restricted in size and scope.[3] Futures markets have emerged to provide an institutional framework that copes with these deficiencies of forward contracts. The organized futures exchange standardizes contract terms and mitigates the credit risk associated with forward contracts. As we will see in Chapter 2, an organized exchange also provides a simple mechanism that allows traders to exit their positions at any time.

OPTIONS

As the name implies, an *option* is the right to buy or sell, for a limited time, a particular good at a specified price. Such options have obvious value. For example, if IBM is selling at $120 and an investor has the option to buy a share at $100, this option must be worth at least $20, the difference between the price at which you can buy IBM ($100) through the option contract and the price at which you could sell it in the open market ($120).

Prior to 1973, options of various kinds were traded over-the-counter. An *over-the-counter market* (OTC) is a market without a centralized exchange or trading floor. In 1973, the Chicago Board Options Exchange (CBOE) began trading options on individual stocks. Since that time, the options market has experienced rapid growth, with the creation of new exchanges and many kinds of new option contracts. These exchanges trade options on assets ranging from individual stocks and bonds, to foreign currencies, to stock indexes, to options on futures contracts.

There are two major classes of options, call options and put options. Ownership of a *call option* gives the owner the right to buy a particular asset at a certain price, with that right lasting until a particular date. Ownership of a *put option* gives the owner the right to sell a particular asset at a specified price, with that right lasting until a particular date. For every option, there is both a buyer and a seller. In the case of a call option, the seller receives a payment from the buyer and gives the buyer the option of buying a particular asset from the seller at a certain price, with that right lasting until a particular date. Similarly, the seller of a put option receives a payment from the buyer. The buyer then has the right to sell a particular asset to the seller at a certain price for a specified period of time. Options, like other financial derivatives, can be written on financial instruments, interest rates, foreign exchange rates, and financial indexes.

In all cases, ownership of an option involves the right, but not the obligation, to make a transaction. The owner of a call option may, for example, buy the asset at the contracted price during the life of the option, but there is no obligation to do so. Likewise, the owner of a put option may sell the asset under the terms of the option contract, but there is no obligation to do so. Selling an option does commit the seller to specific obligations. The seller of a call option receives a payment from the buyer, and in exchange for this payment, the seller of the call option (or simply, the "call") must be ready to sell the given asset to the owner of the call, if the owner of the call wishes. The discretion to engage in further transactions always lies with the owner or buyer of an option. Option sellers have no such discretion. They have obligated themselves to perform in certain ways if the owners of the options so desire.

As Table 1.2 shows, there are five options exchanges in the United States trading options on financial instruments, reference rates, and financial indexes. In many respects, options exchanges and futures exchanges are organized similarly. In the options market, as in the futures market, there is a seller for every buyer, and both markets allow offsetting trades. To buy an option, a trader simply needs to have an account with a brokerage firm holding a membership on the options exchange. The trade can be executed through the broker with the same ease as executing a trade to buy a stock. The buyer of an option will pay for the option at the time of the trade, so there is no more worry about cash flows associated with the purchase. For the seller of an option, the matter is somewhat more complicated. In selling a call option, the seller is agreeing to deliver the stock for a set price if the owner of the call so chooses. This means that the seller may need large financial resources to fulfill his or her obligations. The broker is representing the trader to the exchange and is, therefore, obligated to be sure that the trader has the necessary

TABLE 1.2 U.S. Options Exchanges and Options Traded

Chicago Board Options Exchange
Options on individual stocks
Long-term options on individual stocks
Options on stock indexes
Options on interest rates
American Stock Exchange
Options on individual stocks
Long-term options on individual stocks
Options on stock indexes
Options on exchange traded funds
Philadelphia Stock Exchange
Options on individual stocks
Long-term options on individual stocks
Options on stock indexes
Options on foreign currency
Pacific Exchange
Options on individual stocks
Long-term options on individual stocks
International Securities Exchange
Options on individual stocks

Note: This listing does not include options on futures contracts.

financial resources to fulfill all obligations. For the seller, the full extent of these obligations is not known when the option is sold. Accordingly, the broker needs financial guarantees from option writers. In the case of a call, the writer of an option may already own the shares of stock and deposit these with the broker. Writing call options against stock that the writer owns is called writing a *covered call.* This gives the broker complete protection because the shares that are obligated for delivery are in the possession of the broker. If the writer of the call does not own the underlying stocks, he or she has written a *naked option,* in this case a naked call. In such cases, the broker may require substantial deposits of cash or securities to insure that the trader has the financial resources necessary to fulfill all obligations.

The Option Clearing Corporation (OCC) serves as a guarantor to ensure that the obligations of options contracts are fulfilled for the selling and purchasing brokerage firms. Brokerage firms are either members of the OCC or are affiliated with members. The OCC provides credit risk protection by enforcing rigorous membership standards and margin requirements. The OCC also maintains a self-insurance program that includes a guarantee trust fund. As an additional safeguard, the OCC has the right to assess additional funds from member firms to make up any default losses. As in the futures market, the buyer and seller of an option have no direct obligations to a specific individual but are obligated to the OCC. Later, if an option is exercised, the OCC matches buyers and sellers and oversees the completion of the exercise process, including the delivery of funds and securities.

SWAPS

A *swap* is an agreement between two or more parties to exchange sets of cash flows over a period in the future. For example, Party A might agree to pay a fixed rate of interest on $1 million each year for five years to Party B. In return, Party B might pay a floating rate of interest on $1 million each year for five years. There are five basic kinds of swaps, *interest rate swaps, currency swaps, equity swaps, commodity swaps,* and *credit swaps*. Swaps can also be classified as “plain vanilla” or “flavored.” An example of a plain vanilla swap is the fixed-for-floating swap described earlier. Some types of plain vanilla swaps can be highly standardized, not unlike the standardization of contract terms found on an organized exchange. With flavored swaps, numerous terms of the swap contract can be customized to meet the particular needs of the swap’s counterparties.

Swaps are privately negotiated derivatives. They trade in an off-exchange, over-the-counter environment. Swap transactions are facilitated by dealers who stand ready to accept either side of a transaction (e.g., pay fixed or receive fixed) depending on the customer's demand at the time. These dealers generally run a *matched book,* in which the cash flows on numerous transactions net to a relatively small risk exposure on one side of the market. Many of these matched trades are termed *customer facilitations,* meaning that the dealer serves as a facilitating agent, simultaneously providing a swap to a customer and hedging the associated risk with either an offsetting swap position or with a futures position. The dealer collects a fee for the service and, if the transaction is structured properly, incurs little risk. When exact matching is not feasible for offsetting a position, dealers typically lay off the *mismatch risk* (also known as the *residual risk*) of their dealing portfolio by using other derivatives. Interest rate swap dealers, for example, rely heavily on CME Eurodollar futures to manage the residual risks of an interest rate swap-dealing portfolio. Chapters 6 and 7 explore how swap dealers price their swap transactions and manage the risk inherent in their swap portfolios.

Because dealers act as financial intermediaries in swap transactions, they typically must have a relatively strong credit standing, large relative capitalization, good access to information about a variety of end users, and relatively low costs of managing the residual risks of an unmatched portfolio of customer transactions. Firms already active as financial intermediaries are natural candidates for being swap dealers. Most dealers, in fact, are commercial banks, investment banks, and other financial enterprises such as insurance company affiliates.

Swap customers, called *end users,* usually enter into a swap to modify an existing or anticipated risk exposure. Swaps have also been used to establish unhedged positions allowing the end user an additional means with which to speculate on future market movements. End users of swaps include commercial banks, investment banks, thrifts, insurance companies, manufacturing and other nonfinancial corporations, institutional funds (e.g., pension and mutual funds), and government-sponsored enterprises (e.g., Federal Home Loan Banks). Dealers, moreover, may use derivatives in an end-user capacity when they have their own demand for derivatives exposure. Bank dealers, for example, often have a portfolio of interest rate swaps separate from their dealer portfolio to manage the interest rate risk they incur in their traditional commercial banking practice.

The origins of the swaps market can be traced to the late 1970s, when currency traders developed currency swaps as a technique to evade British controls on the movement of foreign currency. The first interest rate swap occurred in 1981 in an agreement between IBM and the World Bank. Since that time, the market has grown rapidly. Table 1.3 shows the notional amount of swaps outstanding at year-end for 1987 to 2001. By the end of 2001, interest rate and currency swaps with $69.2 trillion in underlying notional principal were outstanding. Over 90 percent of the swaps reported in

TABLE 1.3 Value of Outstanding Interest Rate and Currency Swaps ($ Trillions of Notional Principal)

Year	Total Swaps Outstanding	Year	Total Swaps Outstanding
1987	$ .683	1995	$17.713
1988	1.010	1996	25.453
1989	1.539	1997	29.035
1990	2.312	1998	50.997
1991	3.065	1999	58.265
1992	3.851	2000	63.009
1993	6.177	2001	69.200
1994	11.303		

Note: Figures include interest rate swaps, foreign currency swaps, and interest rate options. ISDA, the Office of the Comptroller of the Currency (OCC), and the Bank for International Settlements (BIS) each conduct surveys of derivatives transactions. The three sources show similar year-to-year changes in activity, but report different absolute levels. The BIS survey, for example, reports a notional principal value of $111 trillion for year-end 2001 compared to ISDA's $69.2 trillion and the OCC's $45 trillion. We report ISDA's results because the data series go back further than the series of either the OCC or BIS.

Source: International Swaps and Derivatives Association (ISDA).

Table 1.3 are interest rate swaps and the remaining are currency swaps. Of these swaps, approximately 90 percent of currency swaps and 30 percent of interest rate swaps involved the U.S. dollar.[4]

Notional principal is simply the total principal amount used to calculate swap cash flows. Currency swaps have principal that actually is exchanged, interest rate swaps do not—hence, the term notional. In most cases, the cash flows actually exchanged are at least an order of magnitude smaller than the notional principal amount. Therefore, the notional amount underlying a swap reveals nothing about the capital actually at risk in that transaction. Despite these flaws, changes in notional principal over time provide a useful measure of growth in the market, if not absolute size.

Table 1.3 shows that swaps grew at a compounded annual rate of 39.1 percent over the 1987 to 2001 period. The growth of the swaps market has been the most rapid for any financial product in history.

Chapter 6 provides a basic introduction to the swaps market. The swaps market is growing rapidly because it provides firms facing financial risks a flexible way to manage that risk. We explore the risk management motivation that has led to this phenomenal growth in some detail.

FINANCIAL ENGINEERING

So far, we have described four types of derivatives—forwards, futures, options, and swaps. These derivatives serve as the financial building blocks for building more complex derivatives. We can view a complex derivative as a portfolio containing some combination of these building blocks. The process of building more complex financial derivatives from the elemental blocks is referred to as *financial engineering*.[5] Financial engineering is most often used to create custom solutions to complex risk management problems and to exploit arbitrage opportunities. But financial engineering can also be used to place leveraged bets on market movements and to engineer around portfolio constraints, tax laws, accounting standards, and government regulations.

Sometimes a combination of elemental building blocks will replicate an already existing building block instead of a new financial instrument. When the net cash flows of two building blocks held in the same portfolio are equivalent to the cash flows on some other building block, the position is called a *synthetic instrument* and the portfolio of original building blocks is said to be "synthetically equivalent" to the resulting building block whose cash flows are replicated. The purpose of creating synthetic instruments is often to exploit arbitrage opportunities between financial positions with equivalent cash flows.

One of the most important applications of financial engineering is to risk management. Some risks can be easily managed using the elemental building block derivatives, but other risks require the services of a financial engineer to design a custom solution. In this section, we show a simple example of how to manage risks with financial derivatives. We then consider some complexities that may call for a custom solution by a financial engineer.

A Simple Risk Management Example Using Building Block Derivatives

Assume that a pension fund expects to receive $1,000,000 in three months to invest in stocks. If the fund manager waits until the money is in hand, the fund will have to pay whatever prices prevail for the stocks at that time. This exposes the fund to risk because of the uncertain value of stocks three months from now. By contrast, the fund manager could use financial derivatives to manage that risk. The manager could buy stock index futures calling for delivery in three months. If the manager buys stock index futures today, the futures transaction acts as a substitute for the cash purchase of stocks and immediately establishes the effective price that the fund will pay for the stocks it will actually purchase in three months. Let us say that the stock index futures trades at a quoted price of 100.00 index units, each unit being worth $1, and the fund manager commits to purchase 10,000 units. The manager now has a $1,000,000 position in stock index futures. This futures commitment does not involve an actual cash purchase. As explained in Chapter 2, purchasing a futures contract commits the buyer to a future exchange of cash for the underlying asset.

Three months later, let us assume that the index stands at 105.00, so the fund manager has a futures position worth $1,050,000 and a futures trading profit of $50,000. The manager can close this position and reap the $50,000 profit. At this time, the pension fund receives the anticipated $1,000,000 for investment. Because the index has risen 5 percent, the stocks the manager hoped to buy for $1,000,000 now cost $1,050,000. By combining the $50,000 futures profit with the $1,000,000 the fund receives for investment, the fund manager can still buy the stocks as planned. If the manager had not entered the futures market, the manager would not have been able to buy all of the shares that were anticipated, as the manager would have $1,000,000 in new investable funds, but the stocks would have risen in value to $1,050,000. By trading the futures contracts, the manager successfully reduced the risk associated with the planned purchase of shares, and the fund is able to buy the shares as it had hoped.

In this example of the pension fund, the stock market rose by 5 percent and the fund generated a futures market profit of $50,000 to offset this rise in the cost of stocks. However, the market could have just as easily fallen by 5 percent over this three-month period. If the stock index fell from 100.00 to 95.00, the fund's futures position would have generated a $50,000 loss. (The fund manager established a $1 million position at an index value of 100.00, so a drop in the index to 95.00 means that the manager's position is worth only $950.000, for a $50,000 loss.) In this case, the manager receives $1,000,000 for investment. The stocks the manager planned to buy now cost only $950,000 instead of the anticipated $1,000,000. Therefore, the manager pays $950,000 for the stocks and uses the remaining $50,000 to cover the losses in the futures market. With a drop in futures prices, the pension fund would have been better off to have stayed out of the futures market. Had it not traded futures, the fund could have bought the desired shares for $950,000 and still had $50,000 in cash.

By trading stock index futures in the way just described, the pension fund manager effectively establishes a price for the shares of $1,000,000. If the stock market rises, the gain on the futures offsets the increase in the cost of the shares, and the pension fund still pays out the $1,000,000 it receives in new funds plus its futures market gains to acquire the shares. If the stock market falls, the loss on the futures is offset by the decrease in the cost of the shares. To acquire the shares and pay its loss in the futures market, the pension fund still pays out the full $1,000,000 it receives. Thus, the pension fund has used the futures market to secure an effective price of $1,000,000 for the shares. Once it enters the futures transaction, the pension fund knows that it will be able to buy the shares that it wants in three months when it receives the $1 million and that it will have no funds left over. Thus, the pension fund has used the futures market to reduce the risk associated with fluctuations in stock prices.

The example of the pension fund illustrates the usefulness of financial derivatives as a risk management tool. At the time the fund entered the market, it could not know whether stock prices would rise or fall. If the fund buys futures as described earlier and the stock market rises, the fund benefits by being in the futures market. However, if the fund buys futures and the stock market falls, the fund suffers by being in the futures market. By trading futures, the fund was effectively ensuring that it would pay $1,000,000 for the stocks it wished to purchase. This decision reduced risk. The decision protected against rising prices, but it sacrificed the chance to profit from falling stock prices.

Complexities in Risk Management and the Financial Engineer

In our example of the pension fund, the risk management problem faced by the pension fund manager was quite simple. A single futures position served to provide a virtually complete solution to manage the risk of an anticipated purchase of stock. Risk management problems are often much more complex. This section introduces some complexities that frequently arise.

Exchange-traded futures and options typically have fairly brief horizons. Financial futures trade actively for maturity dates of only a few months or years into the future. Exchange-traded stock options usually expire within one year. The financial risk facing firms often has a much longer horizon. For example, a firm issuing a bond with a fixed rate of interest may be undertaking a commitment as long as 30 years. The longer the horizon, the less satisfactory are exchange-traded derivatives as risk management tools.

As we describe in detail in the following chapters, exchanges trade derivatives based on a limited array of underlying instruments. Firms often face financial risks that are only partially correlated with the instruments that underlie financial futures or exchange-traded options. Faced with such a situation, using a single financial derivative can be a poor solution to the risk management problem, and even a combination of exchange-traded instruments may not be satisfactory. For example, a U.S. auto firm might consider building a plant in Europe and financing it in euros over the 10 years it will require to build the plant. Such a transaction involves long-term interest rate risk and foreign exchange risk. It would be difficult to manage this risk with exchange-traded instruments alone.

Exchanges trade financial derivatives that are based on well-known and fairly simple instruments. Many times, however, firms encounter financial risks that have complex payoff distributions over an extended period. For example, a firm might issue a callable bond, an instrument that can be retired on demand by the issuer under the terms of the bond covenant. Such a complex security involves complex risks for both the issuer and the purchaser. Fully comprehending the risks associated with such an instrument may require the services of a financial engineer. Managing the risks associated with the bond would likely require an assortment of exchange-traded financial derivatives and perhaps one or more swap agreements as well.

Investing in financial instruments, borrowing, and raising funds through stock offerings all involve financial risk. Investors earn their living by understanding the risks to which they are exposed and managing those risks wisely. When the amounts at risk are small and when the instruments employed are simple, the financial risks can be comprehended readily. However, complex risk exposures involving substantial sums of money can be very important, yet difficult to manage, calling for the services of a financial engineer.

Financial Engineering and Structured Notes

Financial engineers can create new products by combining building-block derivatives with basic (nonderivative) financial instruments. For example, a *structured note* can be created by combining the cash flows on a traditional, corporate bond and a building-block derivative. Structured notes are also sometimes called *hybrid debt* because they are a hybrid combination of debt securities and financial derivatives.

Structured notes can contain embedded building block derivatives. Perhaps the simplest type of structured note is a *floating rate note* (FRN), or a note whose coupon payments are indexed to a floating interest rate such as LIBOR. The cash flows on a FRN can be decomposed into the cash flows on a fixed-coupon bond and a fixed-for-floating interest rate swap whose notional principal is the same as the face value of the bond and whose settlement dates correspond to the bond's coupon dates.

A structured note can also be engineered to include option-like payoffs. For example, the Stock Index Growth Notes (SIGNs) issued by the Republic of Austria several years ago, were five-year notes that paid no coupons and returned a principal value to investors at maturity equal to the face value of the note or the percentage increase in the S&P 500 index of stocks. If the S&P 500 declined in value over the life of the note, investors received only the face value of the note. If the S&P 500 rose, however, investors received the percentage increase in the S&P 500 over the life of the note *plus* the face value of the note. The cash flows on the SIGNs thus were equivalent to the cash flows on a portfolio of a zero-coupon bond and a long, at-the-money call option on the S&P 500.

MARKETS FOR FINANCIAL DERIVATIVES

The broadest way to categorize the market environment for derivatives is to distinguish between those transactions privately negotiated in an off-exchange, over-the-counter environment and those conducted on organized financial exchanges. As we have seen, futures exchanges arose to solve some of the problems associated with over-the-counter trading of forward contracts. By mitigating credit risk exposure, economizing on the cost of searching for trading partners, and providing for an economical means of exiting a position prior to contract termination, the futures market grew to dwarf the forward markets that had existed previously. Similarly, the establishment of exchange-traded options led to an explosion in the volume of option trading and resulted in option markets that are much larger and more robust than the over-the-counter option markets that came before.

Just as organized exchanges emerged to overcome the limitations of over-the-counter markets, the swaps market has emerged to overcome the limitations of organized exchanges. Although only about 20 years old, the swaps market has grown tremendously and now dwarfs organized exchanges that trade financial derivatives. In a certain sense, these markets seem to have come full circle: Over-the-counter markets gave way to organized exchange trading of futures and options, and now the exchanges appear to be giving way to a new over-the-counter market. This section reviews the market forces that led to the introduction of trading on organized exchanges and now seem to be leading to an increasing role for over-the-counter markets.

Exchange versus Over-the-Counter Markets

Over-the-counter markets suffer from problems with credit risk when the trading parties do not know and trust each other. Further, liquidity can be low, due to the search costs in finding trading partners willing to take the other side of a desired transaction. Finally, positions in over-the-counter contracts can be difficult to exit before the prescribed termination date.

Organized exchanges have their own weaknesses. First, for some market participants, the standardized contracts traded on organized exchanges lack flexibility in contract terms. Second, exchanges are regulated by the federal government. While this regulation may provide benefits to some traders, it also restricts the kinds of trading that can be conducted. Third, futures and option exchanges are governed by a set of rules, separate from government rules, aimed at lowering the cost of trading and increasing trading volume. Although these rules help reduce overall trading costs, complying with them can be costly and constraining for many traders. We consider these issues in turn.

Contract standardization is a key feature of the exchange-trading environment. Contract standardization concentrates trading interest, helps lower the cost of trading by promoting market liquidity, and provides for an economical means of exiting a position prior to contract termination. But contract standardization comes at the expense of contract customization. For many traders, the terms specified in standardized exchange-traded contracts are not satisfactory for meeting their unique needs. The contracts available on the exchanges may not have the correct risk exposure characteristics or they may not have the appropriate time horizon. Exchange-traded futures and options have only a limited number of months before they expire, and they do not extend as far into the future as many traders would like. For these traders, the trading cost advantage of using standardized contracts is offset by the cost disadvantage of using an imperfect contract ill-suited for their needs. These traders have an incentive to turn to an over-the-counter derivatives dealer to negotiate the precise contract terms required to meet their customized needs.

Both futures and options exchanges are subject to regulation by the federal government. The Commodity Futures Trading Commission (CFTC) regulates the futures exchanges that trade all futures contracts and options on futures. The Securities Exchange Commission (SEC) regulates the options exchanges. These government regulations may enhance the trustworthiness of the market and may make the market function better in some respects, but complying with these regulations involves costs. Today, many large firms that trade financial derivatives are actively seeking to reduce their trading costs by using over-the-counter markets, particularly the swaps market. To counter this trend, U.S. futures exchanges endorsed the passage of the Commodity Futures Modernization Act of 2000, which, when fully implemented, should put a significant portion of exchange-traded derivatives on a more equal competitive footing with over-the-counter derivatives.

In addition to government regulation, the trading of futures and options is governed by exchange rules. The purpose of these rules is to lower the cost of trading and to increase trading volume. While these rules help reduce overall trading costs and promote efficiency, compliance can be costly and constraining for many traders. For example, futures and options exchanges have rules requiring that all trades be publicly executed on the floor of the exchange. Large traders worry that these rules allow their trading activity to be discerned by rival traders, permitting them to glean confidential information about the large trader's positions and trading strategy. If Merrill Lynch starts to buy, the market may recognize that Merrill is trading and anticipate a very large order. Prices would rise in anticipation of the large order, and the increase in prices would mean that Merrill would have to pay more than expected to complete its purchase. To avoid the price impact of their orders, many large firms seek to arrange privately negotiated transactions away from the exchange. By trading in the over-the-counter market, Merrill might be able to quietly negotiate with a single counterparty and consummate the entire transaction in secrecy. By trading in the over-the-counter market, Merrill can potentially avoid the price impact of its large order, reduce its trading costs, and avoid signaling its trading intentions to the market. Large traders often prefer to trade in an over-the-counter environment where their privacy is maintained and where they can execute large transactions without calling attention to their trading activity.[6]

The choice of executing a transaction on an exchange or in the over-the-counter market ultimately depends on the total all-in cost of completing the transaction. This not only includes explicit trading costs such as fees, but also bid-ask spreads and market impact cost, as well as a calculation concerning the suitability of standardized versus customized contracts. As the cost of using over-the-counter markets has declined over the past decade, more and more traders are finding that they can meet their trading objectives in the over-the-counter market.

THE SOCIAL ROLE OF FINANCIAL DERIVATIVES

One question frequently asked about derivatives is whether these instruments have any redeeming social value. To many observers, derivative transactions appear to be nothing more than an elaborate game of "hide the ball." To these observers, it appears that risk is just being shuffled from one investor to another without creating anything of social value.

Traditionally, two social benefits have been associated with financial derivatives. First, as already seen, financial derivatives are useful in managing risk. Second, the market for financial derivatives generates publicly observable prices containing the market's assessment of the current and future economic value of certain assets. This is true not only for exchange-traded derivatives but also for several benchmark swap transactions conducted in the over-the-counter market. Society as a whole benefits from financial derivatives markets in these two ways. Thus, the financial derivatives markets are not merely a gambling den, as some would allege. While financial derivatives trading *does* provide plenty of opportunity for gambling, these markets create genuine value for society as well.

From the point of view of society as a whole, the risk management and risk transference functions of financial derivatives provide a substantial benefit. Because financial derivatives are available for risk management, firms can undertake projects that might be impossible without advanced risk management techniques. For example, the pension fund manager discussed earlier in this chapter might be able to reduce the risk of investing in stocks and thereby improve the well-being of the pension fund participants. Similarly, the auto firm that seeks to build a plant in Europe might abandon the project if it is unable to manage the financial risks associated with it. Individuals in the economy also benefit from the risk transference role of financial derivatives. Most individuals who want to finance home purchases have a choice of floating rate or fixed rate mortgages. The ability of the financial institution to offer this choice to the borrower depends on the institution's ability to manage its own financial risk through the financial derivatives market.

Financial derivatives markets are instrumental in providing information to society as a whole. Financial derivatives increase trader interest and trading activity in the cash market instrument from which the derivative stems. As a result of greater attention, prices of the derivative and the cash market instrument will be more likely to approximate their true value. Thus, the trading of financial derivatives aids economic agents in *price discovery*—the discovery of accurate price information—because it increases the quantity and quality of information about prices. When parties transact based on accurate prices, economic resources are allocated more efficiently than they would be if prices poorly reflected the economic value of the underlying assets. As discussed in later chapters, the prices of financial derivatives give information about the future direction of benchmark financial instruments, interest rates, exchange rates, and financial indexes. Firms and individuals can use the information discovered in the financial derivatives market to improve the quality of their economic decisions, even if they do not trade financial derivatives themselves.

SUMMARY

This chapter provided a brief overview of financial derivatives, their markets, and applications. We considered futures, forwards, options, options on futures, and swaps. All of these instruments play an important role in risk management, and we explored some simple examples of how traders can use derivatives to manage risks. Often these risks become complex. Financial engineering is a special branch of finance that creates tailor-made solutions to complex risk management problems and other financial problems using financial derivatives as building blocks.

Derivatives trading began with over-the-counter markets. In the early 1970s, futures and options exchanges developed for financial derivatives and these exchanges provided a great impetus to the development of markets for financial derivatives. In the past two decades we have witnessed a re-emergence of over-the-counter markets. We compared the benefits and detriments of exchange trading versus over-the-counter markets. Finally, we considered the social role of financial derivatives and found that these markets contribute to social welfare by providing for a better allocation of resources and by providing more accurate price information on which market participants can base their economic decisions.

QUESTIONS AND PROBLEMS

1. What are the two major cash flow differences between futures and forward contracts?

2. What is the essential difference between a forward contract and a futures contract?

3. What problems with forward contracts are resolved by futures contracts?

4. Futures and options trade on a variety of agricultural commodities, minerals, and petroleum products. Are these derivative instruments? Could they be considered financial derivatives?

5. Why does owning an option only give rights and no obligations?

6. Explain the differences in rights and obligations as they apply to owning a call option and selling a put option.

7. Are swaps ever traded on an organized exchange? Explain.

8. Would all uses of financial derivatives to manage risk normally be considered an application of financial engineering? Explain what makes an application a financial engineering application.

9. List three advantages of exchange trading of financial derivatives relative to over-the-counter trading.

10. Consider again the pension fund manager example in this chapter. If another trader were in a similar position, except the trader anticipated selling stocks in three months, how might such a trader transact to limit risk?

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CHAPTER 2

Futures

In this chapter, we explore the futures markets in the United States and the contracts traded on them. Futures markets have a reputation for being incredibly risky. To a large extent, this reputation is justified. However, futures contracts can also be used to manage many different kinds of risks. The futures markets play a beneficial role in society by allowing the transference of risk and providing information about the future direction of prices on many commodities and financial instruments.

We begin by explaining how a futures exchange is organized and how it helps to promote liquidity by attracting greater trading volume. After explaining how to read futures price quotations, we focus on the principles of futures pricing and some important applications of futures for risk management.

THE FUTURES EXCHANGE

A futures exchange is a corporation established for trading futures contracts. Although some exchanges operate as for-profit business enterprises, most exchanges are organized as nonprofit corporations composed of members holding seats on the exchange. These seats are traded on an open market, so an individual who wants to become a member of the exchange can do so by buying an existing seat from a member and by meeting other exchange-imposed criteria for financial soundness and ethical reputation. Table 2.1 presents recent prices for seats on the major exchanges. These prices fluctuate radically, depending largely on the exchange's level of trading activity.

Exchange members strive to increase the value of their seats by increasing the trading volume at their exchange. Exchanges compete for trading volume in many ways. An obvious and important way for exchanges to compete is through the types of futures contract they offer for trading. But exchanges compete in less obvious ways, too. For example, exchanges invest

TABLE 2.1 Seat Prices for Major U.S. Futures Exchanges

Exchange	Membership Price ($)
Chicago Mercantile Exchange	735,000
New York Mercantile Exchange	650,000
Chicago Board of Trade	255,000
Kansas City Board of Trade	80,000
Coffee, Sugar and Cocoa Exchange	67,000
New York Cotton Exchange	50,000

Source: Futures and Options World, December, 2000, p. 75. Prices represent last sale.

heavily in establishing and maintaining their reputations for offering fair and competitive markets. Exchanges also compete through their trading rules, the transparency of their marketplace, and the technology they employ for order entry and trade execution. As described in detail later, some exchanges compete by catering to specific segments of the industry.

The exchange provides a setting where members, and other parties who trade through an exchange member, can trade futures contracts. The exchange members participate in committees that govern the exchange. Exchanges also employ professional (nonmember) managers to execute the directives of the members. The Commodity Futures Trading Commission (CFTC), an agency of the U.S. government, regulates futures markets in the United States.

FUTURES CONTRACTS AND FUTURES TRADING

Each exchange provides a trading floor where all of its contracts are traded. The rules of an exchange require all of its futures contracts to be traded only on the floor of the exchange during its official hours. Futures exchanges provide an institutional framework for standardizing contract terms and mitigating credit risk. Organized exchanges also provide a simple mechanism that allows traders to exit their positions at any time.

TYPICAL CONTRACT TERMS

Financial futures contracts can be based on underlying assets, reference rates, or indexes. In addition to specifying the underlying, futures contracts contain many other features. They specify, for example, whether the contract is to be physically delivered at contract expiration, or cash settled.

The range of features can be demonstrated by examining the contract specifications of a futures contract. For example, the Chicago Board of Trade (CBOT) trades Treasury bond futures that call for the delivery of U.S. Treasury bonds. The contract specifies that the seller shall deliver $100,000 face value of U.S. Treasury bonds that are not callable and do not mature within 15 years from the first day of the futures' delivery month. The terms of the futures contract regulate the way in which the bonds will be delivered (by wire transfer between approved banks) and the timing of delivery (on a business day of the appropriate delivery month, i.e., March, June, September, or December). This standardization of the contract terms means that all of the traders will know immediately the exact characteristics of the good being traded, without negotiation or long discussion. In fact, the only feature of a futures contract that is determined at the time of the trade is the price.

ORDER FLOW

Futures contracts are created when an order is executed on the floor of the exchange. The order can originate with a member of the exchange trading for his or her own account in pursuit of profit. Alternatively, it can originate with a trader outside the exchange who enters an order through a broker, who has a member of the exchange execute the trade for the client. These outside orders are transmitted electronically to the floor of the exchange, where actual trading takes place in an area called a pit. A trading *pit* is a specific location on the exchange floor designated for the trading of a particular contract. The trading area consists of an oval made up of different levels, like stairs, around a central open space. Traders stand on the steps or in the central part of the pit, which allows them to see each other with relative ease.

This physical arrangement highlights a key difference between futures exchanges and stock exchanges in the United States. In the stock market, there is a designated market maker (called a specialist at the New York Stock Exchange) for each stock, and every trade on the exchange for a particular stock must go through the market maker for that stock. In the futures market, any trader in the pit may execute a trade with any other trader. Exchange rules require, with limited exceptions, that any offer to buy or sell must be made by *open outcry* to all other traders in the pit. Because each trader is struggling to gain the attention of other traders, this form of trading gives the appearance of chaos on the trading floor. One advantage of open outcry is that every order is exposed to the competitive market process. The federal government and the surveillance staffs of the exchanges watch the process to make sure that transactions occur in a competitive manner with no fictitious trades or prearranged trades.

Certain futures transactions may be privately negotiated away from the trading pit. These transactions are called *exchange for physicals* (EFPs). If two traders have previously established positions in futures to offset or hedge an actual physical or financial commitment, those traders may engage in an EFP in conjunction with a spot market transaction to offset simultaneously their cash and futures positions at a known, fixed price. The EFP and its price are then reported to the futures exchange, which processes the transaction as if it were a normal futures trade. This EFP process has become a common way for swap dealers and other traders of financial futures to establish and liquidate market positions.

Another exception to the open-outcry trading process in the United States is electronic trading. Globex, for example, is an electronic trading system maintained by an alliance of several futures exchanges. Like open outcry, however, Globex still ensures that bids and offers are posted publicly on an electronic screen. Although traders do not shout, they are still presumed to have access to the best available prices. Outside the United States, electronic trading systems like Globex account for a large share of futures trading volume.

Once a trade is executed, the trader will receive confirmation of the trade and the information will be communicated to exchange officials who will report the information in real time to vendors such as Reuters. Information vendors pay fees to the exchange for access to real-time quotations and transactions information from the floor of the exchange. In fact, the sale of real-time information is the second largest source of income for futures exchanges after transaction fees. Information vendors then report the real-time market information to their subscribers over a worldwide electronic communication system.[1]

THE CLEARINGHOUSE AND ITS FUNCTIONS

The trade from an outside party must be executed through a broker, and the broker must, in turn, trade through a member of the exchange. Normally, the two parties to a transaction will be located far apart and will not even know each other. This raises the issue of trust and the question of whether the traders will perform as they have promised. We have already seen that this can be a problem with forward contracts.

To resolve this uncertainty about performance in accordance with the contract terms, each futures exchange has a clearinghouse. The *clearinghouse* is a well-capitalized financial institution that guarantees contract performance to both parties. As soon as the trade is consummated, the clearinghouse interposes itself between the buyer and seller. The clearinghouse acts as a seller to the buyer and as the buyer to the seller. At this point, the original buyer and seller have obligations to the clearinghouse and no obligations to each other. This arrangement is shown in Figure 2.1. The top portion of the figure shows the relationship between the buyer and seller when there is no clearinghouse. The seller is obligated to deliver goods to the buyer, who is obligated to deliver funds to the seller. This arrangement raises the familiar problems of trust between the two parties to the trade. In the lower portion, the role of the clearinghouse is illustrated. The clearinghouse guarantees that goods will be delivered to the buyer and that funds will be delivered to the seller.

At this point, the traders need to trust only the clearinghouse, instead of each other. Because the clearinghouse has a large supply of capital, there is little cause for concern. Also, as the bottom portion of Figure 2.1 shows, the clearinghouse has no net commitment in the futures market. After all the transactions are completed, the clearinghouse will have neither funds nor goods. It only acts to guarantee performance to both parties.

The Clearinghouse and the Trader

While the clearinghouse guarantees performance on all futures contracts, it now has its own risk exposure because the clearinghouse will suffer if traders default on their obligations. To protect the clearinghouse and the exchange, traders must deposit funds with their brokers in order to trade futures contracts. This deposit, known as *margin,* must be in the form of cash or short-term U.S. Treasury securities. The margin acts as a good-faith

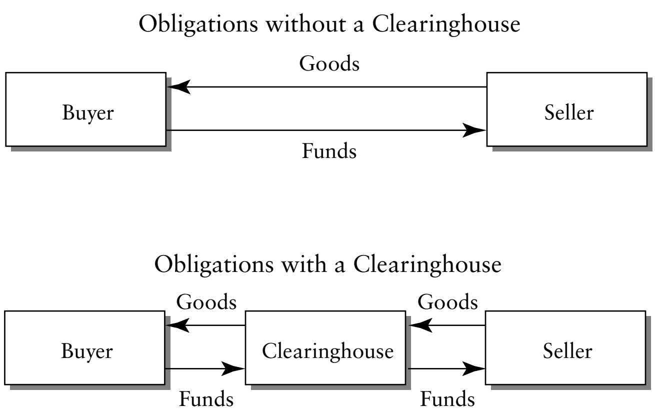

FIGURE 2.1 The function of the clearinghouse in futures markets.

deposit with the broker. If the trader defaults on his or her obligations, the broker may seize the margin deposit to cover the trading losses. This provides a measure of safety to the broker, the clearinghouse, and the exchange.

The margin deposit, however, is normally quite small relative to the value of the goods being traded; it might have a value equal to only 5 to 10 percent of the goods represented by the futures contract. Because potential losses on the futures contract could be much larger than this deposit, the clearinghouse needs other protection from potential default by the trader. To provide protection, futures exchanges have adopted a system known as *daily settlement* or *marking-to-market*. The policy of daily settlement means that futures traders realize their paper gains and losses in cash on the results of each day's trading. The trader may withdraw the day's gains and must pay the day's losses.

The margin deposit remains with the broker. If the trader fails to settle the day's losses, the broker may seize the margin deposit and liquidate the trader's position, paying the losses out of the margin deposit. This practice limits the clearinghouse's exposure to loss from a trader's default. Essentially, the clearinghouse will lose on the default only if the loss on one day exceeds the amount of the margin. This is unlikely to happen and even if it does, the amount lost would probably be very small.

The clearinghouse guarantee extends only to members of the clearinghouse. Such a member may include the broker (who is called a *Futures Commission Merchant* or FCM in futures market lingo). But the clearinghouse's guarantee to the broker does not extend to the customer. The broker could fail to meet his obligations to the customer independent of what happens at the clearinghouse. The March 1985 failure of Volume Investors, a broker and clearing member of the Commodities Exchange, Inc. (now a part of the New York Mercantile Exchange), illustrates the contractual relationships. Some customers of Volume defaulted on a margin call, causing Volume to default on the clearinghouse's margin call, which exceeded the firm's assets. The clearinghouse seized the entire accumulated margin previously posted by Volume on behalf of its customers to pay the other clearinghouse members. This left the nondefaulting customers of Volume with no margin at the clearinghouse and no timely means of obtaining margin and payment on any trading gains. Thus, customers whose only connection with the individuals who defaulted was their use of a common broker found that they had substantial sums at risk. The clearinghouse guarantee did not extend to these customers, who were simply out of luck.

Fulfillment of Futures Contracts

After executing a futures contract, both the buyer and seller have undertaken specific obligations to the clearinghouse. They can fulfill those obligations in two basic ways: First, the trader may actually make or take delivery as contemplated in the original contract, including, where specified, cash settlement. Second, a trader who does not want to make or take delivery can fulfill all obligations by entering a reversing or offsetting trade. More than 99 percent of all futures contracts are settled by a reversing trade.[2]

Delivery

Each futures contract will have its own rules for making and taking delivery. These rules cover the time of delivery, the location of delivery, and the way in which the funds covering the underlying assets will change hands. Investors who do not fully understand the futures market might imagine that they could possibly forget about a futures position and wind up with a boxcar full of porkbellies on the front lawn. However, the delivery process is more complex.

After the clearinghouse interposes itself between the original buyer and seller, each of the trading partners has no obligation to any other trader. As delivery approaches, the clearinghouse supervises the arrangements for delivery. First, the clearinghouse will pair buyers and sellers for the delivery and will identify the two parties to each other. Prior to this time, the two traders had no obligations to each other. Second, the buyer and seller will communicate the relevant information concerning the delivery process to the opposite trading partner and to the clearinghouse. Usually, the seller can choose exactly what features the delivered assets will have. For example, in the T-bond contract, many different bonds qualify for delivery, and the seller has the right to choose which bond to deliver. The seller must tell the buyer which bond will be delivered and the name of the bank account to which the buyer should transmit the funds. Once the funds have been transmitted to the seller's account and this transaction has been confirmed by the seller's bank, the seller will deliver title to the assets to the buyer.

As long as this transaction is proceeding smoothly, which is usually the case, the clearinghouse has little to do. It acts merely as an overseer. If difficulties arise, or if disputes develop, the clearinghouse must intervene to enforce the delivery rules specified in the futures contract.

Reversing Trades

For many futures contracts, physical delivery can be a cumbersome process. In the case of the bond contract, the seller may choose not to deliver the particular bond that the buyer really wants. To avoid taking delivery, most futures traders fulfill their obligations by entering a *reversing trade* prior to the time of delivery. Then, if they need to dispose of their supply of the good, or need to acquire the actual good, they do so in the regular spot market, outside the channels of the futures market.

Prior to the initiation of the delivery process, buyers and sellers are not associated with each other because the clearinghouse has interposed itself between all the pairs of traders. This allows any trader to end a commitment in the futures market without actually making delivery. Figure 2.2 shows the position of three traders assuming that there is no clearinghouse. At time = 0, trader A buys a futures contract, and trader B is the seller. Later, at time = 1, which is still before delivery, trader A decides to liquidate the original position. Accordingly, trader A sells the identical contract that was purchased at time = 0, to trader C, who buys.

In an important sense, trader A no longer has a position in the futures contract, but will merely pass goods from trader B to trader C and will pass funds from trader C to trader B. After time = 1, price fluctuations will not really affect trader A. Traders B and C, however, have a very different perspective. Both have obligations to trader A and expect trader A to perform on the original contracts. This means that trader A is left with duties to perform in the delivery process. As a result, even though there is no longer any risk exposure for trader A, there are still obligations.

From the point of view of trader A, all of this is much simpler if there is a clearinghouse, as shown in Figure 2.2. Because the clearinghouse splits the original trading partners apart as soon as the trade is consummated,

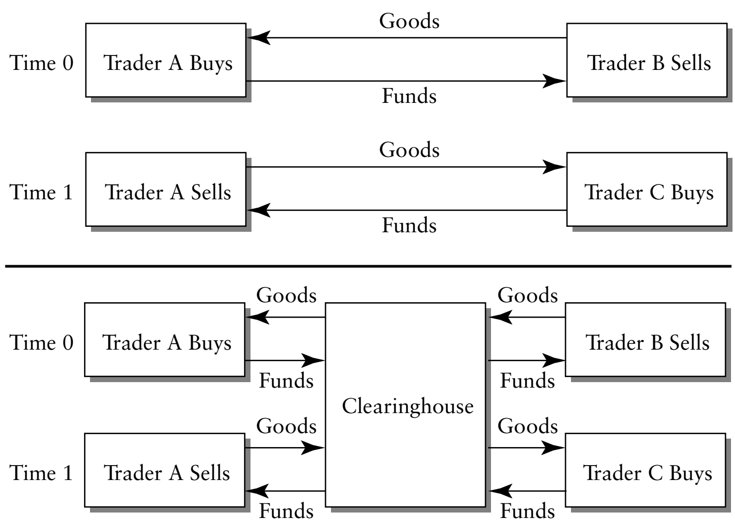

FIGURE 2.2 The mechanism of the reversing trade.

trader A can now execute a reversing trade to get out of the market altogether. After the same trades are made, the clearinghouse can recognize that trader A has no position in the futures market, since the trader has bought and sold the identical futures contract. After time = 1, trader C has assumed the position originally held by trader A. As a result, trader B's position is unaffected, and trader A has no further obligations in the futures market.

It is important to recognize that the reversing trade must be for exactly the same futures contract as originally traded. Otherwise, the trader will have two futures positions rather than none. Also, it should be clear that any trader could execute a reversing trade at any time prior to the contract's expiration. This is exactly what most traders do. As contract expiration approaches, they execute reversing trades to eliminate their futures market commitments. In Figure 2.2, trader C was new to the market, so the same number of futures contracts were still outstanding. However, if trader C had been executing a reversing trade also, the number of contracts outstanding in the marketplace would have decreased.

Cash-Settled Trades

Many financial derivatives are cash settled rather than physically settled. At the maturity of such contracts, the long receives a cash payment if the spot price prevailing at the contract's maturity date is above the purchase price specified in the contract. If the spot price is below the specified purchase price, then the long makes a cash payment. The reverse happens for the short. The short makes a cash payment if the spot price prevailing at the contract's maturity date is above the purchase price specified in the contract. If the spot price is below the specified purchase price, then the short receives a cash payment.

FUTURES PRICE QUOTATIONS

Futures price quotations are freely available over the Internet with a 10-minute delay. Real-time quotes are available from quote vendors on a subscription basis. Brokers, who subscribe to the real-time quote services of vendors, are authorized to transmit the quotes to their customers. Futures quotes are also published each day in the *Wall Street Journal* and other newspapers. Figure 2.3 presents a sample of financial futures price quotations. Although there are far too many different contracts to discuss each in detail, their price quotations are all similar in key respects. For illustrative purposes, we can use the T-bond contract traded by the Chicago Board of Trade (CBOT), as shown in Figure 2.3. The first line of the quotation shows the

[Image not available in this electronic edition.]

FIGURE 2.3 Futures quotations in the *Wall Street Journal. Source: Wall Street Journal,* December 3, 2001.

commodity, followed by the exchange where the futures contract is traded, in this case identified as the CBT. Next, the quotations show the amount of good in a single contract. For the T-bond contract, the contract amount is \$100,000. The last item in this first line is the method of price quotation. For T-bond futures, the price is quoted in points and 32nds of 100% of par. Thus, a quotation of 104-25 means that the futures price is 104 + 25/32 percent of the face value. With a \$100,000 face value, the contract price is \$104,781.25 = [(104 + 25/32)/100](\$100,000). While all of this information is important, it is seriously incomplete. There are additional facts about the T-bond contract that any trader should know before trading T-bond futures, such as the proper way to close a position without delivery, the kinds of T-bonds that are deliverable, and the exact process for making delivery. The CBOT provides all of this detailed information. In the body of the quotation, there is a separate line for each contract maturity. The next contract to come due for delivery is the *nearby contract*. Other contracts, with later delivery dates, are *distant* or *deferred contracts*. The first three columns of figures show the “Open,” “High,” and “Low” prices for the day’s trading.

The fourth column presents the *settlement price* for the day. In most respects, the settlement price is like a closing price, but there can be important differences. Because every trader marks to the market every day, it is important to have an official price to which the trade must be marked. The settlement committee of the exchange stipulates that settlement price. If the markets are active at the close of trading, the settlement price will normally be the closing price. However, if a particular contract has not traded for some time prior to the close of the day’s trading, the settlement committee may believe that the last trade price is not representative of the actual prevailing price for the contract. In this situation, the committee may establish a settlement price that differs from the last trade price. The "Change" column reports the change in the contract's price from the preceding day's settlement price to the settlement price for the day being reported. The next two columns indicate the highest and lowest prices reached by a contract of a particular maturity since the contract began trading.

The last column shows the open interest at the close of the day's trading. The *open interest* is the number of contracts currently obligated for delivery. If a buyer and seller trade one contract, and neither is making a reversing trade, then the open interest is increased by one contract. For example, the transaction shown in Figure 2.1 creates one contract of open interest, since neither party has any other position in the futures market. The trades shown in Figure 2.2, however, also give rise to just one contract of open interest. When traders A and B trade, they create one contract of open interest. When trader A enters a reversing trade and brings trader C into the market, there is no increase in open interest. In effect, trader C has simply taken the place of trader A.

Every contract begins with zero open interest and ends with zero open interest. When the exchange first permits trading in a given contract maturity, there is no open interest until the first trade is made. At the end of the contract's life, all traders must fulfill their obligations by entering reversing trades or by completing delivery. After this process is complete, there is no longer any open interest. Figure 2.4 shows the typical pattern that the open interest will follow. When the contract is first opened for trading, open interest builds slowly and continues to build. In fact, the nearby contract usually has the largest open interest. As the contract nears maturity, however,

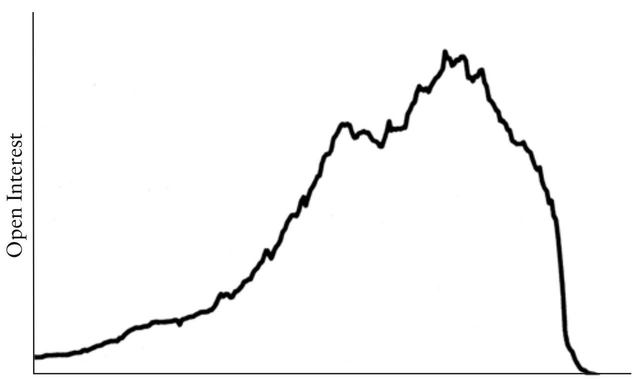

FIGURE 2.4 The typical pattern of open interest over time.

the open interest falls off drastically because many traders enter reversing trades to fulfill their commitments without incurring the expense and bother of actually making delivery. This pattern is uniform and can be seen clearly from the quotations in Figure 2.3.

The final line of the quotations shows the number of contracts that were estimated to have traded on the day being reported and the actual volume for the preceding day's trading. This line also shows the total open interest, which is simply the sum of the open interest for all of the different contract maturities. The very last item in this line is the change in the open interest since the preceding day.

FUTURES PRICING

This section shows that futures prices depend on the cash price of a commodity and the expected cost of storing the underlying good from the present to the delivery date of the futures contract. This Cost-of-Carry Model rests on the idea of arbitrage, and the model defines the price relationship between the spot price of a good and the futures price that precludes arbitrage. Initially, we assume that futures markets are perfect. In this sanitized framework, we can see more clearly the structure of the pricing relationship defined by the Cost-of-Carry Model. Later, we relax the assumption of a perfect market to explore the effect of market imperfections on futures prices.

We focus first on gold as an example of a commodity. While gold is not a financial asset, its simplicity makes it a useful first example. Gold generates no cash flows, such as the coupon payments that are generated by bonds or the dividend payments that are generated by stocks, yet it behaves in most other respects like the financial assets that underlie financial futures contracts. After considering gold, we turn our focus to interest rate futures, stock index futures, and foreign currency futures.

THE COST-OF-CARRY MODEL IN PERFECT MARKETS

We begin by using the concept of arbitrage to explore the Cost-of-Carry Model or carrying charge theory of futures prices. Carrying charges fall into four basic categories: storage costs, insurance costs, transportation costs, and financing costs. Storage costs include the cost of warehousing the commodity in the appropriate facility. While storage seems to apply most clearly to physical goods such as wheat or lumber, it is also possible to store financial instruments. In many cases, the owner of a financial instrument will leave the instrument in a bank vault. For many goods in storage, insurance is also necessary. For example, stored lumber should be protected against fire, and stored wheat should be insured against water damage.[3]

In some cases, the carrying charges also include transportation costs. Wheat in a railroad siding in Kansas must be carried to delivery in two senses. First, it must be stored until the appropriate delivery time for a given futures contract, and second, it must also be physically carried to the appropriate place for delivery. For physical goods, transportation costs between different locations determine price differentials between those locations. Without question, transportation charges play different roles for different commodities. Transporting wheat from Kansas to Chicago could be an important expense. By contrast, a wire transfer costing only a few dollars accomplishes delivery of Treasury bills against a futures contract. In almost all cases, the most significant carrying charge in the futures market is the financing cost. In most situations, financing the good under storage overwhelms the other costs. For financial futures, storage, insurance, and transportation costs are virtually nil, and we ignore them in the remainder of our discussion.

The carrying charges reflect only the charges involved in carrying a commodity from one time or one place to another and do not include the value of the commodity itself. Thus, if gold costs $400 per ounce and the financing rate is 1 percent per month, the financing charge for carrying the gold forward is $4 per ounce per month (1% times $400).

Most participants in the futures markets face a financing charge on a short-term basis that is equivalent to the repo rate, the interest rate on repurchase agreements. In a repurchase agreement, a person sells securities at one time, with the understanding that they will be repurchased at a certain price at a later time. Most repurchase agreements are for one day only and are known, accordingly, as overnight repos. The repo rate is relatively low, exceeding the rate on Treasury bills by only a small amount.[4] The repo rate represents the financing costs of most market participants, who tend to be financial institutions of one type or another and have low financing costs anyway, at least for very short-term obligations.

CASH AND FUTURES PRICING RELATIONSHIPS

The carrying charges just described are important because they play a crucial role in determining pricing relationships between spot and futures prices as well as the relationships among prices of futures contracts of different maturities. For our purposes, assume that the only carrying charge is the financing cost at an interest rate of 10 percent per year. As an example, consider the prices and the accompanying transactions shown in Table 2.2.

The transactions in Table 2.2 represent a successful cash-and-carry arbitrage. This is a cash-and-carry arbitrage because the trader buys the cash

TABLE 2.2 Cash–and–Carry Gold Arbitrage Transactions

Prices for the Analysis:		
Spot price of gold	$400	
Future price of gold (for delivery in one year)	$450	
Interest rate	10%	

	Transaction	Cash Flow
$t = 0$	Borrow $400 for one year at 10%	$+400
	Buy one ounce of gold in the spot market for $400	−400
	Sell a futures contract for $450 for delivery of one ounce in one year	0
	Total Cash Flow	$ 0
$t = 1$	Remove the gold from storage	$ 0
	Deliver the ounce of gold against the futures contract	+450
	Repay loan, including interest.	−440
	Total Cash Flow	$+ 10

good and carries it to the expiration of the futures contract. The trader traded at $t = 0$ to guarantee a riskless profit without investment. There was no investment, because there was no cash flow at $t = 0$. The trader merely borrowed funds to purchase the gold and to carry it forward. The profit in these transactions was certain once the trader made the transactions at $t = 0$. As these transactions show, to prevent arbitrage the futures price of the gold should have been $440 or less. With a futures price of $440, the transactions in Table 2.2 would yield a zero profit. From this example, we can infer the following Cost-of-Carry Rule 1: The futures price must be less than or equal to the spot price of the commodity plus the carrying charges necessary to carry the spot commodity forward to delivery. We can express this rule as follows:

$$F_{0,t} \leq S_0(1 + C) \tag{2.1}$$

where $F_{0,t}$ = the futures price at $t = 0$ for delivery at time = t
S_0 = the spot price at $t = 0$
C = the expected cost of carry, expressed as a fraction of the spot price, necessary to carry the good forward from the present to the delivery date on the futures

As we have seen, if prices do not conform to Cost-of-Carry Rule 1, a trader can borrow funds, buy the spot commodity with the borrowed funds, sell the futures contract, and carry the commodity forward to deliver against the futures contract. These transactions would generate a certain profit without investment, or an arbitrage profit. The certain profit would be guaranteed by the sale of the futures contract. Also, there would be no investment, since the funds needed to carry out the strategy were borrowed and the cost of using those funds was included in the calculation of the carrying charge. Such opportunities cannot exist in a rational market. The cash-and-carry arbitrage opportunity arises because the spot price is too low relative to the futures price.

Whereas an arbitrage opportunity arises if the spot price is too low relative to the futures price, the spot price might also be too high relative to the futures price. If the spot price is too high, we have a reverse cash-and-carry arbitrage opportunity. As the name implies, the steps necessary to exploit the arbitrage opportunity are just the opposite of those in the cash-and-carry arbitrage strategy. As an example of the reverse cash-and-carry strategy, consider the prices for gold and the accompanying transactions in Table 2.3.

In these transactions, the arbitrageur sells the gold short. As in the stock market, a short seller borrows the good from another trader and must later repay it. Once the good is borrowed, the short seller sells it and takes the money from the sale. (The transaction is called short selling because the

TABLE 2.3 Reverse Cash–and–Carry Gold Arbitrage Transactions

Prices for the Analysis:

Spot price of gold	\$420
Future price of gold (for delivery in one year)	\$450
Interest rate	10%

Transaction		Cash Flow
$t = 0$	Sell one ounce of gold short	\$+420
	Lend the \$420 for one year at 10%	–420
	Buy one ounce of gold futures for delivery in one year	0
	Total Cash Flow	\$ 0
$t = 1$	Collect proceeds from the loan (\$420 × 1.1)	\$+462
	Accept delivery on the futures contract	–450
	Use gold from futures delivery to repay short sale	0
	Total Cash Flow	\$+ 12

trader sells a good that he or she does not actually own.) In this example, the short seller has the use of all the proceeds from the short sale, which are invested at the interest rate of 10 percent. The trader also buys a futures contract to ensure that he or she can acquire the gold needed to repay the lender at the expiration of the futures in one year. These transactions guarantee an arbitrage profit. Once the transactions at $t = 0$ are completed, the \$12 profit at $t = 1$ year is certain. Also, the trader had no net cash flow at $t = 0$, so the strategy required no investment. To make this arbitrage opportunity impossible, the spot and futures prices must obey Cost-of-Carry Rule 2: The futures price must be equal to or greater than the spot price plus the cost of carrying the good to the futures delivery date. We can express this rule mathematically with the notation introduced earlier:

$$F_{0,t} \geq S_0(1+C) \tag{2.2}$$

If prices do not obey this Cost-of-Carry Rule 2, there will be an arbitrage opportunity. Table 2.4 summarizes the transactions necessary to conduct the cash-and-carry and the reverse cash-and-carry strategies.

To prevent arbitrage, the two following rules must hold:

Rule 1: To prevent Cash-and-Carry Arbitrage:

$$F_{0,t} \leq S_0(1+C)$$

Rule 2: To prevent Reverse Cash-and-Carry Arbitrage:

$$F_{0,t} \geq S_0(1+C)$$

TABLE 2.4 Transactions for Arbitrage Strategies

Market	Cash–and–Carry	Reverse Cash–and–Carry
Debt	Borrow funds	Lend short sale proceeds
Physical	Buy asset and store; deliver against futures	Sell asset short; secure proceeds from short sale
Futures	Sell futures	Buy futures; accept delivery; return physical asset to honor short sale commiment

Together, equations 2.1 and 2.2 imply Cost-of-Carry Rule 3: The futures price must equal the spot price plus the cost of carrying the spot commodity forward to the delivery date of the futures contract. Expressing Rule 3 mathematically, we have equation 2.3:

$$F_{0,t} = S_0(1+C) \tag{2.3}$$

The relationship of equation 2.3 was derived under the following assumptions: Markets are perfect; that is, they have no transaction costs and no restrictions on the use of proceeds from short sales. It must be acknowledged that this argument explicitly excludes transaction costs. Transaction costs exist on both sides of the market, for purchase or sale of the futures. In many markets, however, transaction costs for short selling are considerably more expensive, which limits the applicability of the reverse cash-and-carry strategy.

We have also assumed, for simplicity, that the asset underlying the futures contract (in our example, gold) does not yield a cash flow to its owner. But for many financial assets, cash flows such as stock dividends and bond coupons will be important considerations. These cash flows can be easily included in the Cost-of-Carry Model as part of a "net" financing cost.

The expected cost of carry defines the economic relationship between cash and futures market. This relationship is expressed in a concept known as *the basis*. The basis is defined as the current cash price of a particular commodity at a specified location minus the price of a particular futures contract for the same commodity. Since the futures price and the cash price must converge at futures expiration, the basis must therefore converge to zero at futures expiration. In the absence of arbitrage opportunities, the value of the basis will equal the expected net cost of carry.

For an arbitrage opportunity to exist, the rate of return on a cost-of-carry strategy must exceed the cost of financing the arbitrage strategy. In trading vernacular, the theoretical rate of return on a cost-of-carry strategy is the *implied repo rate*. Arbitrageurs would therefore calculate the implied repo rate and compare it with their own financing cost (proxied by the actual repo rate) to determine whether an arbitrage opportunity exists. In a well-functioning market without arbitrage opportunities, the implied repo rate is equivalent to the actual repo rate. As we have seen in this section, deviations from this relationship lead to arbitrage opportunities in a perfect market. We now consider the qualifications to the basic conclusion resulting from market imperfections.

THE COST-OF-CARRY MODEL IN IMPERFECT MARKETS

In real markets, the four market imperfections discussed earlier complicate and disturb the relationship of equation 2.3. (These imperfections were transaction costs, restrictions on short selling, unequal borrowing and lending rates, and lack of storability.) The main effect of these market imperfections is to require adjustments in the identity expressed by equation 2.3. Market imperfections do not invalidate the basic framework we have been building. Instead of being able to state an equality as we did in the perfect markets framework leading to equation 2.3, we will find that market imperfections introduce a certain indeterminacy to the relationship.

For financial futures, there are few effective restrictions on short selling. In addition, the goods are essentially storable. However, the coupon payments on bonds and dividend payments on stocks make bonds and stocks somewhat less storable than a commodity like gold. In essence, market imperfections frustrate the cash-and-carry and reverse cash-and-carry strategies that we have been considering. We illustrate these effects by considering direct transaction costs.

In actual markets, traders face a variety of direct transaction costs. First, the trader must pay a fee to have an order executed. For a trader off the floor of the exchange, these fees include brokerage commissions and various exchange fees. Even members of the exchange must pay a fee to the exchange for each trade. Second, in every market there is a bid-asked spread. A market maker on the floor of the exchange must try to sell at a higher price (the asked price) than the price at which he or she is willing to buy (the bid price). The difference between the asked price and the bid price is the bid-asked spread. In our discussion, we assume that these transaction costs are some fixed percentage, T, of the transaction amount. For simplicity, we assume that the transaction costs apply to the spot market but not to the futures market.

To illustrate the impact of transaction costs, we use the same prices with which we began our analysis in perfect markets, but now we consider transaction costs of 3 percent. With transaction costs, our previous arbitrage strategy of buying the good and carrying it to delivery will not work. Table 2.5 shows the results of this effort. With transaction costs, the attempted arbitrage results in a certain loss, not an arbitrage profit.

We would have to pay \$400 as before to acquire the good, plus transaction costs of 3 percent for a total outlay of \$400 $(1 + T)$ = \$412. We would then have to finance this total until delivery for a cost of \$412 (1.1) = \$453.20. In return, we would only receive \$450 on delivery of the futures contract. Given these prices, it does not pay to attempt this "cash-and-

TABLE 2.5 Attempted Cash–and–Carry Gold Arbitrage Transactions

Prices for the Analysis:	
Spot price of gold	\$400
Future price of gold (for delivery in one year)	\$450
Interest rate	10%
Transaction cost (T)	3%

Transaction		Cash Flow
$t = 0$	Borrow \$412 for one year at 10%	\$+412.00
	Buy one ounce of gold in the spot mar-ket for \$400 and pay 3% transaction costs, to total \$412	–412.00
	Sell a futures contract for \$450 for delivery of one ounce in one year	0
	Total Cash Flow	\$ 0
$t = 1$	Remove the gold from storage	\$ 0
	Deliver the ounce of gold to close futures contract	+450.00
	Repay loan, including interest	–453.20
	Total Cash Flow	\$ –3.20

carry" arbitrage. As Table 2.5 shows, these attempted arbitrage transactions generate a certain loss of \$3.20. With transaction costs of 3 percent and the same spot price of \$400, the futures price would have to exceed \$453.20 to make the arbitrage attractive. To see why this is so, consider the cash outflows and inflows. We pay the spot price plus the transaction costs, $S_0(1 + T)$, to acquire the good. Carrying the good to delivery costs $S_0(1 + T)(1 + C)$. These costs include acquiring the good and carrying it to the delivery date of the futures. In our example, the total cost is:

$$S_0(1+T)(1+C) = \$400(1.03)(1.1) = \$453.20$$

Thus, to break even, the futures transaction must yield \$453.20. We can write this more formally as:

$$F_{0,t} \leq S_0(1+T)(1+C) \tag{2.4}$$

If prices follow equation 2.4, the cash-and-carry arbitrage opportunity will not be available. Notice that equation 2.4 has the same form as equation 2.1, but equation 2.4 includes transaction costs.

In discussing the Cost-of-Carry Model in perfect markets, we saw that if futures prices are too high relative to spot prices, arbitrage opportunities will be available, as in Table 2.3. We now explore the transactions of Table 2.3, except we include the transaction costs of 3 percent. Table 2.6 shows these transactions. Including transaction costs in the analysis gives a loss on the same transactions that were profitable with no transaction costs. In the transactions of Table 2.3 with the same prices, the profit was $12. For perfect markets, equation 2.2 gave the no-arbitrage conditions for the reverse cash-and-carry arbitrage strategy:

$$F_{0,t} \geq S_0(1+C) \tag{2.2}$$

Including transaction costs, we have:

$$F_{0,t} \geq S_0(1-T)(1+C) \tag{2.5}$$

Combining equations 2.4 and 2.5 gives:

$$S_0(1-T)(1+C) \leq F_{0,t} \leq S_0(1+T)(1+C) \tag{2.6}$$

TABLE 2.6 Attempted Reverse Cash–and–Carry Gold Arbitrage

Prices for the Analysis:	
Spot price of gold	$420
Future price of gold (for delivery in one year)	$450
Interest rate	10%
Transaction costs (T)	3%

Transaction		**Cash Flow**
$t = 0$	Sell one ounce of gold short, paying 3% transaction costs	
	Receive $420(–.97) = $407.40	$+407.40
	Lend the $407.40 for one year at 10%	–407.40
	Buy one ounce of gold futures for delivery in one year	0
	Total Cash Flow	$ 0
$t = 1$	Collect loan proceeds ($407.40 × 1.1)	$+448.14
	Accept gold delivery on the futures contract	–450.00
	Use gold from futures delivery to repay short sale	0
	Total Cash Flow	$ –1.86

Equation 2.6 defines the no-arbitrage bounds—bounds within which the futures price must remain to prevent arbitrage. In general, transaction costs force a loosening of the price relationship in equation 2.3. In perfect markets, equation 2.3 gave an exact equation for the futures price as a function of the spot price and the cost-of-carry. If the futures price deviated from that no-arbitrage price, traders could transact to reap a riskless profit without investment. For a market with transaction costs, equation 2.6 gives bounds for the futures price. If the futures price goes beyond these boundaries, arbitrage is possible. The futures price can wander within the bounds without offering arbitrage opportunities, however. As an example, consider the bounds implied by the transactions in Table 2.5. With no transaction costs, the futures price must be exactly \$440 to exclude arbitrage. With the 3 percent transaction costs on spot market transactions, the futures price can lie between \$426.80 and \$453.20 without allowing arbitrage, as Table 2.7 shows.

Figure 2.5 illustrates the concept of arbitrage boundaries. The vertical axis graphs futures prices and the horizontal axis shows the time dimension. The solid horizontal line in the graph shows the no-arbitrage condition for a perfect market. In a perfect market, the futures price must exactly equal the spot price times 1 plus the cost of carry, $F_{0,t} = S_0(1 + C)$. With transaction costs, however, we have a lower and an upper bound. If the futures price goes above the upper no-arbitrage bound, there will be a cash-and-carry arbitrage opportunity. This occurs when $F_{0,t} > S_0(1 + T)(1 + C)$. Likewise, if the futures price falls too low, it will be less than the lower

TABLE 2.7 Illustration of No-Arbitrage Bounds

Prices for the Analysis:	
Spot price of gold	\$400
Interest rate	10%
Transaction costs (T)	3%

No-Arbitrage Futures Price in Perfect Markets

$$F_{0,t} = S_0(1 + C) = \$400(1.1) = \$440$$

Upper No-Arbitrage Bound with Transaction Costs

$$F_{0,t} \le S_0(1 + T)(1 + C) = \$400(1.03)(1.1) = \$453.20$$

Lower No-Arbitrage Bound with Transaction Costs

$$F_{0,t} \ge S_0(1 - T)(1 + C) = \$400(.97)(1.1) = \$426.80$$

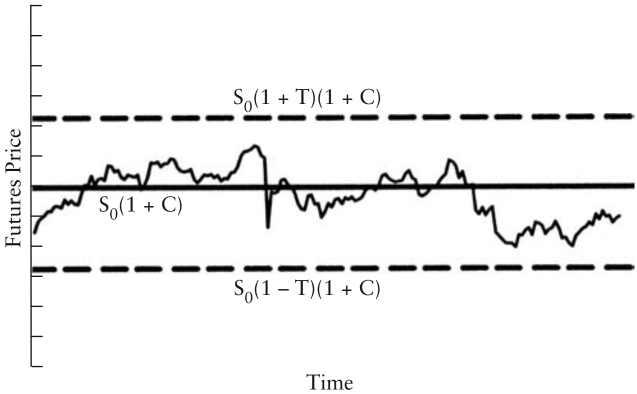

FIGURE 2.5 No-arbitrage bounds.

no-arbitrage bound. Futures prices that are too low relative to the spot price give rise to a reverse cash-and-carry arbitrage. This opportunity arises when $F_{0,t} < S_0(1 - T)(1 + C)$. Figure 2.5 shows these no-arbitrage boundaries as dotted lines.

If the futures price stays between the bounds, no arbitrage is possible. If the futures price crosses the boundaries, arbitrageurs will flock to the market to exploit the opportunity. For example, if the futures price is too high, traders will buy the spot commodity and sell the futures. This action will raise the price of the spot good relative to the futures price, thereby driving the futures price back within the no-arbitrage boundaries. If the futures price stays within the boundaries, no arbitrage is possible, and the arbitrageurs will not be able to affect the futures price.

From Figure 2.5, we can note three important points. First, the greater the transaction costs, *T,* the farther apart are the bounds. With higher transaction costs, the arbitrage relationships we have been exploring are less binding on possible prices. Second, we have been assuming that all traders in the market face the same percentage transaction costs, *T.* Clearly, different traders face different transaction costs. For example, a retail trader (who is not an exchange member) can face much higher transaction costs than those of a floor trader. It is easily possible for the retail trader to pay as much as 100 times the exchange and brokerage fees paid by a floor trader. Therefore, Figure 2.5 really pertains to a particular trader, not to every trader in the market. Consider a trader facing higher transaction costs of $2T$ instead of *T.* For this trader, the no-arbitrage bounds would be twice as wide as those in Figure 2.5. Third, market forces exist to keep the futures price within the no-arbitrage bounds and each trader faces his or her own particular bounds, depending on that trader's transaction costs.

Differences in transaction costs give rise to the concept of quasi-arbitrage. Some traders, such as small retail customers, face full transaction costs. Other traders, such as large financial institutions, have much lower transaction costs. Exchange members pay much lower transaction costs than do outside traders. Therefore, the quasi-arbitrageur is a potential cash-and-carry or reverse cash-and-carry trader with relatively lower transaction costs. The futures price should stay within the bounds of the lowest transaction cost trader. Once the futures price drifts beyond the bounds of the lowest transaction cost trader, he or she will exploit the arbitrage opportunity. Arbitrage activity will drive the futures price back within the no-arbitrage bounds for that trader.

Thus, in the actual market, we expect to see futures prices within the no-arbitrage bounds of the lowest transaction cost trader. This means that traders with higher transaction costs will not be able to exploit any arbitrage opportunities. If prices start to drift away from the perfect markets equality of equation 2.3, the traders with low transaction costs will exploit them first. This exploitation will take place through quasi-arbitrage because the low-transaction-cost trader does not face the full transaction costs of an outside trader. Other market imperfections have a similar effect—they widen the no-arbitrage bounds, such as those illustrated in Figure 2.5.

PRICING INTEREST RATE FUTURES CONTRACTS

In this section, we apply the Cost-of-Carry Model to interest rate futures under the initial assumption of perfect markets. In addition, we assume that the only carrying charge is the interest rate to finance the holding of a good and that we can disregard the special features of a given futures contract. For example, we ignore the options that sellers of futures contracts may hold, such as to substitute various grades of the commodity at delivery or to choose the exact delivery date within the delivery month. We also ignore the differences between forward and futures prices that may result from the daily resettlement cash flows on the futures contract. Later in this chapter, we relax some of these assumptions.

Each interest rate futures contract specifies the maturity of the deliverable bond. For example, the T-bill futures contract requires that a deliverable T-bill must have a maturity of 90 to 92 days. For any given contract, the precise maturity of the underlying T-bill will be known well in advance from the U.S. Treasury's financing schedule. As we saw earlier, the cash-and-carry strategy involves selling a futures contract, buying the spot commodity, and storing it until the futures delivery date. Then the trader delivers the good against the futures contract. For example, if the futures price of gold is too high relative to the cash market price of gold, a trader could engage in a cash-and-carry arbitrage. Part of this strategy would involve buying gold, storing it until the futures expiration, and delivering the gold against the futures contract.

To apply this strategy in the interest rate futures market, we must be very careful. For example, if a T-bill futures contract expires in 77 days, we cannot buy a 90-day T-bill and store it for future delivery. If we attempt to do so, we will find ourselves with a 13-day T-bill on the delivery date, which will not be eligible for delivery against the futures contract. Therefore, to apply a cash-and-carry strategy, a trader must buy a T-bill that will still have or come to have the correct properties on the delivery date. For our T-bill cash-and-carry strategy, the trader must secure a 167-day T-bill to carry for 77 days. Then the bill will have the requisite 90 days remaining until expiration on the delivery date.

We illustrate the cash-and-carry strategy with an example. Consider the data in Table 2.8. The yields used in Table 2.8 are calculated according to the bond pricing formula. The example assumes perfect markets, including the assumption that one can either borrow or lend at any of the riskless rates represented by the T-bill yields. These restrictive assumptions will be relaxed momentarily. The data presented in Table 2.8, and the assumptions just made, mean that an arbitrage opportunity is present. Since the futures contract matures in 77 days, the spot 77-day rate represents the financing cost to acquire the 167-day T-bill, which can be delivered against the MAR futures contract on March 22. This is possible because the T-bill that has 167 days to maturity on January 5 will have exactly 90 days to maturity on March 22.

As the transactions in Table 2.9 indicate, an arbitrage opportunity exists because the prices and interest rates on the three instruments are mutually inconsistent. To implement a cash-and-carry strategy, a trader can sell the MAR futures and acquire the 167-day T-bill on January 5. The trader

TABLE 2.8 Interest Rate Futures and Arbitrage

Today's Date: January 5

Futures	Yield
MAR Contract (Matures in 77 days on March 22)	12.50%
Cash Bills:	
167-day T-bill (Deliverable on MAR futures)	10.00
77-day T-bill	6.00

TABLE 2.9 Cash-and-Carry Arbitrage Transactions

January 5

Borrow $956,750 for 77 days by issuing a 77-day T-bill at 6%.

Buy 167-day T-bill yielding 10% for $956,750.

Sell MAR T-bill futures contract with a yield of 12.50% for $970.984.

March 22

Deliver the originally purchased T-bill against the MAR futures contract and collect $970,984.

Repay debt on 77-day T-bill that matures today for $968,749.

Profit:	
	$970,984
	−968,749
	$ 2,235

then holds the bill for delivery against the futures contract. The trader must finance the holding of the bill during the 77-day interval from January 5 to delivery on March 22. To exploit the rate discrepancy, the trader borrows at the short-term rate of 6 percent and uses the proceeds to acquire the long-term T-bill. At the maturity of the futures, the long-term T-bill has the exactly correct maturity and can be delivered against the futures contract. This strategy generates a profit of $2,235 per contract. Relative to the short-term rate, the futures yield and the long-term T-bill yield were too high. In this example, the trader acquires short-term funds at a low rate (6 percent) and reinvests those funds at a higher rate (10 percent). It may appear that this difference generates the arbitrage profit, but that is not completely accurate, as the next example shows.[5]

Consider the same values as shown in Table 2.8, but assume that the rate on the 77-day T-bill is 8 percent. Now the short-term rate is too high relative to the long-term rate and the futures yield. To take advantage of this situation, we reverse the cash-and-carry procedure of Table 2.9, as Table 2.10 shows. We now exploit a reverse cash-and-carry strategy. With this new set of rates, the arbitrage is more complicated since it involves holding the T-bill that is delivered on the futures contract. In this situation, the arbitrageur borrows $955,131 for 167 days at 10 percent and invests these funds at 8 percent for the 77 days until the MAR futures contract matures. The payoff from the 77-day investment of $955,131 will be $970,984, exactly enough to pay for the delivery of the T-bill on the futures contract.

TABLE 2.10 Reverse Cash-and-Carry Arbitrage Transactions

January 5

Borrow $955,131 by issuing a 167-day T-bill at 10%.

Buy a 77-day T-bill yielding 8% for $955,131 that will pay $970,984 on March 22.

Buy one MAR futures contract with a yield of 12.50% for $970,984.

March 22

Collect $970,984 from the maturing 77-day T-bill.

Pay $970,984 and take delivery of a 90-day T-bill from the MAR futures contract.

June

Collect $1,000,000 from the maturing 90-day T-bill that was delivered on the futures contract.

Pay $998,308 debt on the maturing 167-day T-bill.

Profit:	
	$1,000,000
	−998,308
	$ 1,692

This bill is held for 90 days until June 20 when it matures and pays $1,000,000. On June 20, the arbitrageur's loan on the 167-day T-bill is also due and equals $998,308. The trader repays this debt from the $1,000,000 received on the maturing bill. The strategy yields a profit of $1,692. In this second example, the trader borrowed at 10 percent and invested the funds at 8 percent temporarily. This shows that it is the entire set of rates that must be consistent and that arbitrage opportunities need not only involve misalignment between two rates.

In our previous analysis, the reverse cash-and-carry strategy involves selling an asset short and investing the proceeds from the short sale. In Table 2.10, the short sale is the issuance of debt. By issuing debt, the arbitrageur literally sells a bond. A trader can also simulate a short sale by selling from inventory. The same is true for interest rate futures. For example, a bank that holds investments in T-bills can simulate a short sale by selling a T-bill from inventory.

To this point, we have considered a cash-and-carry strategy in Table 2.9 and a reverse cash-and-carry strategy in Table 2.10. These two examples show that there must be an exact relationship among the rates on the different instruments to exclude arbitrage opportunities. If the yield on the

MAR futures is 12.50 percent and the 167-day spot yield is 10 percent, there is only one yield for the 77-day T-bill that will not give rise to an arbitrage opportunity, and that rate is 7.15 percent. To see why that is the case, consider two ways of holding a T-bill investment for the full 167-day period of the examples:

1. Hold the 167-day T-bill.
2. Hold a 77-day T-bill followed by a 90-day T-bill that is delivered on the futures contract.

Since these two ways of holding T-bills cover the same time period and have the same risk level, the two positions must have the same yield to avoid arbitrage. For the examples, the necessary yield on the 77-day T-bill can be found by solving for a forward rate. This equation expresses the yield on a long-term instrument as being equal to the yield on two short-term positions:

$$(1.10)^{167/360} = (1+x)^{77/360}(1.1250)^{90/360}$$

This equation holds only if the rate, x, on the 77-day T-bill equals 7.1482 percent.

We can also express the same idea in terms of the prices of the bills. To illustrate this point, consider the prices of three securities. The first is a 167-day bill that yields 10 percent and pays $1 on maturity. The second is a T-bill futures with an underlying bill having a $1 face value. With a yield of 12.50 percent, the futures price will be $.970984. Finally, the third instrument matures in 77 days, has a face value of $.970984, and yields 7.1482 percent.

$$P_{167} = \frac{\$1}{(1+r_{167})^{167/360}} = \frac{\$1}{1.1^{167/360}} = .956750$$

$$P_F = \frac{\$1}{(1+r_{fut})^{90/360}} = \frac{\$1}{1.1250^{90/360}} = .970984$$

$$P_{77} = \frac{\$.970984}{(1+r_{77})^{77/360}} = \frac{\$.970984}{1.071482^{77/360}} = .956750$$

The third instrument is peculiar, with its strange face value. However, this is exactly the payoff necessary to pay for delivery on the futures contract in 77 days. Notice also that the 77-day bill and the 167-day bill have the same price. They should, because both prices of $.956750 are the investment now that is necessary to have a $1 payoff in 167 days. The futures yield and the 167-day yield were taken as fixed. The yield on the 77-day bill, 7.1482 percent, is exactly the yield that must prevail if the two strategies are to be equivalent and to prevent arbitrage.

THE FINANCING COST AND THE IMPLIED REPO RATE

With these prices, and continuing to assume that the only carrying cost is the financing charge, we can also infer the implied repo rate. We know that the ratio of the futures price divided by the spot price equals 1 plus the implied repo rate. As shown, the correct spot instrument for our example is the 167-day bill, because this bill will have the appropriate delivery characteristics when the futures matures. Thus, we have:

$$1 + C = \frac{P_F}{P_{167}} = \frac{.970984}{.956750} = 1.014878$$

Thus, the implied repo rate, *C,* is 1.4878 percent. This covers the cost-of-carry for 77 days from the present to the expiration of the futures. We can annualize this rate as follows:

$$1.014878^{360/77} = 1.071482$$

The annualized repo rate is 7.1482 percent. This exactly matches the interest rate on the 77-day bill that will prevent arbitrage. Therefore, assuming that the interest cost is the only carrying charge, the cost-of-carry equals the implied repo rate. This equivalence between the cost-of-carry and the implied repo rate also leads to two rules for arbitrage.

Rule 1: If the implied repo rate exceeds the financing cost, then exploit a cash-and-carry arbitrage strategy:

Borrow funds.

Use the funds to buy the bond in the cash market.

Sell futures to cover the cash market bond.

Hold the bond and deliver it against the futures at the futures expiration to secure the arbitrage profit.

Rule 2: If the implied repo rate is less than the financing cost, then exploit a reverse cash-and-carry arbitrage strategy:

Buy futures.

Sell the cash market bond short.

Invest the short sale proceeds until the futures expiration.

Accept delivery on the futures.

Repay short sale obligation and keep the arbitrage profit.

THE FUTURES YIELD AND THE FORWARD RATE OF INTEREST

The futures price of an interest rate futures contract implies a yield on the instrument that underlies the futures contract. We call this the implied yield the futures yield. Now we continue to assume that the financing cost is the only carrying charge, that markets are perfect, that we can ignore the options that the seller of a futures contract may possess, and that the price difference between forward contracts and futures contracts is negligible. Under these conditions, we can show that the futures yield must equal the forward rate of interest.

We continue to use the T-bill futures contract as our example. The T-bill futures, like many other interest rate futures contracts, has an underlying instrument that will be delivered when the contract expires. The SEP 2002 contract calls for the delivery of a 90-day T-bill that will mature in December 2002. The futures yield covers the 90-day time span from delivery in September to maturity in December 2002. It is possible to compute forward rates from the term structure. Given the necessary set of spot rates, we can compute a forward rate to cover any given period.

To illustrate the equivalence between futures yields and forward rates under our assumptions, we continue to use our example of a T-bill with a 167-day holding period. Assume the following spot yields:

For a 167-day bill	10.0000%
For a 77-day bill	7.1482

These two spot rates imply a forward rate to cover the period from day 77 to day 167:

$$\left(1+r_{0,167}\right)^{167/360}=\left(1+r_{0,77}\right)^{77/360}\left(1+r_{77,167}\right)^{90/360}$$

Substituting values for the spot bills and solving for the forward rate, $r_{77,167}$, gives:

$$(1.10)^{167/360} = (1.071482)^{77/360}\left(1+r_{77,167}\right)^{90/360}$$

$$\left(1+r_{77,167}\right)^{90/360} = \frac{(1.10)^{167/360}}{(1.071482)^{77/360}} = \frac{1.045205}{1.014877} = 1.029884$$

$$\left(1+r_{77,167}\right) = 1.1250$$

$$\left(r_{77,167}\right) = .1250$$

Therefore, the forward rate, to cover day 77 to day 167, is 12.50 percent. As we saw earlier, the futures yield is also 12.50 percent for the T-bill futures that expires on day 77. Therefore, the futures yield equals the forward rate for the same period. In deriving this result, we must bear our assumptions in mind: Markets are perfect, the financing cost is the only carrying charge, and we can ignore the seller's options and the difference between forward and futures prices.

THE COST-OF-CARRY MODEL FOR T-BOND FUTURES

In this section, we apply the Cost-of-Carry Model to the T-bond futures contract. In essence, the same concepts apply, with one difference. The holder of a T-bond receives cash flows from the bond. This affects the cost-of-carry that the holder of the bond actually incurs. For example, assume that the coupon rate on a $100,000 face value T-bond is 6 percent and the trader finances the bond at 6 percent. In this case, the net carrying charge is zero—the earnings offset the financing cost.

To illustrate, assume that, on January 5, a T-bond that is deliverable on a futures contract has a 6 percent coupon as quoted at 100 percent of par value. The trader faces a financing rate of 7.1482 percent (quoted for a 360-day year) for the 77 days until the futures contract is deliverable. Because the T-bond has a 6 percent coupon rate, the conversion factor is 1.0 and plays no role.[6]

With a 6 percent coupon, the accrued interest from the date of purchase to the delivery date on the futures is:

$$\left(\frac{77}{182}\right)(.03)(100{,}000) = \$1{,}269$$

Therefore, the invoice amount will be $101,269. If this is the invoice amount in 77 days, the T-bond must cost the present value of that amount, discounted for 77 days at the 77-day rate of 7.1482 percent. This implies a cost for the T-bond of $99,785. If the price is less than $99,785, a cash-and-carry arbitrage strategy will be available. Under these circumstances, the cash-and-carry strategy would have the cash flows shown in Table 2.11. The transactions in Table 2.11 show that the futures price must adjust to reflect the accrual of interest. The bond in Table 2.11 had no coupon payment during the 77-day interval, but the same adjustment must be made to account for cash flows that the bondholder receives during the holding period.

The instrument underlying the T-bond futures contract is a U.S. government bond with $100,000 face value and a remaining time to maturity (or time to first call if callable) of at least 15 years, as of the first day of the delivery month. Of course, there are several bonds meeting these criteria that are therefore eligible for delivery. At any given time, probably 25 or so bonds are eligible for delivery. This means that T-bond futures are based on a basket of Treasury issues with widely different price and yield characteristics. This is by deliberate design. One of the characteristics of good contract design is that corners and squeezes cannot influence the delivery process. Given that there is a wide variety of eligible bonds, which bond does a holder of a short T-bond futures position choose to deliver to the long? The answer is that the holder will deliver the one that is most profitable, or least costly, to deliver. This bond is called the *cheapest-to-deliver* bond.

Selecting the cheapest-to-deliver bond from the basket of eligible bonds depends on the invoicing convention used at the CBOT. The invoice amount is the amount the short invoices the long for the delivery of a bond against the futures contract. For a particular bond, it is computed as:

$$\text{Invoice Amount} = \left(\text{Future Price} \times \text{Conversion Factor}\right) + \text{Accrued Interest}$$

TABLE 2.11 Cash-and-Carry Transactions for a T-Bond

January 5
Borrow $99,785 for 77 days at the 77-day rate of 7.1482%.
Buy the 6% T-bond for $99,785.
Sell one T-bond futures contract for $101,269.

March 22
Deliver T-bond; receive invoice amount of $101,269.
Repay loan of $101,269.

Profit: 0

In our example, the conversion factor (used to approximate benchmark 6 percent coupon bond for invoicing purposes) is 1.0 since the bond is already a 6 percent coupon bond. Accrued interest is the accrued coupon on the bond between the last coupon date and the delivery date.

Determining the cheapest-to-deliver bond is of interest prior to delivery as well as at delivery. Implementing a cost-of-carry arbitrage strategy prior to delivery requires knowledge of which bond is likely to be delivered.

Interest Rate Futures and the Yield Curve

The yield curve is extremely important for bond investing and bond portfolio management. The different maturities of bonds and their commensurate yields allow investors to commit their money for various periods of time to take advantage of a particular shape that the yield curve might possess at any given moment.

In the interest rate futures markets, the exchanges have made a conscious effort to offer interest rate futures that cover the yield curve. For example, the Chicago Mercantile Exchange (CME) has specialized in the shorter maturity instruments. The CME currently offers interest rate futures contracts on Treasury bills and Eurodollar deposits, all with maturities of about three months. By contrast, the Chicago Board of Trade (CBOT) has focused on the longer maturities. The CBOT trades a contract on long-term T-bonds, the most successful futures contract ever introduced. It also offers contracts on 2-, 5-, and 10-year Treasury notes.

The CME's Eurodollar contract illustrates the connection between the yield curve and interest rate futures. This contract is the largest and most liquid of all short-term interest rate markets. The contract is based on a 3-month Eurodollar time deposit with a face value of one million dollars. The contract is cash settled depending on the 3-month London Interbank Offer Rate (LIBOR) at contract maturity. Prices for Eurodollar futures are quoted according to a system known as the CME Index. The CME Index is simply the yield on the Eurodollar futures subtracted from 100. So, for example, a quoted futures settlement value of 94.00 means that the futures yield would be 6 percent. Eurodollar futures yields are quoted on an "add-on" basis. Each basis point change in the futures yield has a dollar value of $25 under the contract's terms.

As noted in our discussion of arbitrage, there is an important relationship between yields implied by futures prices and the yields on spot market instruments. Essentially, interest rate futures yields may be interpreted as forward rates of interest. For example, the yield of the March 2002 Eurodollar futures contract is the forward rate of interest for a 90-day Eurodollar time deposit to run from March to June 2002. If we calculated from the spot market the forward rate for the same period as that covered by the March 2002 Eurodollar futures contract, we should find a result that almost exactly matches the yield on the Eurodollar futures. If that were not the case, and markets were perfect, it would be possible to take advantage of the yield discrepancy and generate arbitrage profits. Actual markets, however, are not perfect, so the relationship would not have to hold exactly. If we take into account transaction costs, though, the difference between the forward rate of interest calculated from the spot market and the rate of interest implied by the futures contract would still have to be very close.

STOCK INDEX FUTURES

Active futures markets exist for nearly every major stock index across the globe. In the United States, the S&P 500 contract traded at the CME is far and away the most actively traded. The S&P 500 futures contract is based on a portfolio of 500 individual stocks: 400 industrials, 40 utilities, 20 transportation companies, and 40 financial institutions. This contract is cash settled, meaning that delivery of the actual underlying stocks does not occur. Instead, the contract is simply marked-to-market with the cash index on the expiration date of the futures contract and the differences are settled in cash. Neither the S&P futures contract nor the cash index is quoted in dollars. Instead, they are quoted in index points. Each index point is worth $250. For example, if the futures price is quoted at 1250 index points, then the futures contract invoice value would be equivalent to $312,500.

The Cost-of-Carry Model can be applied directly to stock index futures. The price of the futures contract in equilibrium will be the "no arbitrage" price consistent with the model. The main cost of carrying stocks to the expiration date of the futures contract is the financing cost since there are no warehousing fees or insurance premiums. Determination of the carrying cost must include the fact that the stocks in the underlying index pay dividends.

The Cost-of-Carry Model provides a virtually complete understanding of stock index futures pricing. When the conditions of the model are violated, arbitrage opportunities arise. For a cash-and-carry strategy, a trader would buy the stocks that underlie the futures contract, in proportion to their weight in the index, and sell the futures. The trader would then carry these stocks until the futures expiration, at which time the stock position would be liquidated and the futures position settled in cash. The cash-and-carry strategy is attractive when stocks are priced too low relative to the futures. In a reverse cash-and-carry strategy, the trader would sell the stocks short and invest the proceeds, in addition to buying the futures. The reverse cash-and-carry strategy is attractive when stocks are priced too high relative to the futures. Thus, any discrepancy between the futures and cash market prices would lead to a profit at the expiration of the futures simply by exploiting the appropriate strategy.

THE COST-OF-CARRY MODEL FOR STOCK INDEX FUTURES

To fit stock index futures, the Cost-of-Carry Model as expressed in equation 2.3 must be adjusted to include the dividends that will be received between the present and the expiration of the futures. In essence, the chance to receive dividends lowers the cost of carrying the stocks. Carrying stocks requires that a trader finance the purchase price of the stock from the present until the expiration of the futures. For stocks, the cost-of-carry is the financing cost for the stock, less the dividends received while the stock is carried forward.

From the arguments advanced earlier, we know that equation 2.3 holds as an equality with perfect markets and unrestricted short selling. The cash-and-carry trading opportunity requires that the futures price must be less than or equal to the cash inflows at the futures expiration. Similarly, the reverse cash-and-carry trading opportunity requires that the futures price must equal or exceed the cash inflows at the futures expiration. Therefore, the stock index futures price must equal the cost of the stocks underlying the stock index, plus the cost of carrying those stocks to expiration, $S_0(1 + C)$, minus the future value of all dividends to be received, $D_i(1 + r_i)$. The future value of dividends is measured at the time the futures contract expires. More formally:

$$F_{0,t} = S_0\left(1+C\right) - \sum_{i=1}^{n} D_i\left(1+r_i\right)$$

where $F_{0,t}$ = the stock index futures price at $t = 0$ for a futures contract that expires at time t

S_0 = the value of the stocks underlying the stock index at $t = 0$

C = the percentage cost of carrying the stocks from $t = 0$ to the expiration at time t

D_i = the ith dividend

r_i = the interest earned on carrying the i^{th} dividend from its time of receipt until the futures expiration at time t

FAIR VALUE FOR STOCK INDEX FUTURES

The futures price that conforms with the cost-of-carry model is called the *fair-value futures price*. In this section, we consider an example of determining the fair value of the December 2001 S&P 500 stock index futures contract traded on November 30, 2001.

The December 2001 futures contract closed at 1140.00 index points on November 30. The cash index price on this date was 1139.45. The value of the compounded dividend stream expected to be paid out between November 30 and December 21, the expiration date of the December contract, totaled .9 index points.[7] The financing cost prevailing at the time for large, creditworthy borrowers was approximately 1.90 percent annualized over a 365-day year, or .1093 percent over the 21 days between November 30 and the December 21 expiration date for the futures contract. Using this information, we can apply the Cost-of-Carry Model to determine the fair-value futures price:

$$F_{0,t} = 1139.45(1 + .001093) - .9 = 1139.80 \text{ index points}$$

This is the estimated fair value of the December 2001 futures contract at the close of trading on November 30, 2001. Given the design of the S&P 500 futures contract, each index point is worth $250. This means the expected invoice price of the contract is $284,950 (i.e., $250 per index point times 1139.80 index points).

STOCK INDEX ARBITRAGE AND PROGRAM TRADING

In the preceding section, we saw how to derive the fair value futures price from the Cost-of-Carry Model. We know that deviations from the theoretical price of the Cost-of-Carry Model give rise to arbitrage opportunities. If the futures price exceeds its fair value, traders will engage in cash-and-carry arbitrage. If the futures price falls below its fair value, traders can exploit the pricing discrepancy through a reverse cash-and-carry trading strategy. These cash-and-carry strategies in stock index futures are called index arbitrage. This section presents an example of index arbitrage using the S&P 500. Because index arbitrage can require the trading of many stocks, computer programs are often used to automate the trading.

To continue with the previous example, since the closing futures price on this day is 1140, it would appear that an arbitrage opportunity does not exist. The annualized implied repo rate from a cash-and-carry arbitrage strategy would only be .3054 percent. This is determined by calculating the rate of return for the strategy over the 21-day arbitrage period and then annualizing the 21-day rate of return over a 365-day year (i.e., $(1140/1139.80)^{365/21} - 1$). The fact that the implied repo rate is less than the financing cost confirms that an arbitrage opportunity does not exist on this date. The *basis error* for this contract, that is, the actual futures price minus the fair-value futures price, is .20 index points.

Like all stock index futures, our simple example uses cash settlement. Therefore, at expiration on December 21, the final futures settlement price is set equal to the cash market index value. This ensures that the futures and cash prices converge and that the *basis*—the cash price minus the futures price—goes to zero.[8]

To illustrate how stock index arbitrage works, suppose that the December 2001 futures price on November 30, 2001, had been 1143.00 instead of the actual 1140.00 The implied repo rate then would be 4.99 percent annualized over a 365-day year (i.e., $(1143/1139.80)^{365/21} - 1$). This implied repo rate is well above the annualized financing cost of 1.90 percent. In this case, the futures price is above its fair market value as determined by the Cost-of-Carry Model by 3.20 index points. To exploit this apparent arbitrage opportunity, the trader would simultaneously sell the relatively overvalued futures and buy the relatively undervalued cash index: The trader would buy low

TABLE 2.12 Cash-and-Carry Index Arbitrage

Date	Cash Market	Futures Market
November 30	Borrow \$284,862.5 (1139.45 × \$250) 21 days at 1.9%. Buy stocks in the S&P 500 for \$284,862.5.	Sell one DEC S&P 500 index futures contract for 1143.00.
December 21	Receive accumulated proceeds from invested dividends of \$225 (.9 index points × \$250). Sell stock for \$285,000 (1140 index points × \$250). Total proceeds are \$285,225. Repay debt of \$285,173.9.	At expiration, the futures price is set equal to the spot index value of 1140.00. This gives a profit of 3.00 index units. In dollar terms, this is 3.00 index points times \$250 per index point.
	Gain: \$311.40	Gain: \$750
Total Profit: \$311.40 + \$750 = \$1,061.40		

and sell high using a cash-and-carry arbitrage strategy. The cash flows for this cash-and-carry strategy are summarized in Table 2.12.

Suppose instead that the December 2001 futures price on November 30, 2001, had been 1138.00. The implied repo rate then would be 2.78 percent annualized over a 365-day year (i.e., $(1139.80/1138)^{365/21} - 1$). This implied repo rate is above the trader's annualized financing cost of 1.90 percent. In this case, the futures price is below its fair market value as determined by the Cost-of-Carry Model by –1.80 index points. To exploit this apparent arbitrage opportunity, the trader would simultaneously buy the relatively undervalued cash futures and sell the relatively overvalued cash index. The stocks would either be sold short or sold out of inventory. The trader would buy low and sell high using a reverse cash-and-carry arbitrage strategy. The cash flows for this cash-and-carry strategy are summarized in Table 2.13.

Identifying an apparent arbitrage opportunity does not depend on the price prevailing at expiration on December 21 (which happens to be 1140 index points). Instead, the arbitrage opportunity arises solely from a discrepancy between the current futures price and its fair value. The arbitrage gain is locked in no matter what happens to stock prices between November 30 and December 21.

The success of the arbitrage depends on identifying the misalignment between the actual futures price and the fair value futures price. At a given moment, however, the fair value futures price depends on the current price

TABLE 2.13 Reverse Cash-and-Carry Index Arbitrage

Date	Cash Market	Futures Market
November 30	Sell stock in S&P 500 for \$284,862.5 (1139.45 × \$250). Lend \$284,862.5 for 21 days at 1.9%.	Buy one DEC index futures contract for 1138.00.
December 21	Receive proceeds from investment of \$285,173.9. Buy stocks in S&P 500 index for \$285,000 (1140.00 × \$250). Return stocks to repay short sale.	At expiration, the futures price is set equal to the spot index value of 1140.00. This gives a profit of 2.00 index points. In dollar terms, this is 2.00 index points times \$250 per index.
	Gain: \$173.90	Profit: \$500
Total Profit: \$173.90 + \$500 = \$673.90		

of 500 different stocks. Identifying an index arbitrage opportunity requires the ability to instantly find pricing discrepancies between the futures price and the fair futures price reflecting 500 different stocks. In addition, exploiting the arbitrage opportunity requires trading 500 stocks at the prices that created the arbitrage opportunity. Enter the computer!

By using their computers, large financial institutions can communicate orders to trade stock for very rapid execution. Faced with a cash-and-carry arbitrage opportunity, one of these large traders could execute a computer order to buy each and every stock represented in the S&P 500. Simultaneously, the institution would sell the S&P 500 futures contract. The execution of large and complicated stock market orders with computers is called *program trading*. While computers are used for other kinds of stock market transactions, index arbitrage is the main application of program trading. Often the terms "index arbitrage" and "program trading" are used interchangeably.

STOCK INDEX ARBITRAGE IN THE REAL WORLD

The Cost-of-Carry Model is an essential tool for determining whether arbitrage opportunities exist between the cash market and the futures market. However, the model needs to be refined to account for real-world impediments to arbitrage strategies. For example, our model assumes a known dividend stream between the date the arbitrage strategy is implemented and the date of futures expiration. Since most corporations have stable dividend policies and declare dividends well in advance of actual payment, treating dividends as known is justifiable. On occasion, however, dividend payments may be suspended or altered. In the fall of 2001, Enron Corporation declared a quarterly dividend to be paid in December of that year. But between the date the dividend was declared and date it was to be paid, Enron filed for bankruptcy protection and suspended the dividend payment.

Another potential real-world problem of conducting stock index arbitrage is that the composition of the underlying index continually changes through time. For arbitrage strategies of short duration, this is not much of a problem. But for longer-term strategies, the composition of the index can noticeably change, especially during periods of frequent corporate takeovers and spin-offs.

A study conducted by George Sofianos provides direct evidence on how index arbitrageurs behave in the real world. Sofianos examines 2,659 S&P 500 index arbitrage trades over a 6-month period, explicitly accounting for commissions, margins, bid-ask spread costs, and other factors, such as the *early-closing option*. The early-closing option accounts for the fact that in the real world index arbitrage strategies can be unwound prior to futures expiration when it is profitable to do so. The early-closing option, like any option, is valuable to the option holder and this value will be reflected in the arbitrageur's calculation of the arbitrage strategy's profitability.

Sofianos finds that the existence of arbitrage opportunities depends on the level of transaction costs. As expected, lower transaction costs are associated with more frequent arbitrage opportunities. Sofianos finds that the duration of arbitrage opportunities range from 2.5 to 4.5 minutes. This means that arbitrage opportunities must be quickly exploited to be profitable.

Sofianos finds differences in profitability between cash-and-carry arbitrage and reverse cash-and-carry arbitrage. Because of stock exchange restrictions on short sales of stock, arbitrage strategies requiring short sales of stock can be more costly than an otherwise equivalent strategy requiring the purchase of stock or the sale of stock out of existing inventory. Orders requiring short selling of stocks take longer to fill, resulting in higher risks in trade execution. To compensate for this increased execution risk, reverse cash-and-carry strategies require higher expected returns before a profitable position can be established.

Sofianos finds that arbitrageurs often use surrogate stock baskets containing a subset of the index stocks instead of trading all the stocks in the index. He finds that the average number of stocks used in S&P 500 arbitrage strategies is only 280 of the 500 members of the index. By using fewer stocks, arbitrageurs can reduce transaction costs, but they introduce the risk that the surrogate portfolio may not precisely track the entire index. This *tracking risk* means that the arbitrage strategy is no longer risk free. To compensate for this risk, arbitrageurs will use surrogate baskets only when the difference between the futures price and the fair value futures price is large.

Sofianos also finds that arbitrageurs frequently establish (or liquidate) their futures and cash positions at different times. In over one-third of the trades he studied, arbitrageurs did not establish the cash and futures legs of the transaction simultaneously. This practice, known as *legging,* is risky because until the arbitrageur establishes both legs, the arbitrage profit is not locked in.

Sofianos points out some other real-world problems of conducting stock index arbitrage. For example, the prices observed on trading screens may not reflect the actual state of the market at the time arbitrageurs make their decisions. The staleness problem is particularly pronounced at the opening. S&P starts reporting S&P 500 index values before all component stocks have opened for trading. For stocks that have not opened, the index uses the previous day's closing price. Apparent arbitrage opportunities may therefore be illusory. The cost associated with stale prices is really a subset of the execution risk problem.

INDEX ARBITRAGE AND STOCK MARKET VOLATILITY

Since the advent of stock index futures in 1982, stock index arbitrage has been a controversial trading practice because of its alleged role in contributing to stock market volatility. Episodes of extreme market volatility in 1987 and 1989 heightened the controversy, leading many policymakers and market professionals to request that the New York Stock Exchange (NYSE) take steps to restrict the practice. In response to these requests, the NYSE amended its trading rules in July 1990 to restrict—or "collar"—stock index arbitrage on days of large price movements. Index arbitrage continues to be controversial and the subject of continuing investigation. Therefore, the following summary of current thinking must be regarded as provisional.

GENERAL STOCK MARKET VOLATILITY

There has been a general impression of increased volatility in the stock market. This view has been strengthened by occurrences such as the October 1987 crash and the frequency of days with large price swings. However, most studies of stock market volatility conclude that volatility in the 1980s (the period in which stock index futures trading was introduced) was not noticeably greater than during other periods. Thus, there does not seem to be a general increase in the overall volatility in the stock market attributable to stock index futures trading.

EPISODIC VOLATILITY

While overall volatility may not have increased, some scholars have suggested that short-term volatility may be higher due to stock index futures trading. For example, the massive orders associated with program trading might lead to temporary episodes of extremely high volatility, even though the month-to-month volatility shows no real change. This concern focuses on bursts of volatility that occur for a day or even an hour; these periods have become known as *episodic volatility* or *jump volatility*. The bulk of evidence suggests a connection between stock index futures trading and jump volatility. However, the evidence does not suggest that the temporary increase in volatility associated with futures trading impairs the functioning of the stock market.[9] In fact, volatility can be a result of a well-functioning market if the volatility reflects genuine market information. As noted, stock market volatility and the role of futures markets in contributing to that volatility remain controversial issues.

Stock exchange restrictions on index arbitrage appear to have had little impact on episodes of extreme stock market volatility. However, the restrictions have significantly curtailed index arbitrage activity on days of extreme price movements. Despite the curtailment in index arbitrage trading volume, its primary effect on the cost-of-carry relationship between cash and futures seems to be only that arbitrage opportunities persist longer than when arbitrage restrictions are not in place.[10]

SINGLE STOCK FUTURES

The Commodity Futures Modernization Act of 2000 officially ended a two-decade ban on the trading of single stock futures in the United States. Several exchanges announced plans to enter this market beginning in 2002. In the spring of 2001, three exchanges, the Chicago Board of Trade, the Chicago Mercantile Exchange, and the Chicago Board Options Exchange formed a joint venture to trade single stock futures. This means, for example, that it will be possible to trade individual futures contracts on shares of IBM stock. The prohibition on single stock futures trading in the United States meant that traders had to use stock index futures or stock options to manage risks generated by individual stocks.

Single stock futures have been traded in countries other than the United States. These contracts have been reasonably successful in Sweden, Canada, and Asia. Because of these successes, futures exchanges have been cautiously optimistic about the prospects of single stock futures in the United States.

FOREIGN CURRENCY FUTURES

Foreign exchange futures were the first financial futures traded on organized futures exchanges. Even though trading in foreign exchange (FX) forwards had gone on for centuries, it was not until 1972 that the CME began trading FX futures. Although fabulously successful, FX futures still do not approach the volume observed in the interbank over-the-counter forward FX market. Futures contracts written on the European Union's euro and the Japanese yen are the most actively traded.

THE COST-OF-CARRY MODEL AND FOREIGN CURRENCY FUTURES

The best way to understand futures FX pricing is to study forward FX pricing. Foreign currency forward contracts are traded in most major currencies, with bid-ask spreads quoted in standard maturities of 1, 2, 3, 6, 9, and 12 months. Moreover, for the major currencies—the European Union's euro, British pound sterling, and Japanese yen—quotes for 4 months, 5 months, or other intervals are also available. On a negotiated basis, FX forward contracts are available in major currencies for "odd dates" (also referred to as "broken dates") as well. The forward FX markets—like the spot FX markets—are liquid and efficient and are used by sophisticated participants, generally large, internationally active banks.

The cost-of-carry model for exchange rates is known as the *Interest Rate Parity Model*. Cost-of-carry arbitrage is called *covered interest arbitrage*. Even though the names are different from the ones we have used previously, the same basic principles apply. Working with FX rates can be confusing because traders must always keep straight whether price quotations are expressed as the domestic currency in terms of the foreign currency (e.g., dollars per euro) or the foreign currency in terms of the domestic currency (e.g., euros per dollar). Particularly for FX contracts, it helps to label the dimensions of every price quotation. To see how covered interest arbitrage works, consider the four markets that interact in determining this FX rate between the euro and U.S. dollars:

1. The U.S. credit market (determines r_{US}, the rate at which dollars at time 0, $\$_0$, are converted to dollars at time T, $\$_T$).
2. The European Union credit market (determines r_{euro}, the rate at which euros at time 0, $euro_0$, are converted to euros at time T, $euro_T$).
3. The spot FX market, S.
4. The forward FX market, F.

The force of arbitrage means that the foreign exchange rate must be related to the spot rate via the domestic and foreign interest rates. To see how arbitrage works in this market, consider the two ways in which a person with dollars can obtain forward euros:

Route 1: Go from future dollars to future euros via the forward currency market.

Route 2:
- A. Borrow dollars today in the U.S. credit market, and
- B. Trade the current dollars for current euros in the spot FX market, and
- C. Lend the euros today in the European Union credit markets for future euros.

If the exchange rate is expressed as dollars per euro, then the cost-of-carry arbitrage relation produces the forward FX rate, $F_{0,t}$:

$$F_{0,t} = S_0 \times \frac{(1 + r_{\text{US}})}{(1 + r_{\text{euro}})}$$

Where r_{US} is the interest rate in the United States and r_{euro} is the interest rate in Europe. If the exchange rate is expressed as euros per dollar, then the cost-of-carry relation produces the following forward FX rate:

$$F_{0,t} = S_0 \times \frac{(1 + r_{\text{euro}})}{(1 + r_{\text{US}})}$$

For example, if the current FX spot rate is .8954 dollars per euro and the one-year dollar LIBOR is 3.5 percent (annual) and euro LIBOR (euribor) is 4.33 percent (annual), then the one-year fair value forward rate can be calculated as:

$$F_{0,t} = S_0 \times \frac{(1 + r_{\text{US}})}{(1 + r_{\text{euro}})} = .8954 \times \left(\frac{1.035}{1.0433} \right) = .8883 \left(\frac{\$}{\text{euro}} \right)$$

Continuing with our example, we can use a shorthand approximation of the cost-of-carry relationship between U.S. dollars and euros:

$$F_{0,t} = S_0 \times (1 + C)$$

where C is the percentage cost of carrying foreign currency (in our example, euros) forward from $t = 0$ to time t:

$$C = (r_{\text{US}} - r_{\text{euro}})$$

The advantage of this shorthand notation is that it conforms with the more general Cost-of-Carry Model used throughout this chapter. Applying this shorthand cost-of-carry relationship to our example, we now have a one-year fair value forward rate of

$$F_{0,t} = 8954 \times [1 + (.035 - .0433)] = .8880 \left(\frac{\$}{\text{euro}} \right)$$

SUMMARY

This chapter provided a general introduction to financial futures markets. We examined the institutional features of futures markets, including the flow of orders, the role of the clearinghouse, and the fulfillment of commitments in the futures market.

Futures pricing was also explored in this chapter, with a focus on the cost-of-carry relationship between futures and spot prices. If this cost-of-carry relationship is violated, arbitrage opportunities will exist. Although there are many different kinds of futures contracts, including those on agricultural commodities, petroleum products, and metals, this chapter has emphasized financial futures—particularly interest rate, stock index, and foreign currency futures.

QUESTIONS AND PROBLEMS

1. What are the two most important functions of the clearinghouse of a futures exchange?

2. What is the investment for a trader who purchases a futures contract? Justify your answer.

3. What is open interest? What happens to open interest over the life of a futures contract?

4. What are the two ways to fulfill a futures contract commitment? Which is used more frequently? Why?

5. Explain why the futures price might reasonably be thought to equal the expected future spot price.

6. What is the implied repo rate? What information does the implied repo provide about the relationship between cash and futures prices?

7. What kinds of transaction costs do traders face in conducting cost-of-carry arbitrage?

8. As part of a cash-and-carry arbitrage strategy, you are obligated to make delivery of a T-bill against the nearby T-bill futures contract when it expires in 30 days. Which T-bill should you purchase today?

9. How can the Cost-of-Carry Model be modified to account for dividends?

10. What is the fair value futures price? What market forces drive the futures price toward fair value?

11. What real-world complications can frustrate a stock index arbitrage strategy?

12. How is the spot FX rate related to the futures FX rate? What market forces keep spot and futures FX rates in proper alignment?

13. According to widely held belief, an upward sloping yield curve generally implied that spot interest rates were expected to rise. If this is so, does it also imply that futures prices are expected to rise? Does this suggest a trading strategy? Explain.

14. Assume that the spot corn price is $3.50, that it costs $.017 cents to store a bushel of corn for 1 month, and that the relevant cost of financing is 1 percent per month. If a corn futures contract matures in 6 months and the current futures price for this contract is $3.95 per bushel, explain how you would respond. Explain your transactions for one contract, assuming 5,000 bushels per contract and assuming that all storage costs must be paid at the outset of the transaction.

15. Explain the risks inherent in a reverse cash-and-carry strategy in the T-bond futures market.

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CHAPTER 3

Risk Management with Futures Contracts

This chapter explores how futures contracts can be used as part of a risk management strategy. People often talk about futures trading as if it were synonymous with gambling. In many instances, however, futures can enable the conservative management of preexisting risk. Offsetting such risks with futures contracts reduces overall risk as well as expected return. Using futures in this way is called *hedging*. Hedging with interest rate futures, for example, can help banks efficiently manage the asset/liability mismatches inherent in their funding of long-term assets, such as fixed-rate mortgages, with short-term liabilities that reprice more frequently, such as certificates of deposits. Pension fund managers can use stock index futures to manage the risk in their equity portfolios ahead of predetermined distribution dates. Foreign currency futures can help importers, exporters, and multinational corporations manage the foreign exchange risk inherent in their ordinary business operations.

Futures contracts can also be used to exploit perceived profit opportunities. Traders with a view on the direction of the market can use futures (in addition to other derivatives and cash market instruments) to exploit that view under the subjective belief that they possess better information than what is reflected in market prices. Using futures in this way is called *speculation*.

In addition to hedging and speculation, futures contracts provide a low-cost and effective means for both corporations and institutional investors to respond quickly and cheaply to new information and manage their portfolios of assets and liabilities more efficiently as a result. A fully invested equity fund can reduce its market exposure quickly and cheaply by using futures on stock indexes. Corporate borrowers can use interest rate futures to effectively manage their liability structure. Futures and other financial derivatives can be less expensive to trade than the underlying instruments. Without access to futures markets, it would be much more costly to alter risk exposure in response to new information.

In this chapter, we examine how to use futures for hedging and speculation. First, we discuss hedging, providing examples with interest rate futures, stock index futures, and foreign currency futures. We next introduce the concept of *spread trading* and apply the concept to some simple speculation strategies using financial futures.

HEDGING WITH FUTURES

Hedging typically refers to a transaction on a futures exchange undertaken to reduce a preexisting risk inherent in an underlying business activity. Futures contracts are tools for managing these preexisting risks. Futures are not the only tools available for hedging: Many different financial instruments, including cash market instruments, can form effective hedges.

Insurance and other risk management tools can also be used to manage preexisting risks. But in this discussion, we consider only futures hedges.

Futures hedges can be characterized in several ways depending on the risk being hedged and the construction of the hedge. A firm that knows it will sell an asset in the future can hedge the price of this asset by taking a short futures position. This is known as a *short hedge*. A firm that knows it will buy an asset in the future can hedge by taking a long position. This is known as a *long hedge*.

Traders can also distinguish between a futures position they establish to hedge an existing position in the cash market and a futures position that hedges a cash position they expect to take in the future. The former situation is called an *inventory hedge,* whereas the latter is known as an *anticipatory hedge*. The distinction is important because, although traders conduct hedging in commodity markets primarily to protect an actual position in the cash market, hedging in the financial markets is mostly anticipatory (e.g., hedging the interest rate at which the trader will borrow or lend in the future).

Hedges can also be characterized by the scope of the underlying risks subject to the hedge. The term *micro hedge* describes a futures position that is matched against a specific asset or liability item on the balance sheet (e.g., a bank hedging rates on one-year certificates of deposit from the liability side of its balance sheet). The term *macro hedge* describes a hedge that is structured to offset the net (i.e., combined) risk associated with the hedger's overall asset/liability mix. An example of a macro hedge would be a bank that uses interest rate futures to equate the interest rate exposure of its assets with the interest rate exposure of its liabilities.

Sometimes the risk being hedged is long-term, such as when a swap dealer seeks to hedge the sequence of floating-rate payments over the entire 10-year life of a swap. There are two ways to implement a hedge for this type of long-term risk. First, futures positions can be established in a series of futures contracts of successively longer expirations. This is called a *strip hedge*. Second, the entire futures position can be stacked in the front month and then rolled forward (less the portion of the hedge that is no longer needed) into the next front month contract. This is called a *stack hedge*.

Each strategy involves trade-offs. The strip hedge has a higher correlation with the underlying risks than the stack hedge (i.e., has lower *tracking error*), but may have higher liquidity costs because the more distant contracts may be thinly traded and may have high bid/ask spreads accompanied by high trade-execution risk. The stack hedge has lower liquidity costs but higher tracking error. Swap dealers typically hedge their long-term LIBOR exposure using strips of Eurodollar futures because this contract has active trading (i.e., low liquidity costs) out to 10 years. The stack hedge made news in 1993 when Metallgesellschaft's U.S. subsidiary, MG Corporation, was forced to unwind a substantial stack hedge in energy futures with disastrous results.[1]

Often, the portfolio's underlying risk cannot be precisely hedged using available futures contracts. For example, a bond portfolio manager seeking to hedge a long-term IBM corporate bond will find that IBM corporate bond futures do not exist. The manager may end up using T-bond futures to construct the hedge, even though IBM corporate bonds and U.S. Treasury bonds are not perfectly correlated. This type of hedge is called a *cross-hedge:* The characteristics of the instrument being hedged and the instrument underlying the futures contract do not perfectly match.

HEDGING WITH SHORT-TERM INTEREST RATE FUTURES

Interest rate futures can be used to hedge risks posed by fluctuating interest rates. Hedges constructed with interest rate futures are useful for banks facing an interest-sensitivity mismatch between their assets and liabilities. Suppose that in March a bank customer demands a $1 million fixed-rate loan for nine months. The problem the bank faces is estimating its cost of funds over the life of the loan. If the bank could issue a $1 million fixed-rate certificate of deposit (CD) for nine months, it would have a precise match between the interest sensitivity of its asset (the loan) and the interest sensitivity of its liability (the CD). Suppose, however, the bank can only lock in its funding for six months at 3.00 percent. To fund the loan for the entire nine months, the bank will need to issue a $1 million three-month

CD in September at the interest rate prevailing at that time. The September Eurodollar futures yield quoted in March is 3.5 percent. This yield is the market's assessment of what prevailing three-month CD rates will be in September and is helpful to the bank in determining an expected cost of funds over the life of the loan. However, it still leaves the bank vulnerable to rates rising above the expected rate.

To hedge this risk, the bank can establish a short position in SEP Eurodollar futures in March. In constructing the hedge, the bank will use a single futures contract. If rates unexpectedly rise, the Eurodollar futures price will fall and the bank's short position will become more valuable. Suppose, in fact, that the three-month rate rises to 4.5 percent in September. We know that the Eurodollar futures yield must converge to the prevailing three-month rate at contract expiration. This means that the bank's Eurodollar futures position has gained $2,500. This amount precisely offsets the bank's increase in its cost of funds over and above the 3.5 percent rate it expected to prevail at the time it priced the bank loan and constructed the hedge. Table 3.1 displays the cash flows associated with the construction of this hedge. By hedging, the bank has given up the opportunity to make extra money if its cost of funds should fall unexpectedly. But hedging and locking in the cost of funds has enabled the bank to price the nine-month fixed-rate loan with certainty and lock in an acceptable profit.

TABLE 3.1 Hedging a Bank's Cost of Funds Using Interest Rate Futures

Date	Cash Market	Futures Market
March	Bank makes nine-month fixed rate loan financed by a six-month CD at 3.0% and rolled over for three months at an expected rate of 3.5%.	Establish a short position in SEP Eurodollar futures at 96.5% reflecting a 3.5% futures yield.
September	Three-month LIBOR is now at 4.5%. The bank's cost of funds are one percent above its expected cost of funds of 3.5%. The additional cost equals $2,500 (i.e., 90/360 × .01 × $1 million).	Offset one SEP Eurodollar futures contract at 95.5% reflecting a 4.5% futures yield. This produces a profit of $2,500 = 100 basis points × $25 per basis point × 1 contract.
	Total additional cost of funds: $2,500.	Futures profit: $2,500.
	Net interest expense after hedge: 0	

HEDGING WITH LONG-TERM INTEREST RATE FUTURES

In the previous example, the instrument being hedged was precisely matched by the characteristics of the instrument underlying the futures contract. In this section, we consider an example of a cross-hedge, where the characteristics of the instrument being hedged do not match the characteristics of the instrument underlying the futures contract. Often the need arises to hedge an instrument very different from those underlying the futures contract. The effectiveness of a hedge depends on the gain or loss on both the spot and futures sides of the transaction. But the change in the price of any bond depends on the shifts in the level of interest rates, changes in the shape of the yield curve, the maturity of the bond, and its coupon rate. A long maturity/low coupon bond will be much more sensitive to a given change in interest rates than a short maturity/high coupon bond.

To illustrate the effect of the maturity and coupon rate on hedging performance, consider the following example. A portfolio manager learns on March 1 that he will receive $5 million on June 1 to invest in AAA corporate bonds paying an annual coupon of 5 percent and having 10 years to maturity. The yield curve is flat and is assumed to remain so over the period from March 1 to June 1. The current yield on AAA corporate bonds is 9.5 percent. Fearing a drop in rates, the portfolio manager decides to hedge with T-bond futures to lock in the forward rate of 9.5 percent. The T-bond futures contract is written on a benchmark 20-year bond with a 6 percent coupon rate.

Because $5 million is becoming available for investment, the portfolio manager implements the hedge using $5 million face value of T-bond futures. Table 3.2 presents the transactions and results. By June 1, yields have fallen by 42 basis points on both the corporate bonds and T-bond futures. The total futures gain is $169,203.5, more than offsetting the loss on the corporate bonds and generating a net wealth change of $18,464.5.

If the goal of the hedge is to secure a net wealth change of zero, a gain is appropriately viewed as no better than a loss. If rates had risen, the loss on the futures would have exceeded the gain on the cash instrument. This discrepancy arose even though the yield curve was flat and rates on both instruments changed in the same direction and by the same amount. Because the coupon and maturity of the hedged and hedging instruments do not match, prices on the two instruments changed by different amounts. Therefore, a simple hedge of $1 in the futures market per $1 in the cash market is unlikely to produce satisfactory results.

For satisfactory hedging results, it is necessary to adjust the futures position so it has the same interest rate sensitivity as the cash market position

TABLE 3.2 A Cross Hedge Between Corporate Bonds and T-Bond Futures

Date	Cash Market	Futures Market
March 1	A portfolio manager learns he will receive $5 million to invest in 5%, 10-year AAA bonds in 3 months, with an expected yield of 9.5% and a price of $717.45. The manager expects to buy 6,969 bonds.	The portfolio manager buys $5 million face value of T-bond futures (50 contracts) to mature on June 1 with a futures yield of 8.5% and a futures price, per contract, of $76,153.41.
June 1	AAA yields have fallen to 9.08%, causing the price of the bonds to be $739.08. This represents a loss, per bond, of $21.63. Since the plan was to buy 6,969 bonds, the total loss is (6,969 × $21.63) = −$150,739.	The T-bond futures yield has fallen to 8.08%, so the futures price = spot price = $79,537.48 per contract, or a profit of $3,384.07 per contract. Since 50 contracts were traded, the total profit is $169,203.5.
	Loss = −$150,739	Gain = $169,203.5
	Net wealth change = +$18,464.5	

being hedged. Two typical techniques for making this adjustment are the Basis Point Model and the Price Sensitivity Model. A common intuition underlies both approaches.

BASIS POINT (BP) MODEL

The BP Model focuses on the price effect of a change in yield of 1 basis point on the cash market instrument and on the futures market-hedging instrument. The futures market position necessary to hedge a given cash market risk position depends on the ratio of the dollar price change for the cash market instrument to the futures instrument for a common interest rate change of 1 basis point. Therefore, the hedge ratio is given by equation 3.1:

$$HR = \frac{BPC_C}{BPC_F} \tag{3.1}$$

where BPC_C = dollar price change for a 1 basis point change in the cash instrument

BPC_F = dollar price change for a 1 basis point change in the futures instrument

The ratio BPC_C/BPC_F indicates the relative number of contracts to trade.

PRICE SENSITIVITY (PS) MODEL

The PS Model has been designed explicitly for interest rate hedging.[2] The PS Model assumes that the goal of hedging is to eliminate unexpected wealth changes at the hedging horizon, as defined in equation 3.2:

$$dP_i + dP_F(N) = 0 \tag{3.2}$$

where dP_i = unexpected change in the price of the cash market instrument
dP_F = unexpected change in the price of the futures instrument
N = number of futures contracts to hedge a single unit of the cash market asset

Equation 3.2 expresses the goal that the unexpected change in the value of the spot instrument, denoted by *i,* and the futures position, denoted by *F,* should together equal zero. If this is achieved, the wealth change, or hedging error, is zero. Like the BP Model, the PS Model uses a hedge ratio that reflects the relative price sensitivities of the cash and futures market instruments.

The problem for the hedger is to choose the correct number of contracts, denoted by N in equation 3.2, to achieve a zero hedging error. Equation 3.3 gives the correct number of contracts to trade (N), per spot market bond:

$$N = -\left(\frac{R_F P_i D_i}{R_i FP_F D_F}\right) RV \tag{3.3}$$

where R_F = 1 + the expected futures yield
R_i = 1 + the expected yield to maturity on asset *i*
FP_F = the futures contract price
P_i = the price of asset *i* expected to prevail at the hedging horizon
D_i = the duration of asset *i* expected to prevail at the hedging horizon
D_F = the duration of the asset underlying futures contract *F* expected to prevail at the hedging horizon[3]
RV = the volatility of the cash market asset's yield relative to the volatility of the futures instrument's yield

In nontechnical terms, equation 3.3 says that the number of futures contracts to trade for each cash market instrument to be hedged is the number

TABLE 3.3 Data for the Price Sensitivity Hedge

Cash Instrument		T-Bond Futures	
P_i	$717.45	FP_F	$76,153.41
D_i	7.709	D_F	10.6789
R_i	1.095	R_F	1.085
		N	.0066695
		Number of contracts to trade	46

that should give a perfect hedge, assuming that yields on the cash and futures instrument change by the same amount. To explore the meaning and application of this technique, consider again the AAA bond hedge of Table 3.2. The large hedging errors resulted from the different price sensitivities of the futures instruments and the AAA bonds.

Table 3.3 presents the data needed to calculate the hedge ratios for hedging the AAA bonds with T-bond futures. Here we assume that the cash and futures market assets have the same volatilities, so RV = 1.0. Applying the PS Model to our example:

$$N = -\frac{(1.085)(-\$717.45)(7.709)}{(1.095)(\$76{,}153)(10.679)} = .0066695$$

With 6,969 bonds to hedge, the portfolio manager should buy 46 (actually, 46.4797) T-bond futures.

Table 3.3 presents the performance of our original one-for-one hedge with the PS model for the same 42 basis point drop in rates used in Table 3.2.

With the given hedges and the same drop in yields, the one-for-one hedge gave a futures gain of $169,203.5 to offset the loss on the AAA bonds of $150,742. The futures gain on the PS Model hedge is $155,667.2. The

TABLE 3.4 Performance Analysis of Price Sensitivity Future Hedge

Hedging Error	Cash Market	One-for-One Hedge	PS Model Hedge
Gain/loss	−$150,742	+$169,203.5	+$155,667.20
Hedging error	–	$18,464.5	$4,928.22

bottom line of Table 3.4 shows the size of the hedging error for the one-for-one and PS model hedges. In this example, the PS Model worked very effectively, greatly improving on the results achieved using a one-for-one hedge ratio. In actual hedging situations, traders could not hope for such nearly perfect results since yields need not change by the same amount on all instruments all of the time. The most widely used technique for hedging with interest rate futures is some version of the BP or PS model.

PORTFOLIO IMMUNIZATION

The previously described examples illustrate how interest rate futures can hedge interest rate risks. In these examples, interest rate risk is caused by a single rate. Banks, corporate treasury departments, and bond portfolio managers often are interested in hedging not just against a single rate, but against shocks to the entire yield curve. Hedging whole yield curve risk is called *immunization*—a complex subject that has received abundant attention from finance professionals. Implementing such hedges may involve using interest rate futures in conjunction with other interest rate derivatives. The Suggested Readings for this chapter include publications on this subject for the interested reader.

HEDGING WITH STOCK INDEX FUTURES

In this section, we analyze stock index futures hedging. We begin by finding the futures position to establish a combined stock and futures portfolio with the lowest possible risk. We illustrate this hedging technique with actual market data. It is also possible to use futures to alter the beta of an existing portfolio. For example, if a stock portfolio has a beta of 0.8 and the desired beta is 0.9, it is possible to trade stock index futures to make the combined stock and futures portfolio behave like a stock portfolio with a beta of 0.9.

THE MINIMUM RISK HEDGE RATIO

For any two-asset portfolio, comprising assets A and B, with weights W_A and W_B, respectively, the risk of the portfolio is given by:

$$\sigma_P^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 - 2\,\rho_{A,B} W_A \sigma_A W_B \sigma_B \tag{3.4}$$

where σ_P^2 = variance of the portfolio
σ_A^2 = variance of asset A
σ_B^2 = variance of asset B
$\rho_{A,B}$ = correlation between assets A and B
σ_A = standard deviation of asset A
σ_B = standard deviation of asset B

We can use this basic result from portfolio theory to guide a hedging strategy. Assume that we have a cash market stock portfolio C that is fixed. Thus, we assume that the commitment of funds to this portfolio is 1.0, so that $W_C = 1.0$. We can use futures to hedge the risk associated with this portfolio by adding the right amount of futures to create a two-asset stock/futures portfolio. Denoting the futures position by F, and denoting the weight given to the futures as W_F, the variance of the stock/futures portfolio is:

$$\sigma_P^2 = \sigma_C^2 + W_F^2 \sigma_F^2 - 2\, W_F \rho_{CF} \sigma_C \sigma_F$$

We can minimize the risk of the stock/futures portfolio by choosing W_F to make the variance as small as possible. Denoting the minimum-risk hedge ratio as HR, it can be proven that:[4]

$$HR = \frac{\rho_{CF}\sigma_C\sigma_F}{\sigma_F^2} = \frac{COV_{CF}}{\sigma_F^2} \tag{3.5}$$

where COV_{CF} = the covariance between C and F

As a practical matter, the easiest way to find the risk-minimizing hedge ratio is to estimate the following regression:

$$C_t = \alpha + \beta_{RM} F_t + \varepsilon_t \tag{3.6}$$

where C_t = the returns on the cash market position in period t
F_t = the returns on the futures contract in period t[5]
α = the constant regression parameter
$ß_{RM}$ = the slope regression parameter for the risk-minimizing hedge
ε = an error term with zero mean and standard deviation of 1.0

The estimated beta from this regression is the risk-minimizing hedge ratio, because the estimated $ß_{RM}$ equals the sample covariance between the independent (F) and dependent (C) variables divided by the sample variance of the independent variable. The R^2 from this regression shows the percentage of risk in the cash position that is eliminated by holding the futures position.

Having found the risk-minimizing hedge ratio, $ß_{RM}$ we need to compute the number of contracts to trade. Notice that the regression of equation 3.6 uses the returns on the portfolio and the futures contract, so it does not reflect differences in the size of the portfolio and the futures contract. To find the right number of futures contracts to trade, we need to adjust for the relative sizes of the portfolio and the futures contract. Therefore, the risk-minimizing number of futures contracts is:

$$\left(\frac{V_P}{V_F}\right)\beta_{RM} = \text{number of contracts} \tag{3.7}$$

A MINIMUM-RISK HEDGING EXAMPLE

In this section, we consider an example of a minimum-risk hedge in stock index futures. Let us assume a trader has a portfolio worth $10 million on November 28. The portfolio is invested in the 30 stocks in the Dow Jones Industrial Average. The portfolio manager will hedge this cash market portfolio using the S&P 500 JUN futures contract. We consider each step that the portfolio manager follows to compute the hedge ratio and to implement the hedge.

Organizing Data and Computing Returns

The manager plans to hedge according to equation 3.7. Therefore, she needs to find the beta for the hedge ratio. Accordingly, she collects data for her portfolio value for 101 days from July 6 through yesterday, November 27. She also finds the price of the S&P 500 JUN futures for each day. There is nothing magic about using 101 days, but these data are available and she believes that this procedure will provide a sufficient sample to estimate the hedging beta. From the 101 days of prices, she computes the daily percentage change in the value of the cash market portfolio and the futures price. This gives 100 paired observations of daily returns data.

Estimating the Hedging Beta

With the data in place, the portfolio manager regresses the cash market returns on the returns from the futures contract as shown in equation 3.6. From this regression, the estimated beta is .8801, so $\beta_{RM} = .8801$. This indicates that each dollar of the cash market position should be hedged with $.8801 dollars in the futures position. The R^2 from the regression is .9263, and this high R^2 encourages the belief that the hedge is likely to perform well. Again, for emphasis, the estimated beta from regressing the portfolio's returns on the stock index futures returns is not the same as the portfolio's CAPM beta; β_P does not equal β_{RM}.

Computing the Futures Position

In this example, the portfolio manager administers a stock portfolio worth $10,000,000 in November. The portfolio manager anticipates distributing the fund in May of the following year and wants to hedge against a decline in the value of the stock portfolio over the next six months with stock index futures. If she establishes a short position in stock index futures, the value of the futures position will go up if the value of the stock portfolio goes down. The portfolio manager chooses to hedge the portfolio with the S&P JUN futures contract. Having found the risk-minimizing hedge ratio, she needs to translate the hedge ratio into the correct futures position that takes into account the size of the futures contract. On November 27, the S&P futures closed at 1200.00. The futures contract value is for the index times $250. Therefore, applying equation 3.7, she computes the number of contracts as:

$$\left(\frac{V_P}{V_F}\right)\beta_{RM} = \left(\frac{\$10,000,000}{(1200.00)(\$250)}\right).8801 = 29.3367$$

The estimated risk-minimizing futures position is 29.3367 contracts, so the portfolio manager decides to sell 29 contracts.

Table 3.5 shows the results of the hedge. In May, the S&P 500 has declined 3 percent, confirming the portfolio manager's fears. The manager's portfolio has fallen 2.64 percent (i.e., 3% times the portfolio beta of .8801). The dollar loss in the portfolio comes to $264,030. This amount is nearly offset by a $261,000 gain on the short futures position. By hedging, the net value of the portfolio is largely unaffected by the decline in the stock market.

Of course, if the stock market had gone up, the futures hedge would have produced a loss that would have exactly offset the gain in the stock

TABLE 3.5 Hedging a Stock Portfolio with Stock Index Futures

Date	Cash Market	Futures Market
November 27	Hold $10 million stock portfolio.	Establish a short futures position with 29 S&P JUN futures contracts at 1200.
May 15	Stock portfolio falls by 2.64 percent (.8801 × 3%).	Offset 29 JUN futures contracts at 1164 (a 3% decline).
	Loss: $264,030	Gain: $261,000
	Net loss after hedging: –$3,030	

portfolio. By hedging, the portfolio manager has deliberately chosen to forgo gains in the portfolio in order to avoid losses.

ALTERING THE BETA OF A PORTFOLIO

Portfolio managers often adjust the CAPM betas of their portfolios in anticipation of bull and bear markets. If a manager expects a bull market, he might increase the beta of the portfolio to take advantage of the expected rise in stock prices. Similarly, if a bear market seems imminent, the manager might reduce the beta of a stock portfolio as a defensive maneuver. If the manager trades only in the stock market itself, changing the beta of the portfolio involves selling some stocks and buying others. To reduce the beta of the portfolio, the manager would sell high beta stocks and use the funds to buy low beta stocks. With transaction costs in the stock market being relatively high, this procedure can be expensive.

Portfolio managers have an alternative. They can use stock index futures to create a combined stock/futures portfolio with the desired response to market conditions. In this section, we consider techniques for changing the risk of a portfolio using stock index futures.

In the Capital Asset Pricing Model, all risk is either systematic or unsystematic. *Systematic risk* is associated with general movements in the market and affects all investments. By contrast, *unsystematic risk* is particular to a certain investment or a certain range of investments. Diversification can almost completely eliminate unsystematic risk from a portfolio. The remaining systematic risk is unavoidable. Studies show that a random selection of 20 stocks will create a portfolio with very little unsystematic risk. In this section, we restrict our attention to portfolios that are well diversified and consequently have no unsystematic risk.

Starting with a stock portfolio that has systematic risk only and combining it with a risk-minimizing short position in stock index futures creates a combined stock/futures portfolio with zero systematic risk. According to the Capital Asset Pricing Model, a portfolio with zero systematic risk should earn the risk-free rate of interest. Instead of eliminating all systematic risk by hedging, it is possible to hedge only a portion of the systematic risk to reduce, but not eliminate, the systematic risk inherent in the portfolio. Similarly, a portfolio manager can use stock index futures to increase the systematic risk of a portfolio.

For a risk-minimizing hedge, the manager attempts to match a long position in stock with a short position in stock index futures to create a portfolio whose value will not change with fluctuations in the stock market. To reduce, but not eliminate the systematic risk, a portfolio manager could sell some futures, but less than the risk-minimizing amount. For example, the portfolio manager could sell half the number of contracts stipulated by the risk-minimizing hedge. The combined stock/futures position would then have a level of systematic risk equal to half of the stock portfolio's systematic risk.

Trading stock index futures also can increase the systematic risk of a stock portfolio. Traders who buy stock index futures increase their systematic risk. Thus, if a portfolio manager holds a stock portfolio and buys stock index futures, the resulting stock/futures position has more systematic risk than the stock portfolio alone. Assume a portfolio manager buys, instead of sells, the risk-minimizing number of stock index futures. Instead of eliminating the systematic risk, the resulting stock/long futures position should have twice the systematic risk of the original portfolio.

HEDGING WITH FOREIGN CURRENCY FUTURES

Consider a small import/export firm that is negotiating a large purchase of Japanese watches from a firm in Japan. The Japanese firm, being a tough negotiator, has demanded payment in yen on delivery of the watches. (If the contract had called for payment in dollars, rather than yen, the Japanese firm would bear the exchange risk.) Delivery will take place in seven months, but the price of the watches is agreed today to be Yen 2,850 per watch for 15,000 watches. This means that the purchaser will have to pay Yen 42,750,000 in about seven months. Table 3.6 shows the current exchange rates on April 11. With the current spot rate of .004173 dollars per yen, the purchase price for the 15,000 watches would be $178,396. If the

TABLE 3.6 $/Yen Foreign Exchange Rates, April 11

Spot	.004173
JUN Futures	.004200
SEP Futures	.004237
DEC Futures	.004265

futures prices on April 11 are treated as a forecast of future exchange rates, it seems that the dollar is expected to lose ground against the yen. With the DEC futures trading at .004265, the actual dollar cost might be closer to $182,329. If delivery and payment are to occur in December, the importer might reasonably estimate the actual dollar outlay to be about $182,000 instead of $178,000.

To avoid any worsening of his exchange position, the importer decides to hedge the transaction by trading foreign exchange futures. Delivery is expected in November, so the importer decides to trade the DEC futures. By selecting this expiration, the hedger avoids having to roll over a nearby contract, thereby reducing transaction costs. Also, the DEC contract has the advantage of being the first contract to mature after the hedge horizon, so the DEC futures exchange rate should be close to the spot exchange rate prevailing in November when the yen are needed.

The importer's next difficulty stems from the futures contract being written for Yen 12.5 million. If he trades three contracts, his transaction will be for 37.5 million. If he trades four contracts, however, he would be trading 50 million, when he really only needs coverage for 42.75 million. No matter which way he trades, the importer will be left with some unhedged exchange risk. Finally, he decides to trade three contracts. Table 3.7 shows his transactions. On April 11, he anticipates that he will need Yen 42.75 million, with a current dollar value of $178,396 and an expected future value of $182,329, where the expected future worth of the yen is measured by the DEC futures price. This expected future price is the most relevant price for measuring the success of the hedge. In the futures market, the importer buys three DEC yen contracts at .004265 dollars per yen.

On November 1, the watches arrive, and the importer purchases the yen on the spot market at .004273. Relative to his anticipated cost of yen, he pays $342 more than expected. Having acquired the yen, the importer offsets his futures position. Since the futures has moved only .000005, the futures profit is only $187. This gives a total loss on the entire transaction of $155. Had there been no hedge, the loss would have been the full change of the price in the cash market, or $342. This hedge was only partially effective

TABLE 3.7 The Importer's Hedge

Date	Cash Market		Futures Market
April 11	The importer anticipates a need for Yen 42,750,000 in November, the current value of which is $178,396, and which have an expected value in November of $182,329.		The importer buys 3 DEC yen futures contracts at .004265 for a total commitment of $159,938.
November 1	Receives watches; buys Yen 42,750,000 at the spot market rate of .004273 for a total of $182.671.		Sells 3 DEC yen futures contracts at .004270 for a total value of $160,125.
	Spot market results:		Futures market results:
	Anticipated cost	$182,329	Profit = $187
	–Actual cost	–182,671	
		$ –342	
		Net loss: –$155	

for two reasons. First, the futures price did not move as much as the cash price. The cash price changed by .000008 dollars per yen, but the futures price changed by only .000005 dollars per yen. Second, the importer was not able to fully hedge his position because his needs fell between two contract amounts. Since he needed Yen 42.75 million and only traded futures for Yen 37.5 million, he was left with an unhedged exposure of Yen 5.25 million.

IS HEDGING DESIRABLE?

Does hedging with futures contracts and other derivative instruments make sense for a publicly held company? It is tempting to say that since risk is bad, then reducing it must be good. However, this argument has at least two flaws. First, reducing risk also means reducing expected return. Whether hedging improves the trade-off between risk and expected return depends on the risk preferences of individual traders. Second, when applied to publicly held corporations, hedging may not add to shareholder value. These companies are organized using the corporate form specifically to spread the risk of corporate investments across many shareholders who further spread the risk through their individual ownership of diversified portfolios of stocks from many corporations. In a sense, a publicly held corporation is hedged naturally through its ownership structure. Shareholders are therefore likely to be at best indifferent to hedges constructed at the corporate level since such hedges can be replicated or undone by the portfolio composition of individual shareholders. The shareholders' indifference means that they are unwilling to pay a premium for shares of stock where earnings are hedged at the corporate level. Yet despite this indifference, many publicly held corporations are observed to hedge. We must assume that since capital market discipline creates powerful incentives for corporations to make value-maximizing decisions, that not all observed hedging is being done over the objections of shareholders.[6]

SPREAD STRATEGIES WITH FINANCIAL FUTURES

Forming a spread strategy with futures means constructing a portfolio of futures contracts where the payoff depends entirely on relative price changes (e.g., the DEC futures price relative to the MAR futures price). Broadly speaking, the concept of spreading need not be confined to futures but can be extended to any set of prices. Spread strategies can be confusing if traders lose sight of a basic rule: Buy low, sell high. Armed with this rule, however, understanding spread strategies is straightforward. The only trick is that "low" and "high" prices are expressed in terms of one price relative to another price as opposed to absolute levels. A spread strategy is designed to isolate relative price relationships and by doing so provides an opportunity to speculate on relative movements in price. By engaging in such a strategy, the trader deliberately avoids expressing a view on the absolute level of market prices.

Through the years, terms have evolved to characterize spread strategies. In this section, we focus on a few of these strategies. There are many variations on the theme, but once you understand the theme, the variations should be easy.

The first type of spread is known as an *intracommodity spread* (sometimes called a *time spread*). This type of spread involves futures contracts written on the same underlying instrument but with different futures delivery dates. For example, consider the simultaneous sale of the SEP Eurodollar contract and the purchase of the DEC Eurodollar contract. The motivation behind such a spread would be to place a bet (practitioners prefer to use the polite term "taking a view" instead of "bet") on the future relationship between September and December Eurodollar yields. In the case of Eurodollar futures, the bet is really about the future shape of the yield curve relative to the current shape. Selling the SEP contract and buying the DEC contract means that the trader is expecting the September price to fall *relative* to the December price. Another way of saying the same thing is that the trader is expecting the September yield to rise *relative* to the December yield—the trader is expecting the yield curve to flatten. After yields move, the SEP and DEC positions are unwound. Table 3.8 displays the results of such a strategy and shows how the profits are determined. In June, when the spread position is established, the spread between the DEC futures yield and the SEP futures yield is 50 basis points. By July, the spread has narrowed to 30 basis points and the position is unwound, yielding a profit of $500. The key thing to understand about this strategy is that it is a bet on relative prices and yields. The payoff to this strategy is independent of the absolute direction of rates and prices.

Table 3.9 shows another example of a time spread. In this case, the trader devises a strategy in June to reflect the view that the euro will fall relative to the dollar. Since the euro FX futures contract traded at the CME is expressed in terms of dollars per euro, such a bet can be made using a time spread (i.e., buying a SEP euro contract and simultaneously selling the DEC contract). After rates move, the futures positions are unwound. The table shows that the DEC contract falls in price more than the SEP contract (i.e., 1 euro buys fewer dollars with the DEC contract than with the SEP contract). Each euro contract is written for 125,000 euros, yielding a profit of $700.

TABLE 3.8 Time Spread Using Eurodollar Futures

Date	Futures Market	
June	Sell the SEP Eurodollar futures contract at 96.00 reflecting a 4.00% futures yield.	
	Buy the DEC Eurodollar futures contract at 95.50 reflecting a 4.5% futures yield.	
July	Buy the SEP Eurodollar futures contract at 96.50 reflecting a 3.5% futures yield.	
	Sell the DEC Eurodollar futures contract at 96.20 reflecting a 3.8% futures yield.	
	SEP Contract	**DEC Contract**
	96.00	96.20
	−96.50	−95.50
	−.50	.70
	Total gain: 20 basis points × $25 = $500	

TABLE 3.9 Time Spread Using Euro FX Futures

Date	Futures Market
June	Buy 1 September Euro FX futures at .8924. Sell 1 December Euro FX futures at .8900.
July	Sell 1 September Euro FX futures at .8880. Buy 1 December Euro FX futures at .8800.

September Contract	December Contract
.8880	.8900
−.8924	−.8800
−.0044	.0100

Total gain: .0056 ($/euro) × 125,000 euros/contract = $700

Another variation on the theme of the intracommodity spread is the so-called *butterfly spread*. This frequently used spread is best understood by example. Assume that today is March 1 and the prices for S&P 500 futures are as shown in Table 3.10. In comparing the price for a September contract, 1150 index points, with the prices on the adjacent contract months of JUN and DEC, a trader determines, based on subjective analysis, that the SEP price is out of line. To this trader, it appears that the SEP price should be about halfway between the July and December prices, but it is currently below that level. The trader, who does not really know whether stock prices are going to rise or fall in general, only wants to take advantage of the apparent relative discrepancy between contracts.

Because this trader expects the price of the SEP contract to rise relative to the JUN and DEC contracts (i.e., the SEP contract is currently underpriced relative to the other contracts, or equivalently, the JUN and DEC contracts are overpriced relative to the SEP contract), he buys the underpriced contract and sells the overpriced contracts. The trick to constructing a butterfly spread is that the trader needs to take positions in the proper proportion. In this

TABLE 3.10 Prices for S&P 500 Futures on March 1

Contract Month	Price (Index Points)
JUN	1100
SEP	1150
DEC	1300

case, the trader would sell one JUN contract, sell one DEC contract, and buy two SEP contracts. Suppose that by April 15, the prices of all the contracts have fallen, but their price relationships are much closer to what the trader believed to be correct. On April 15, the SEP price has risen, relative to the other contracts, to a point about halfway between them. To unwind the position, the trader would merely take offsetting futures positions. The net result is a gain of 100 index points, which at $250 per index point yields a profit of $25,000. The results are shown in Table 3.11.

A second type of spread strategy is known as an *intercommodity spread.* This strategy involves the simultaneous purchase and sale of futures contracts written on different underlying commodities. For example, the T-bill/Eurodollar spread is a way of betting on the future risk structure of interest rates. The idea is that a T-bill represents a risk-free asset (i.e., its yield depends strictly on the time value of money without any premium added for risk). Therefore, any crisis that upsets the market, such as the September 11 terrorist attacks, triggers a "flight to quality." This means that investors are willing to pay a premium for safe assets like T-bills and demand a steeper discount for investing in risky assets like Eurodollars. By bidding up T-bill prices (relative to Eurodollars) and bidding down Eurodollar prices (relative to T-bills), T-bill yields fall relative to Eurodollar yields. A T-bill/Eurodollar spread (called a TED spread by practitioners) is designed to take advantage of this new view on the future risk structure between T-bill rates and Eurodollar

TABLE 3.11 Butterfly Spread Using S&P 500 Futures

Date	Futures Market
March 1	Sell 1 JUN S&P 500 futures contract at 1100.
	Buy 2 SEP S&P 500 futures at 1150.
	Sell 1 DEC S&P 500 futures contract at 1300.
April 15	Buy 1 JUN S&P 500 futures contract at 1000.
	Sell 2 SEP S&P 500 futures at 1100.
	Buy 1 DEC S&P 500 futures contract at 1200.

JUN Contract	SEP Contract	DEC Contract
1100	1100	1300
–1000	–1150	–1200
100	–50	100

Total gain: (100 – 2 × 50 + 100) × $250 = $25,000 rates. Applying the buy-low/sell-high principle, a trader would sell Eurodollar futures and simultaneously purchase T-bill futures. After rates move, the T-bill and Eurodollar futures positions are unwound.

The payoff to such a strategy is described in Table 3.12. In March, when the position is established, the Eurodollar futures yield reflected in the JUN contract is 3.71 percent and the T-bill futures yield is 2.82 percent. Eurodollars pay a .89 percent risk premium over T-bills. In April, the risk premium has widened to 1.16 percent and the spread position is unwound. The spread position has produced a net gain of 27 basis points, which at $25 per basis point yields a total profit of $675. Again, the payoff is independent of the absolute level of rates. It depends only on relative rates.

The last spread strategy we examine is what Chicago traders call an "O'Hare" spread. With an O'Hare spread, the trader takes a large, unhedged position in a futures contract, preferably with someone else's money, and simultaneously purchases a one-way plane ticket to an exotic foreign destination (preferably a place without an extradition treaty with the United States). If the trader wins his futures bet, then he keeps the money and lives happily ever after. If he loses the bet, then he quickly grabs his bags, dashes off a note to his wife and kids, and hops on a commuter train for O'Hare International Airport.

TABLE 3.12 Intercommodity Spread in Short-Term Rates

Date	Futures Market	
March	Sell 1 JUN Eurodollar futures contract at 96.29 reflecting a futures yield of 3.71%.	
	Buy 1 JUN T-bill futsures contract at 97.18 reflecting a futures yield of 2.82%.	
April	Buy 1 JUN Eurodollar futures contract at 95.91 reflecting a futures yield of 4.09%.	
	Sell 1 JUN T-bill futures contract at 97.07 reflecting a futures yield of 2.93%.	
	Eurodollar	**T-bill**
	96.29	97.07
	−95.91	−97.18
	.38	−.11
	Total profit: 27 basis points × $25 = $675	

OTHER USES OF FINANCIAL FUTURES

In addition to hedging and speculation strategies, there are other ways to use financial futures in portfolio management such as to change the effective maturity of a debt instrument. Many investors find themselves with an existing portfolio that has undesirable maturity characteristics. A firm might hold a six-month T-bill and realize that it will have a need for funds in three months. By the same token, another investor might hold the same six-month T-bill and fear that those funds might have to face lower reinvestment rates on maturity in six months. This investor might prefer a one-year maturity. Both the firm and the investor could sell the six-month bill and invest for the preferred maturity. However, spot market transactions costs are relatively high, and many investors prefer to alter the maturities of investment by trading futures. The following examples show how to use futures to both shorten and lengthen maturities.

SHORTENING THE MATURITY OF A T-BILL INVESTMENT

Consider a firm that has invested in a T-bill. Now, on March 20, the T-bill has a maturity of 180 days, but the firm learns of a need for cash in 90 days. Therefore, it would like to shorten the maturity so it can have access to its funds in 90 days, around mid-June.

For simplicity, we assume that the short-term yield curve is flat with all rates at 10 percent on March 20. For convenience, we assume a 360-day year to match the pricing conventions for T-bills. The face value of the firm's T-bill is $10 million. With 180 days to maturity and a 10 percent discount yield, the price of the bill is:

$$P = FV - \frac{\left[DY(FV)(DTM)\right]}{360}$$

where P = bill price
FV = face value
DY = discount yield
DTM = days until maturity

Therefore, the 180-day bill is worth $9,500,000. If the yield curve is flat at 10 percent, the futures yield must also be 10 percent and the T-bill futures price must be $975,000 per contract. Starting from an initial position of a

TABLE 3.13 Transactions to Shorten Maturities

Date	Cash Market	Futures Market
March 20	Holds six-month T-bill with a face value of $10,000,000, worth $9,500,000. Wishes a three-month maturity.	Sell 10 JUN T-bill futures contracts at 90.00, reflecting the 10% discount yield.
June 20		Deliver cash market T-bills against futures; receive $9,750,000.

six-month T-bill, the firm can shorten the maturity by selling T-bill futures for expiration in three months, as Table 3.13 shows. On March 20, there was no cash flow because the firm merely sold futures. On June 20, the six-month bill is now a three-month bill and can be delivered against the futures. In Table 3.13, the firm delivers the bills and receives the futures invoice amount of $9,750,000. (Although we have assumed the futures price did not change, this does not limit the applicability of our results. No matter how the futures price changed from March to June, the firm would still receive a total of $9,750,000. We assume that this occurs in June instead of over the period.) The firm has effectively shortened the maturity from six months to three months.

LENGTHENING THE MATURITY OF AN INVESTMENT

Consider now, on August 21, an investor who holds a $100 million face-value T-bill that matures in 30 days on September 20. She plans to reinvest for another three months after the T-bill matures. However, she fears that interest rates may fall unexpectedly. If so, she would be forced to reinvest at a lower rate than is now reflected in the yield curve. The SEP T-bill futures yield is 9.8 percent, as is the rate on the current investment. She finds this rate attractive and would like to lengthen the maturity of the T-bill investment. She knows that she can lengthen the maturity by buying a September futures contract and taking delivery. She will then hold the delivered bills until maturity in December.

With a 9.8 percent discount futures yield, the value of the delivery unit is $975,500. With $100 million becoming available on September 20, the investor knows she will have enough funds to take delivery of

TABLE 3.14 Transactions to Lengthen Maturities

Date	Cash Market	Futures Market
August 21	Holds 30-day T-bill with a face value of $100,000,000. Wishes to extend the maturity for 90 days.	Buy 102 SEP T-bill futures contracts, with a yield of 9.8%.
September 20	30-day T-bill matures and investor receives $100,000,000. Invest $499,000 in money market fund.	Accept delivery on 102 SEP futures, paying $99,501,000.
December 19	T-bills received on SEP futures mature for $102,000,000.	

($100,000,000/$975,500) = 102.51 futures contracts. Therefore, she initiates the strategy presented in Table 3.14.

On August 21, she held a bill worth $99,183,333, assuming a yield of 9.8 percent. With the transactions of Table 3.14, she had no cash flow on August 21. With the maturity of the T-bill in September, the investor received $100,000,000 and used almost all of it to pay for the futures delivery. She also received $499,000, which we assume she invested at 9.8 percent for three months. In December, this investment would be worth $499,000 + $499,000(.098)(90/360) = $511,226. With the T-bills maturing in December, the total proceeds will be $102,511,226, from an investment that was worth $99,183,333 on August 21. This gives her a discount yield of 9.8 percent over the four-month horizon from August to December.

This transaction locked in the 9.8 percent on the futures contract, which, in this example, happens to match the spot rate of interest. However, the important point is that lengthening the maturity locks in the futures yield, no matter what that yield may be. Thus, for the period covered by the T-bill delivered on the futures contract, the investment will earn the futures yield at the time of contracting.

SUMMARY

In this chapter, we examined how futures can be used for implementing risk management strategies. We looked at several examples of hedging risk with interest rate futures, stock index futures, and foreign currency futures. In addition to managing risks, traders also use futures contracts to exploit perceived profit opportunities. Using futures in this way is called speculation. We demonstrated some simple speculation strategies using spread positions constructed with financial futures. We also described ways to transform the maturity of an investment with financial futures.

QUESTIONS AND PROBLEMS

1. Assume that you believe the futures prices for corn are too low relative to wheat prices. Explain how you could take advantage of this belief.

2. Assume that you are a bond portfolio manager and that you anticipate an infusion of investable funds in three months. How could you use the futures market to hedge against unexpected changes in interest rates?

3. You believe that the yield difference between T-bills and Eurodollar CDs will widen. How would you use the futures market to take advantage of this belief?

4. Which is likely to have the greater variance—the basis between cash and futures, or the cash price of the good? Why?

5. Describe the difference between a stack hedge and strip hedge. What are the advantages and disadvantages of each?

6. Why might it be inappropriate for a corporation to hedge?

7. What risks are associated with using a cross hedge?

8. Assume you hold a T-bill that matures in 90 days, when the T-bill futures expires. Explain how you could transact to effectively lengthen the maturity of the bill.

9. Assume that you will borrow on a short-term loan in six months, but you do not know whether you will be offered a fixed rate or a floating-rate loan. Explain how you can use futures to convert a fixed rate to a floating-rate loan and to convert a floating rate to a fixed-rate loan.

10. Assume you hold a well-diversified portfolio with a beta of 0.85. How would you trade futures to raise the beta of the portfolio?

11. An index fund is a mutual fund that attempts to replicate the returns on a stock index, such as the S&P 500. Assume you are the manager of such a fund and are fully invested in stocks. Measured against the S&P

500 index, your portfolio has a beta of 1.0. How could you transform this portfolio into one with a zero beta without trading stocks?

12. You expect a steepening yield curve over the next few months, but you are not sure whether the level of rates will increase or decrease. Explain two ways you can trade to profit if you are correct.

13. The Iraqi invasion of Alaska has financial markets in turmoil. You expect the crisis to worsen more than other traders suspect. How could you trade short-term interest rate futures to profit if you are correct? Explain.

14. You believe that the yield curve is strongly upward sloping and that yields are at very high levels. How would you use interest rate futures to hedge a prospective investment of funds that you will receive in nine months? If you faced a major borrowing in nine months, how would you use futures?

15. For the most part, the price of oil is denominated in dollars. Assume that you are a French firm that expects to import 420,000 barrels of crude oil in six months. What risks do you face in this transaction? Explain how you could transact to hedge the currency portion of those risks.

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CHAPTER 4

Options

Because options markets are diverse and have their own particular jargon, understanding them requires a grasp of institutional details and market terminology. This chapter begins with a discussion of the institutional background of options markets, including the kinds of contracts traded and the price quotations for various options.

The successful option trader must also understand the pricing relationships that prevail in the options market. For example, how much should an option to buy IBM at $100 be worth if IBM is selling at $120? With IBM trading at $120, how much more would an option be worth if it required a payment of only $90 instead of $100? Similarly, how much would an option to sell IBM for $115 be worth if IBM were trading at $120? Prospective option investors need answers to these kinds of questions. Fortunately, the pricing principles for options are well developed. Although the answers to such questions may sometimes be surprising, they are logical on reflection.

As options markets have proliferated, the ways to contract for the same good have grown. For example, there are futures contracts on foreign currencies, such as the euro. There are also options on euros, and even an option on the euro futures contract. The chapter concludes by exploring options on stock indexes, options on futures, and options on foreign currencies.

CALL AND PUT OPTIONS

As discussed in Chapter 1, there are two major classes of options, call options and put options. Ownership of a *call option* gives the owner the right to buy a particular good at a certain price, with that right lasting until a particular date. Ownership of a *put option* gives the owner the right to sell a particular good at a specified price, with that right lasting until a particular date. For every option, there is both a buyer and a seller. For a call option, the seller receives a payment from the buyer and gives the buyer the option of buying a particular good from the seller at a certain price, with that right lasting until a particular date. Similarly, the seller of a put option receives a payment from the buyer. The buyer then has the right to sell a particular good to the seller at a certain price for a specified period of time.

In all cases, ownership of an option involves the right, but not the obligation, to make a certain transaction. The owner of a call option may buy the good at the contracted price during the life of the option, but there is no obligation to do so. Likewise, the owner of a put option may sell the good under the terms of the option contract, but there is no obligation to do so. Selling an option does commit the seller to specific obligations. The seller of a call option receives a payment from the buyer, and in exchange, the seller must be ready to sell the given good to the owner of the call, if the owner wishes. The discretion to engage in further transactions always lies with the owner, or buyer, of an option. Option sellers have no such discretion. They have obligated themselves to perform in certain ways if the owners of the options so desire. Later in this chapter, we discuss the conditions under which buyers and sellers find it reasonable to act in different ways.

OPTION TERMINOLOGY

A great deal of special terminology is associated with the options market. The seller of an option is also known as the *writer* of an option, and the act of selling an option is called *writing an option*. If the owner of the call takes advantage of the option, he or she is said to *exercise* the option. An owner would exercise a call option by buying a good under the terms of an option contract. Each option contract stipulates a price that will be paid if the option is exercised, and this price is known as the *exercise price, strike price,* or the *striking price*. In our first example of the call option to buy IBM at $100 when it is selling at $120, the exercise price would be $100, because this is the amount that must be paid at exercise.

Every option involves a payment from the buyer to the seller. This payment is simply the price of the option, but it is also called the *option premium*. Also, every option traded on an exchange is valid for only a limited period. An option on IBM might be valid only through August of the present year. The option has no validity after its *expiration date* or *maturity*. This special terminology is used widely in the options market and throughout the rest of this chapter.

TYPES OF OPTION

Many different kinds of options are traded actively on a variety of option exchanges. Perhaps the best known options are those on individual stocks. However, options also trade on stock indexes, exchange-traded funds, interest rate instruments, precious metals indexes, foreign currencies, and futures contracts. Table 4.1 lists the major option exchanges in the United States and Europe as well as the instruments that they trade.

TABLE 4.1 Principal Option Exchanges

Exchange	Key Instruments on which Options Trade
Panel 1: US Option Exchanges	
Chicago Board Options Exchange (CBOE)	Individual stocks, stock indexes, Treasury securities
Philadelphia Stock Exchange (PHLX)	Individual stocks, stock indexes, currencies, and precious metal index
American Stock Exchange (AMEX)	Individual stocks, stock indexes, exchange traded funds
Pacific Stock Exchange (PSE)	Individual stocks
International Securities Exchange (ISE)	Individual stocks
Chicago Mercantile Exchange	commodity futures, stock index futures, interest rate futures, and FX futures
New York Board of Trade (NYBT)	commodity futures, stock index futures, interest rate futures, and FX futures
Kansas City Board of Trade (KCBT)	commodity futures
MidAmerica Commodity Exchange (MIDAM)	commodity futures
Minneapolis Grain Exchange (MGE)	commodity futures
New York Mercantile Exchange (NYMEX)	commodity futures
Panel 2: Key Option Exchanges Outside of the United States	
Eurex-Germany and Switzerland	Individual stocks, stock index futures, interest rate futures, and commodity futures
Marche a Terme International de France (MATIF)	Interest rate futures and commodity futures
London International Financial Futures and Options Exchange (LIFFE) United Kingdom	Individual stocks, stock indexes, interest rates, and commodity futures

OPTION QUOTATIONS

No matter what the exchange or the good underlying the option, the quotations are similar. Because the market for individual stocks is the oldest and has the most overall trading activity, we use the quotations for IBM to illustrate the basic features of the prices. Figure 4.1 shows the quotations for call and put options on individual stocks from *The Wall Street Journal,* including options on IBM in particular. Options on IBM trade on the Chicago Board Options Exchange (CBOE) and the quotations pertain to the close of trading on the previous trading day.

Beneath the identifier "IBM," the quotations list the closing price of IBM stock for the day, while the second column lists the various striking

[Image not available in this electronic edition.]

FIGURE 4.1 Quotations for options on individual stocks. *Source: Wall Street Journal,* February 15, 2002.

prices or exercise prices that are available for IBM. The striking prices are kept fairly near the prevailing price of the stock. As the stock price fluctuates, new striking prices are opened for trading, at intervals of $5. As a consequence, volatile stocks are likely to have a greater range of striking prices available for trading at any one time. Each contract is written on 100 shares, but the prices quoted are on a per share basis. On payment, the owner of the call would have the right to purchase 100 shares of IBM for the exercise price of, we assume, $100 per share, and this right would last until the expiration date. For the purchaser of the option, the total price to acquire a share of IBM would be the option premium plus the exercise price. The option writer would receive the premium as soon as the contract is initiated, and this amount belongs to the option writer no matter what develops. The option writer, however, is obligated to sell 100 shares of IBM to the call purchaser for $100 per share if the option purchaser chooses to exercise the option. The purchaser must exercise the option before it expires.[1]

Obviously, the right to buy IBM at $100 per share, when the market price of IBM is above $100, is valuable. By contrast a put option also is traded on IBM, which allows the owner to sell a share of IBM for, we assume, $100. Investors are not willing to pay very much for the right to sell IBM at $100 via an options contract if it could be sold for more than $100 in the marketplace.

Important features about options can be illustrated from price quotations, such as the ones shown in Figure 4.1. First, for any given expiration, the lower the striking price for a call, the greater will be the price. Similarly, the longer the time to expiration, the higher will be the price of an option. The same relationship holds true for put options. In the following section, we explain why these kinds of pricing relationships must be obtained in the marketplace.

OPTION PRICING

Option pricing affords one of the showcase results of research in modern finance. The pricing models that have been developed for options perform very well, and a study of these models is useful for the trader. In fact, traders on the options exchanges have immediate access to the information provided by option pricing models through machines located on the floors of the exchanges.

Prices of options on stocks without cash dividends depend on five factors:

Stock Price	S
Exercise Price	E
Time until Expiration	T
Volatility of the Underlying Stock	σ
Risk-Free Interest Rate	R_f

Initially, we consider the effects of just the first three factors, the stock price (S), the exercise price (E), and the time until expiration (T). Later, we look at the more complicated situations that arise from taking into account different interest rate environments and different risk levels.

For a call option, we can express the call price as a function of the stock price, the exercise price, and the time until expiration using this compact notation:

$$C(S, E, T)$$

For example, the equation:

$$C(\$120, \$100, .25) = \$22.75$$

says that a call option on a share trading at \$120, with an exercise price of \$100 and one quarter of a year to expiration, has a price of \$22.75.

THE PRICING OF CALL OPTIONS AT EXPIRATION

The term *at expiration* refers to the moment just prior to expiration. If the option is not exercised at this time, it will expire immediately and have no value. The value of options at expiration is an important topic because many complications that ordinarily affect option prices disappear when the option is about to expire. With this terminology in mind, consider the value of a call option at expiration, where $T = 0$. In this case, only two possibilities may arise regarding the relationship between the exercise price (E) and the stock price (S). Either $S > E$ or $S \leq E$. If the stock price is less than or equal to the exercise price ($S \leq E$), the call option will have no value. To see why this is the case, consider a call option with an exercise price of \$80 on a stock trading at \$70. Since the option is about to expire, the owner of the option has only two alternatives—to exercise the option, or to allow it to expire.[2] If the option is exercised in this situation, the holder of the option must pay the exercise price of \$80 and receive a stock trading in the market for only \$70. In this situation, it does not pay to exercise the option and the owner will allow it to expire worthless. Accordingly, this option has no value and its market price will be zero. Employing our notation, we can say:

$$\text{If } S \leq E,\ C(S, E, 0) = 0 \tag{4.1}$$

If an option is at expiration and the stock price is less than or equal to the exercise price, the call option has no value. This equation simply summarizes the conclusion we have already reached.

The second possible relationship that could exist between the stock price and the exercise price at expiration is for the stock price to exceed the exercise price ($S > E$). Again, in our notation:

$$\text{If } S > E,\ C(S, E, 0) = S - E \tag{4.2}$$

If the stock price is greater than the exercise price, the call option must have a price equal to the difference between the stock price and the exercise price.

If this relationship did not hold, there would be arbitrage opportunities. Assume for the moment that the stock price is \$50 and the exercise price is \$40. If the option were selling for \$5, an arbitrageur would make the following trades:

Transaction	**Cash Flow**
Buy a call option	\$–5
Exercise the option	–40
Sell the stock	50
Net Cash Flow	\$ 5

As these transactions indicate, there will be an arbitrage opportunity if the call price is less than the difference between the stock price and the exercise price.

What if the price of the call option is greater than the difference between the stock price and the exercise price? Using our example of a stock priced at \$50 and the exercise price of the option being \$40, assume now that the call price is \$15. Faced with these prices, an arbitrageur would make the following transactions:

Transaction	Cash Flow
Sell a call option	$+15
Buy the stock	−50
Initial Cash Flow	$−35

The owner of this call option must then immediately exercise the option or allow it to expire. If the option is exercised, the seller of the call has these additional transactions:

Transaction	Cash Flow
Deliver stock	0
Collect exercise price	$+40
Total Cash Flow	$ 5

In this case, there is still a profit of $5. Alternatively, the owner of the option may allow the option to expire. In that event, the arbitrageur would simply sell the stock as soon as the option expires and would receive $50. The profit would be $15, since the arbitrageur simply keeps the option premium. In this second situation, in which the call price is greater than the stock price minus the exercise price, the holder of the call option would exercise the option. The important point is that the arbitrageur would make a profit no matter what the holder of the call might do.

If the stock price exceeds the exercise price at expiration, the price of the call must equal the difference between the stock price and the exercise price. Combining these two relationships allows us to state the first basic principle of option pricing:

$$C(S, E, 0) = \max(0, S - E) \tag{4.3}$$

At expiration, a call option must have a value that is equal to zero or to the difference between the stock price and the exercise price, whichever is greater. This condition must hold, otherwise there will be arbitrage opportunities awaiting exploitation.[3]

OPTION VALUES AND PROFITS AT EXPIRATION

In this discussion, it is important to keep separate the option's value or price and the profit or loss that a trader might experience. A concrete example easily shows the value of options at expiration. Consider both a call and a put option, each having a striking price of $100. Figure 4.2 shows the value of these options at expiration for various stock prices. The graph shows the value of call and put options at expiration on the vertical axis as a function of the stock price, which is shown on the horizontal axis. The call price is shown as a solid line, and the put price is shown as a dotted line.

If the stock price is less than or equal to the exercise price of $100, the value of the call must be zero, as shown in Figure 4.2. For stock prices above the exercise price, the call price equals the difference between the stock price and the exercise price. This is reflected by the graph of the call option's value, which rises at a 45-degree angle for stock prices above $100. The graph presents a similar analysis for a put option. Although we have not discussed the pricing of put options in detail, the reader can conclude that this is the correct graph by the same argument given earlier for call options.

Now consider the same situation, with put and call options each having an exercise price of $100, but assume that trades had taken place for the options with a premium of $5 on both the put and the call options. Knowing the price that was paid allows us to calculate the profits and losses at expiration for the sellers and buyers of both the put and call options. Alternative outcomes for all of these trading parties are shown in Figure 4.3. The top panel shows the profit and loss positions for the call option. The solid line pertains to the buyer of the call, and the dotted line to the seller.[4] For any stock price less than or equal to the exercise price of $100, the option will expire worthless and the purchaser of the call will lose the full purchase

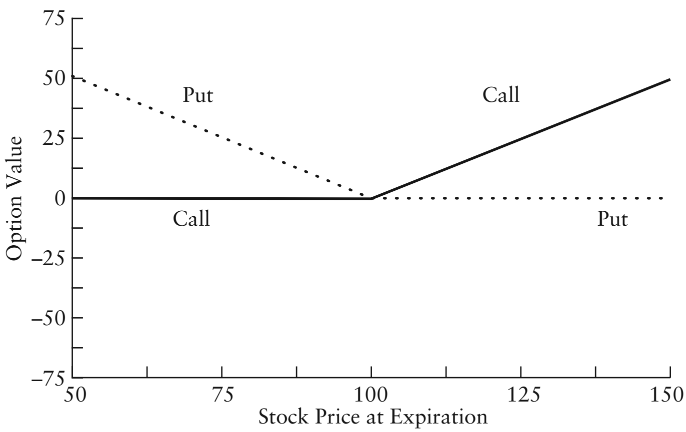

FIGURE 4.2 Values of call and put options at expirations when the striking price equals $100.

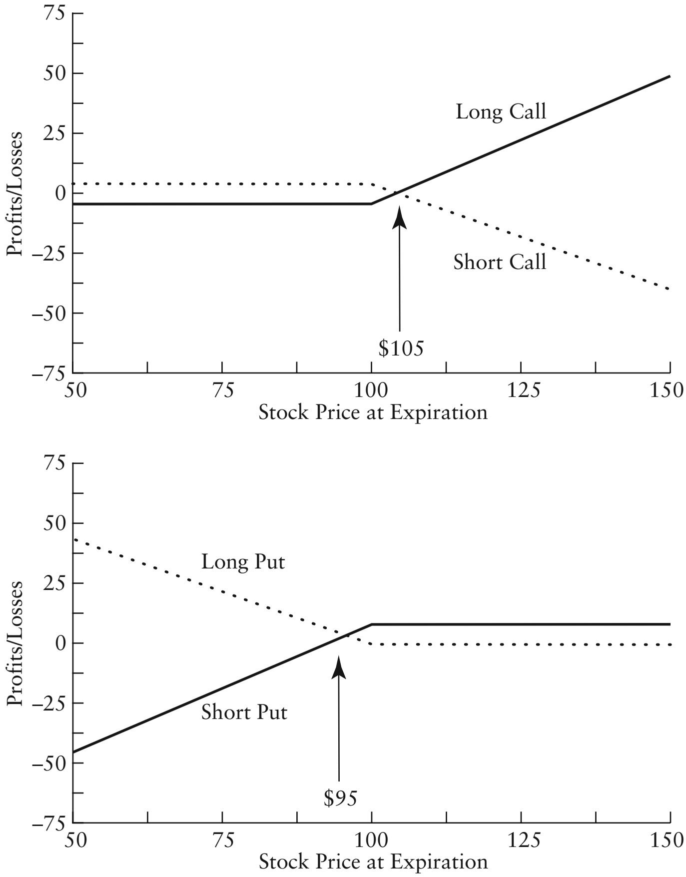

FIGURE 4.3 Profits for call and put options at expiration when the striking price equals $100 and the premium is $5.

price. If the stock price exceeds $100, say reaching $105, the owner of the call will exercise the call, paying $100 for the share and receiving a share worth $105. With a share price of $105, the call owner breaks even exactly. The entire cash flow has been the $5 for the option plus the $100 exercise price. The receipt of the share that is worth $105 exactly matches this total outflow of $105. For the call owner, any stock price less than $100 results in the loss of the total amount paid for the option. For stock prices greater than the exercise price, the call owner will exercise the option. The call owner may still lose money even with exercise. In this example, the stock price must be greater than $105 to generate any net profit for the call owner.

For the writer of the call, the profit picture is exactly opposite that of the call owner's. The best situation for the writer of the call is for the stock price to stay at or below \$100. In this situation, the call writer keeps the entire option premium and the call option will not be exercised. If the stock price is \$105, the option may be exercised and the writer of the call must deliver shares now worth \$105 and receive only \$100 for them. At this point, the loss on the exercise exactly equals the premium that was already received, so the call writer breaks even. If the stock price is greater than \$105, the call writer will have a net loss. Notice that the buyer's profits exactly mirror the seller's losses and vice versa. The options market is a *zero sum game:* The buyer's gains are the seller's losses, and the seller's gains are the buyer's losses. If we add up all of the gains and losses in the options market, ignoring transaction costs, the total will equal zero.

The second panel of Figure 4.3 shows the profit-and-loss positions for the put traders. If the put buyer pays \$5 for a put with an exercise price of \$100, he or she will break even at \$95. The writer of a put also breaks even at \$95. These graphs indicate the many possible profit-and-loss patterns that traders may create by using the options market. This kind of graph is useful for analyzing market strategies.

THE PRICING OF A CALL OPTION WITH A ZERO EXERCISE PRICE AND INFINITE TIME UNTIL EXPIRATION

It may appear unimportant to consider an option with a zero exercise price and an infinite time until expiration because such options are not traded in the options market. However, this kind of option represents an extreme situation and, as such, can set boundaries on possible option prices. An option on a stock that has a zero exercise price and an infinite time to maturity can be surrendered at any time, without any cost, for the stock itself. Since such an option can be transformed into the stock without cost, it must have a value as great as the stock itself. Similarly, an option on a good can never be worth more than the good itself. This allows us to state a second principle of option pricing:

$$C(S, 0, \infty) = S \tag{4.4}$$

A call option with a zero exercise price and an infinite time to maturity must sell for the same price as the stock. Together, these first two principles allow us to specify the upper and lower possible bounds for the price of a

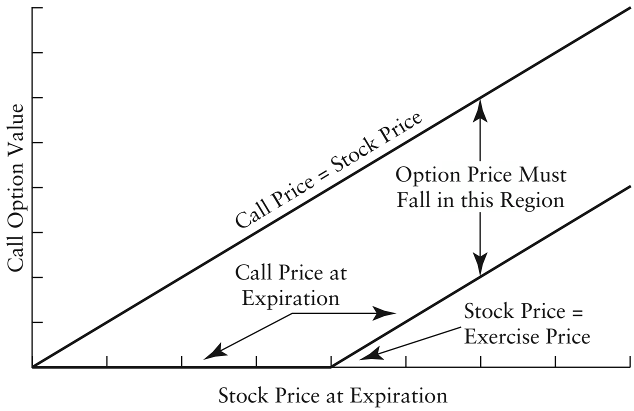

FIGURE 4.4 Boundaries for call option prices.

call option as a function of the stock price, the exercise price, and the time to expiration. These boundaries are shown in Figure 4.4. If the call has a zero exercise price and an infinite maturity, the call price must equal the stock price, and this situation is shown as the 45-degree line from the origin. This represents the upper bound for an option's price. Alternatively, if the option is at expiration, the price of the option must lie along the horizontal axis from the origin to the point at which the stock price equals the exercise price ($S = E$), and then upward at a 45-degree angle. If the stock price is less than or equal to the exercise price, the call price must be zero, as shown in the graph. If the stock price exceeds the exercise price, the option must trade for a price that is equal to the difference between the stock price and the exercise price. Other options such as those with some time remaining until expiration and with positive exercise prices would have to lie in the shaded region between these two extremes. To further our understanding of option pricing, we need to consider other factors that put tighter restrictions on the permissible values of option prices.

RELATIONSHIPS BETWEEN CALL OPTION PRICES

Numerous striking prices and expiration dates are available for options on the same stock. Not surprisingly, definite relationships must be maintained between these different kinds of options, if there are not to be arbitrage opportunities:

$$\text{If } E_1 < E_2,\ C(S, E_1, T) \geq C(S, E_2, T) \tag{4.5}$$

If two call options are alike, except that the exercise price of the first is less than that of the second, then the option with the lower exercise price must have a price that is equal to or greater than the price of the option with the higher exercise price.

In this situation, both options allow the owner of the option to acquire the same share of stock for the same period. However, the option with the lower exercise price allows the owner of that option to acquire the stock for a lower price. Therefore, the option with the lower exercise price should have a greater value. To see why this rule must hold, imagine a situation in which two options are just alike, except that the first has an exercise price of

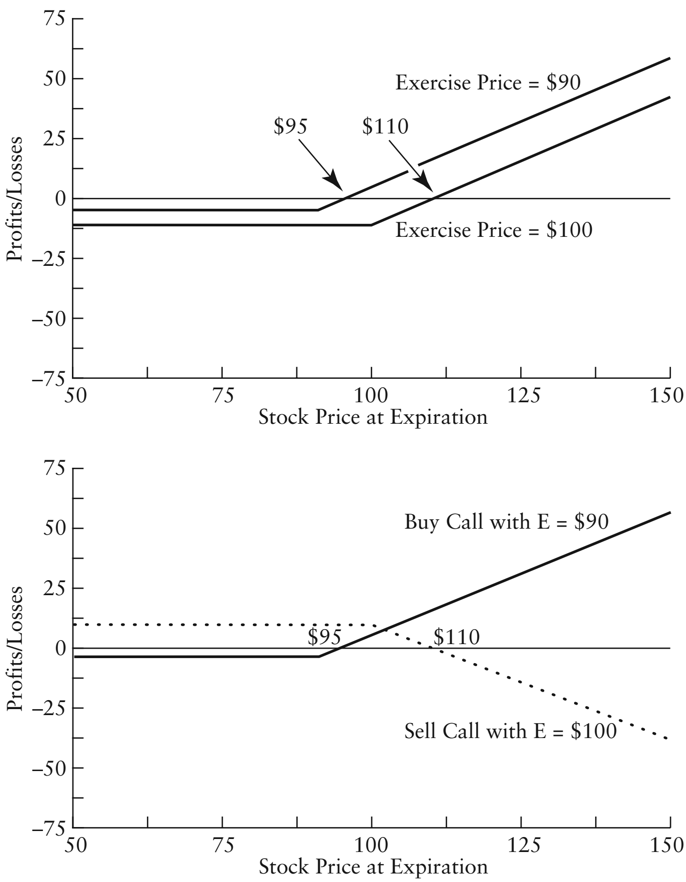

FIGURE 4.5 Why options with lower exercise prices cannot have lower prices than options with higher exercise prices.

$100 and sells for $10. The second option has an exercise price of $90 and a premium of $5. The profit-and-loss graphs for both options are shown in Figure 4.5. The option with the $90 exercise price has a much better profit-and-loss profile than the option with the $100 exercise price. No matter what the stock price might be at expiration, the option with the $90 exercise price will perform better.

This is already an impossible pricing situation because it represents a disequilibrium result. With these prices, all participants in the market would want the option with the exercise price of $90. This would cause the price of the option with the $100 exercise price to fall until investors were willing to hold it, too. But this could only occur if it were not inferior to the option with the $90 exercise price.

The same point can be made in the following context because the profit-and-loss opportunities in the first panel of Figure 4.5 create an arbitrage opportunity. Faced with these prices, the arbitrageur would simply transact as follows:

Transaction	Cash Flow
Sell the option with the $100 exercise price	$10
Buy the option with the $90 exercise price	−5
Net Cash Flow	$ 5

This gives a combined position that is graphed in the second panel of Figure 4.5. Here, the sale of the option with the $100 striking price is shown as the dotted line. To see why this would be a desirable transaction, consider the profit-and-loss position on each option and the overall position for alternative stock prices that might prevail at expiration (see Table 4.2).

TABLE 4.2 Profit and Loss Position on Each Option

	Profit or Loss on the Option Position		
Stock Price at Expiration	**For E = $90**	**For E = $100**	**For Both**
80	$−5	$+10	$+5
90	−5	+10	+5
95	0	+10	+10
100	+5	+10	+15
105	+10	+5	+15
110	+15	0	+15
115	+20	−5	+15

For any stock price, there will be some profit. If the stock price is \$90 or less, the profit will be \$5 from the options position, plus the net cash inflow of \$5 that was received when the position was initiated. As the stock price at expiration goes from \$90 to \$100, the profit goes up, until the maximum profit of \$15 on the options position is achieved at a stock price of \$100. When stock prices at expiration are greater than \$100, the profit on the options position remains at \$15.

With the prices in the example, it is possible to trade to guarantee a total profit of at least \$10 and perhaps as much as \$20 without risk or investment, so it is an example of arbitrage. If option prices are to be rational, they cannot allow arbitrage. To eliminate the arbitrage opportunity, the price of the option with a striking price of \$90 must be at least as large as the price of the option with the striking price of \$100.[5]

A similar principle refers to the expiration date:

$$\text{If } T_1 > T_2,\ C(S, T_1, E) \geq C(S, T_2, E) \tag{4.6}$$

If two options are otherwise alike, the option with the longer time to expiration must sell for an amount equal to or greater than the option that expires earlier.

Intuitively, this principle must hold, because the option with the longer time until expiration gives the investor all of the advantages that the one with a shorter time to expiration offers. But the option with the longer time to expiration also allows the investor to wait longer before exercising the option. In some circumstances, the extra time for the option to run will have positive value.[6]

If the option with the longer period to expiration sold for less than the option with the shorter time to expiration, there would also be an arbitrage opportunity. To conduct the arbitrage, assume that two options are written on the same stock with a striking price of \$100. Let the first option have a time to expiration of six months and assume it trades for \$8, while the second option has three months to expiration and trades for \$10. In this situation, the arbitrageur would make the following transactions:

Transaction	Cash Flow
Buy the six-month option for \$8	\$− 8
Sell the three-month option for \$10	+10
Net Cash Flow	\$ 2

By buying the longer maturity option and selling the shorter maturity option, the option trader receives a net cash flow of \$2. It appears there could be some risk however, because the option that was sold might be exercised. To see that the trader's position is secure, consider that if the buyer exercises the option, the arbitrageur can simply exercise the six-month option previously purchased and use the stock that is received to deliver on the three-month option. This will guarantee that the seller can keep the \$2, so there will be a \$2 profit no matter what happens to the stock price. Since this profit is certain and was earned without investment, it is an arbitrage profit.[7] The option with the longer time to expiration cannot be worth less than the option with the shorter time to expiration. Otherwise, there will be arbitrage opportunities.

Generally, the option with the longer time to expiration will actually be worth more than the option with the shorter time to expiration. We have seen that any option must be worth at least the difference between the stock price and the exercise price ($S - E$) at expiration. If the stock price is greater than the exercise price ($S > E$), the call option is said to be *in-the-money,* but if the stock price is less than the exercise price ($S < E$), the option is *out of-the-money.* If the stock price equals, or nearly equals, the exercise price ($S = E$), the option is *at-the-money.* Prior to expiration, an in-the-money option will normally be worth more than $S - E$. This difference ($S - E$) is known as the *intrinsic value* of the option, which is simply the value of the option if it were exercised immediately. An in-the-money option prior to expiration can be worth more than $S - E$ because the value of being able to wait to exercise is generally positive. If the option is exercised prior to expiration, the trader will receive only the amount $S - E$ for the option. By selling the option in the market, the trader will get the market price of the option, which normally exceeds $S - E$. So it generally will not pay to exercise an option early.[8]

Thus far, we have set bounds for option prices and we have established relationships between pairs of options, as shown in Figure 4.6. There, the two options, C_1 and C_2, are alike except that option C_1 has a lower exercise price. Accordingly, the price of C_1 is more tightly bounded than that of option C_2. The two options in the second pair, C_3 and C_4, differ only by the time to expiration. Consistent with this fact, the price of the option with the longer time to expiration, C_4, has a higher price. While we can now put bounds on the overall price of options and establish which of two options should have the higher price, we need to be able to put further restrictions on the price of a call option. To do this, we need to consider the impact of interest rates.

CALL OPTION PRICES AND INTEREST RATES

Assume that a stock now sells for \$100 in the marketplace and that over the next year its value can change by 10 percent in either direction. For a round

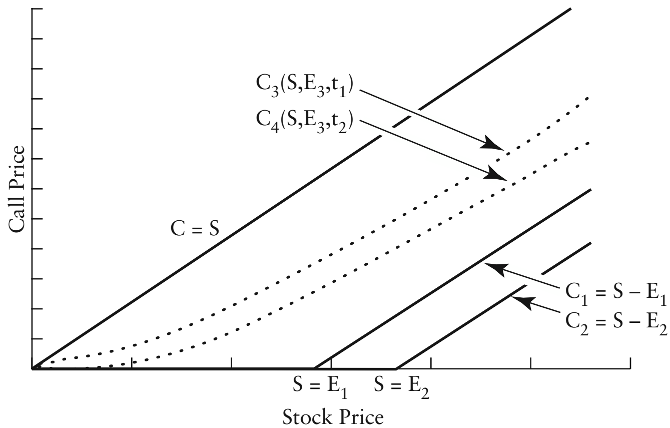

FIGURE 4.6 Bounds on option prices and permissable relationships between pairs of option prices.

lot of 100 shares, the value one year from now will be either \$9,000 or \$11,000. Assume also that the risk-free rate of interest is 12 percent and that a call option exists on this stock with a striking price of \$100 per share and an expiration date one year from now. With these facts in mind, imagine two portfolios constructed in the following way:

Portfolio A	100 shares of stock, current value \$10,000.
Portfolio B	A \$10,000 pure discount bond maturing in one year, with a current value of \$8,929, which is consistent with the 12 percent interest rate. One option contract, with an exercise price of \$100 per share, or \$10,000 for the entire contract.

Which portfolio is more valuable, and what does this tell us about the price of the call option? In one year, the stock price for the round lot will be either \$11,000 if the price goes up by 10 percent, or \$9,000 if the price goes down by 10 percent. This result is shown for Portfolio A in Table 4.3. For Portfolio B, there are both bonds and the call option to consider. As is also shown in Table 4.3, the bonds will mature in one year and will be worth \$10,000 no matter what happens to the stock price. The stock price will have a strong effect on the value of the call option, however. If the stock price goes up by 10 percent, the call option will be worth exactly \$1,000, the difference between the stock price and the exercise price ($S - E$). If the stock price goes down by 10 percent, the option will expire worthless. So, if

TABLE 4.3 Portfolio Values in One Year: Scenario 1

	Stock Price	
	Rises 10%	**Falls 10%**
Portfolio A		
Stock	$11,000	$ 9,000
Portfolio B		
Maturing bond	10,000	10,000
Call option	1,000	0

the stock price goes down, Portfolio B will be worth $10,000; whereas if the stock price goes up, Portfolio B will be worth $11,000.

In this situation, Portfolio B is the better portfolio to hold. If the stock price goes down, Portfolio B is worth $1,000 more than Portfolio A. But if the stock price goes up, Portfolios A and B have the same value. An investor could never do worse by holding Portfolio B, and there is some chance of doing better. Therefore, the value of Portfolio B must be at least as great as the value of Portfolio A.

This tells us something very important about the price of the option. Since Portfolio B is sure to perform at least as well as Portfolio A, it must cost at least as much. Further, the value of Portfolio A is $10,000, so the price of Portfolio B must be at least $10,000. The bonds in Portfolio B cost $8,929, so the option must cost at least $1,071. This means that the value of the call must be worth at least as much as the stock price minus the present value of the exercise price. If the call did not meet this condition, any investor would prefer to purchase Portfolio B in the example, rather than Portfolio A. Further, there would be an arbitrage opportunity.[9] Previously, we were able to say only that the price of the call must be either zero or $S - E$ at expiration. Based on the reasoning from the example, we can now say the following:

$$C \geq S - \text{Present Value}(E) \tag{4.7}$$

The call price must be greater than or equal to the stock price minus the present value of the exercise price. This substantially tightens the bounds on the value of a call option.[10]

As the next example indicates, it must also be true that the higher the interest rate, the higher will be the value of the call option, if everything else is held constant. In the previous example, the interest rate was 12 percent, and we were able to conclude that the price of the call option must be at least \$1,071, because:

$$C \geq \$10,000 - \frac{\$10,000}{(1.12)} = \$1,071$$

For the same portfolio, imagine that the interest rate had been 20 percent instead of 12 percent. In that case, the value of the call option must have been at least \$1,667, as shown by the following equation:

$$C \geq \$10,000 - \frac{\$10,000}{(1.20)} = \$1,667$$

From this line of reasoning, we can assert the following principle:

$$\text{If } R_{f1} > R_{f2},\ C(S, E, T, R_{f1}) \geq C(S, E, T, R_{f2}) \tag{4.8}$$

Other things being equal, the higher the risk-free rate of interest, the greater must be the price of a call option.

PRICES OF CALL OPTIONS AND THE RISKINESS OF STOCKS

Surprisingly enough, the riskier the stock on which an option is written, the greater will be the value of the call option. For example, consider a stock trading at \$10,000 that will experience either a 10 percent price rise or a 10 percent price decline over the next year. As in Table 4.3, a call option on such a stock with an exercise price of \$10,000 and a risk-free interest rate of 12 percent would be worth at least \$1,071. Now consider a new stock that trades at \$10,000, but that will experience either a 20 percent price increase or a 20 percent price decrease over the next year. If we hold the other factors constant by assuming that interest rates are 12 percent per year, and focus on an option with a striking price of \$10,000, what can we say about the value of the call option?

As Table 4.4 shows, the call option on the stock that will go up or down by 10 percent must be worth at least \$1,071. If the stock price goes down, the call will be worth zero. If the stock price goes up, the call will be worth \$1,000. In the bottom panel of Table 4.4, the stock will go up or down by

TABLE 4.4 Portfolio Values in One Year: Scenario 2

	Stock Price	
	Rises 10%	**Falls 10%**
Portfolio B		
Maturing bond	\$10,000	\$10,000
Call option	1,000	0
	Stock Price	
	Rises 20%	**Falls 20%**
Portfolio A		
Stock	\$12,000	\$ 8,000
Portfolio B		
Maturing bond	10,000	10,000
Call option	2,000	0

20 percent. If the stock price goes down, the call in this case will be worth zero. This is the same result as the call in the top panel. If prices go up, the call in the bottom panel will be worth \$2,000, which is the difference between the exercise price and the stock price.

In this scenario, any investor would prefer the option in the bottom panel, because it cannot perform worse than the call in the top panel, and it might perform better if the stock price goes up. This means that the value of the call in the bottom panel must be at least as much as the value of the call in the top panel, but it will probably be worth more. The only difference between the two cases is the risk level of the stock. In the top panel, the stock will move up or down by 10 percent, but the stock in the bottom panel is riskier, because it will move 20 percent. From this example, we can derive the following principle:

$$\text{If } \sigma_1 > \sigma_2,\ C\left(S, E, T, R_f, \sigma_1\right) \geq C\left(S, E, T, R_f, \sigma_2\right) \tag{4.9}$$

Other things being equal, a call option on a riskier good will be worth at least as much as a call option on a less risky good.

CALL OPTIONS AS INSURANCE POLICIES

In Table 4.3, the call option will be worth either \$1,000 or zero in one year, and the value of that option must be at least \$1,071. At first glance, it is a terrible investment to pay $1,071 or more for something that will be worth either zero or $1,000 in a year. However, the option offers more than a simple investment opportunity; it also involves an insurance policy. The insurance character of the option can be seen by comparing the payoffs from Portfolio A and Portfolio B. If the stock price goes down by 10 percent, Portfolio A will be worth $9,000 and Portfolio B will be worth $10,000. If the stock price goes up by 10 percent, both portfolios will be worth $11,000. Holding the option insures that the worst outcome from the investment will be $10,000. This is considerably safer than holding the stock alone. Under these circumstances, it would make sense to pay $1,071 or more for an option that has a maximum payoff of $1,000. Part of the benefit from holding the option portfolio is the insurance that the total payoff from the portfolio will be at least $10,000. This also explains why the riskier the stock, the more the option will be worth. This relationship results because the riskier the stock, the more valuable will be an insurance policy against particularly bad outcomes.

Previously, we said that the price of the option must be at least as great as the stock price minus the present value of the exercise price. However, this formulation neglects the value of the insurance policy inherent in the option. If we take that into account, we can say that the value of the option must be equal to the stock price minus the exercise price, plus the value of the insurance policy inherent in the option. Or, where the value of the insurance policy is denoted by I, the value of the call option is given by:

$$C\left(S, E, T, R_f, \sigma\right) = S - \text{Present Value}(E) + I \tag{4.10}$$

However, we have no way, thus far, of putting a numerical value on the insurance policy denoted by I. That task requires an examination of the option pricing model.

THE OPTION PRICING MODEL

To this point, by reasoning about option prices and finding the boundaries for option prices that rule out arbitrage opportunities, we have learned a great deal about call option prices and the relationship of these prices to other variables. In the preceding discussion, we identified five variables that affect the value of a call option. In the following list, a plus sign (+) by a variable indicates that the price of a call option is larger, the larger the value of the associated variable:

+	Stock Price	S
–	Exercise Price	*E*
+	Time to Expiration	*T*
+	Risk-Free Interest Rate	R_f
+	Variability of the Stock's Returns	σ

There is a great deal to learn in addition to the basic factors that affect the prices of call options and the direction of their influence. For example, in exploring the bounds of option pricing, we considered an example in which the stock price could move by 10 percent up or down in a year. This is obviously a great simplification of reality. In a given time period, stock prices can take on a virtually infinite number of values. Also, stock prices change continuously for all practical purposes. To be able to put an exact price on a call option requires a much more realistic model of stock price behavior.

This is exactly the approach that Fischer Black and Myron Scholes took when they developed their option pricing model (OPM).[11] Strictly speaking, the model applies to European options on non-dividend-paying stocks, although adjustments can be made to the model to deal with other cases.[12] The mathematics of their model is extremely complex, but they were able to derive the model by assuming that stock prices follow a certain kind of path through time called a stochastic process. A *stochastic process* is simply a mathematical description of the change in the value of some variable

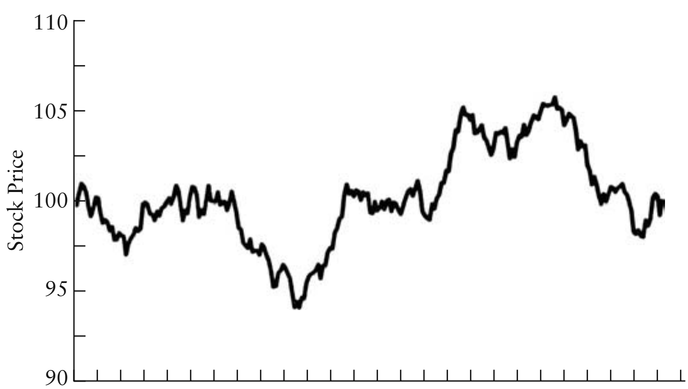

FIGURE 4.7 One possible realization of a Wiener process.

through time. The particular stochastic process that Black and Scholes used is known as a *Wiener process*. The key features of the Wiener process are that the variable changes continuously through time and that the changes that it might make over any given time interval are distributed normally. Figure 4.7 shows a graph of the path that stock prices might follow in a Wiener process.

Essentially, the difference between our discussion to this point and the achievement of the OPM is that the OPM gives a mathematical expression to the value of a call option. Whereas we were unable to say what the call price should equal, Black and Scholes present a theoretical formula for that price. If we know the values of the five variables listed earlier, we can use the OPM to calculate the theoretical price of an option. Further, while we cannot consider the mathematics that Black and Scholes used, we can understand how to calculate option values according to their model, and we can understand the relationship between the OPM and the conclusions we have reached in previous sections.

The formula for the Black-Scholes OPM is given by:

$$C = SN(d_1) - Ee^{-R_f T} N(d_2) \tag{4.11}$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S}{E}\right) + \left[R_f + \left(\frac{1}{2}\right)\sigma^2\right]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$N(d_1)$, $N(d_2)$ = cumulative normal probability values of d_1 and d_2
S = stock price
E = exercise price
R_f = the risk-free rate of interest
σ = instantaneous variance rate of the stock's return
T = time to expiration of the option

The most difficult part of this formula to understand is the normal cumulative probability function. However, this is exactly the part of the OPM that takes account of the risk and allows the model to give such good results for option prices. The best way to understand the application of the model is with an example. Let us assume values for the five parameters and calculate the Black-Scholes value for an option. For purposes of the example, assume the following:

$$
\begin{aligned}
S &= \$100 \\
E &= \$100 \\
T &= 1 \text{ year} \\
R_f &= 12\% \\
\sigma &= 10\%
\end{aligned}
$$

These values make it possible to calculate the Black-Scholes theoretical option value, and the first task is to calculate values for d_1 and d_2:

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{S}{E}\right) + \left[R_f + \left(\frac{1}{2}\right)\sigma^2\right]T}{\sigma\sqrt{T}} \\
&= \frac{\ln\left(\frac{100}{100}\right) + \left[.12 + \frac{1}{2}(.01)\right]1}{(.1)(1)} = \frac{0 + .1250}{.1} \\
&= 1.25 \\
d_2 &= d_1 - \sigma\sqrt{T} \\
&= 1.25 - (.1)(1) = 1.15
\end{aligned}
$$

Having calculated the values of d_1 and d_2, the next step is to calculate the cumulative normal probability values of these two results. Essentially, these two values are simply z-scores from the normal probability function, such as the one shown in Figure 4.8. In this graph, the two values of interest, 1.15 and 1.25, are shown. In calculating the cumulative normal probability values of $d_1 = 1.25$ and $d_2 = 1.15$, we simply need to determine the proportion of the area under the curve that lies to the left of the value in question. For example, if we were interested in a z-score of 0.00, we would know that 50 percent of the area under the curve lies to the left of a z-score of 0.00. This is because the normal probability distribution is symmetrical about its mean, and the z-scores are standardized so that they have a mean of 0.00.

Because the standardized normal probability distribution is so important and so widely used, tables of its values are included in virtually every statistics textbook. The Appendix at the end of this book provides a typical

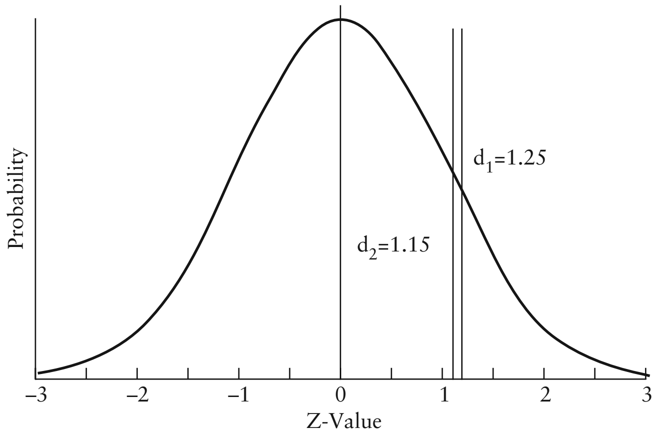

FIGURE 4.8 The normal probability function.

table. As is shown, the probability of drawing a value from this distribution that is less than or equal to $d_1 = 1.25$ is .8944. So, the two values we seek are:

$$N(d_1) = N(1.25) = .8944$$
$$N(d_2) = N(1.15) = .8749$$

Returning to the OPM, we can now make the final calculation:

$$C = S\,N(d_1) - E\,e^{-R_f T}\,N(d_2)$$
$$C = \$100(.8944) - \$100e^{-(.12)(1)}(.8749)$$
$$= \$89.44 - \$100(.8869)(.8749)$$
$$= \$89.44 - \$77.60$$
$$= \$11.84$$

In this calculation, the term $e^{-R_f T} = .8869$ is simply the discounting factor for continuous time with an interest rate of 12 percent and a period of one year. So, according to the OPM, the call option should be worth \$11.84.

The calculation of the value of this option by the OPM corresponds closely to the logic used in our earlier example from Table 4.3. There we concluded that an option with similar characteristics must be worth at least the difference between the stock price and the present value of the option's exercise price, which in this example is \$10.71. The result from the OPM is consistent with this analysis, but it is much more exact. The difference between the OPM value of \$11.84 and the minimum value of \$10.71 is due to the value of the insurance policy that we were unable to capture without the sophisticated approach of the OPM.[13]

Also the OPM result is very close to the result that we reached by just a process of reasoning. We were able to conclude that:

$$C = S - \text{Present Value}(E) + I \tag{4.12}$$

and the OPM says that:

$$C = S\,N(d_1) - E\,e^{-R_f T} N(d_2)$$

The term, Ee^{-RfT} is simply the present value of the exercise price when continuous discounting is used. This means that the OPM is saying:

$$C = S\,N(d_1) - \text{Present Value}(E)\,N(d_2)$$

The terms involving the cumulative probability function are the terms that take account of risk. Coupled with the rest of the formula, they capture the value of the insurance policy. If the stock involved no risk, the calculated values for d_1 and d_2 would be very large and the subsequent calculated cumulative functions would both approach a value of 1. If $N(d_1)$ and $N(d_2)$ both equal 1, the OPM could be simplified to:

$$C = S - \text{Present Value(E)}$$

which is very close to the result we were able to reach without the OPM. This expression simply does not reflect the value of the option as an insurance policy, a value we know it has and that we can measure by using the OPM.

Many people initially think that the OPM is too complicated to be useful. Nothing could be further from the truth. Of all of the models in finance, the OPM is among those receiving the widest acceptance by actual investors. Handheld devices give traders online access to OPM prices for all options using instantaneously updated information on all of the parameters in the model. In most investment banking houses, staffs that specialize in options use the OPM on a daily basis. The OPM has achieved such widespread acceptance that some spreadsheet software manufacturers include special functions to calculate OPM values automatically.

This popularity is due in large part to the OPM's excellent results. The Black-Scholes theoretical model price is usually very close to the market price of the option. Without doubt, the OPM has contributed greatly to our understanding of option pricing and many traders use it as a key tool in their trading strategies.

THE VALUATION OF PUT OPTIONS

Although the OPM pertains specifically to call options, it can also be used to price put options, through the principle of *put-call parity*.[14] Assume that an investor makes the following transactions and that the put and call options are on the same stock:

$$\text{Buy one share of stock } S = \$100$$

$$\text{Buy one put option with price } P = ?,\ E = \$100 \text{ and } T = 1 \text{ year}$$

$$\text{Sell one call option with price } C = \$11.84,\ E = \$100 \text{ and } T = 1 \text{ year}$$

At expiration, the stock price could have many different values, some of which are shown in Table 4.5. The interesting feature about this portfolio is that its value will be the same, $\$100 = E$, no matter what the stock price is at expiration. Consistent with Table 4.5, no matter what the stock price at expiration might be, the value of the entire portfolio will be $\$100 = E$. Holding these three instruments in the way indicated gives a risk-free investment that will pay $\$100 = E$ at expiration, so the value of the whole

TABLE 4.5 Possible Outcomes for Put-Call Parity Portfolio

Stock Price	Call Value	Put Value	Portfolio Value
$ 80	$ 0	$20	$100
90	0	10	100
100	0	0	100
110	−10	0	100
120	−20	0	100

portfolio must equal the present value of the riskless payoff at expiration. This means that we can write:

$$S - C + P = \frac{E}{\left(1 + R_f\right)^T} \tag{4.13}$$

The value of the put-call portfolio equals the present value of the exercise price discounted at the risk-free rate.

Since it is possible to know all of the other values, except for the price of the put *P,* we can use this put-call parity relationship to calculate *P.* To see how this is done, assume, as before, that $R_f = 12$ percent and that the call value is \$11.84, as was calculated according to the OPM. Rearranging the put-call parity formula gives a put value of \$1.13:

$$P = \frac{E}{\left(1 + R_f\right)^T} - S + C$$

$$P = \frac{\$100}{(1.12)} - \$100 + \$11.84 = \$1.13$$

DIVIDEND ADJUSTMENTS

Our discussion to this point omits any reference to dividends because the original Black-Scholes Model was applied only to non-dividend-paying stocks. Therefore, the inclusion of dividends is an extension to that model.

To understand how option modelers treat dividends, it is important to distinguish between discrete dividends and continuous dividends. A discrete dividend is what you normally think of as a dividend: a cash dividend of a set amount paid once over a discrete interval (e.g., a quarterly cash dividend). This could also include a special cash dividend such as the type that is often paid in a corporate restructuring. With a continuous dividend, we calculate the dividend as a percentage yield in the same way that interest on a loan is calculated as a percentage yield. Although no stocks actually pay such a dividend, the approach has clear advantages in many modeling situations. First, it can serve as a simplifying assumption that helps facilitate the application of option pricing models. Second, there are some underlying assets (on which options are written) where the cash flow accruing to the asset holder can be thought of *as if* it were continuous. Options written on futures contracts are valued using just such an assumption: For modeling purposes, an option on a futures contract can be thought of as an option on a stock paying a continuous dividend yield equal to the risk-free rate of interest. Also, options written on some stock indexes are valued using such an assumption.

Even though the dividend is not paid to the option holder (it goes to the holder of the underlying asset) the payment will affect the option price. This is because the underlying asset price is affected by the dividend. Stocks, for example, decline in value by (roughly) the amount of the declared dividend on the ex-dividend day. Since the stock price declines, this reduces the price of calls and increases the price of puts.

Most corporations pay regularly quarterly (i.e., discrete) dividends. Although dividend surprises often occur, it is still a reasonable assumption to say that investors have a good expectation about what dividends will be over the life of an option. This expectation can be accounted for in the option pricing model. The simplest way to include dividends is to subtract the present value of the future dividend stream (the term "stream" reflects that more than one dividend may be paid during the life of the option) from the currently observed stock price. The dividend-adjusted stock price is then used in the Black-Scholes formula in place of the observed stock price.

Consider our previous example of a call option with an exercise price of \$100, a current stock price of \$100, one year to expiration, with an annual volatility rate of 10 percent and a risk-free interest rate of 12 percent. Suppose the underlying stock now pays a \$3 dividend in 90 days. The present value of this dividend is \$2.91 (i.e., $\$3e^{-.12(90/365)}$). This means the dividend-adjusted stock price is 97.09. Inputting this value into equation 4.11 yields a dividend-adjusted call option value of \$9.32 versus the value of \$11.84 we obtained without a dividend adjustment.

Adjustments for continuous dividends operate on the same principle as with discrete dividends, that is, the stock price is adjusted. We define the continuous dividend rate as δ. To obtain the continuous dividend version of the Black-Scholes Model, we replace S in the regular model with:

$$Se^{(-\delta \times T)}$$

Continuing with our example, suppose the stock pays annual dividend rate of 1.5 percent. The adjusted stock price would be:

$$\$100e^{(-.015 \times 1)} = \$98.51$$

And the resulting call price would be \$10.53.

Our discussion of dividend adjustments has assumed European-style options. For American-style options, option pricing in the presence of dividends is not as simple. This is because, with dividends, it may be optimal to exercise American calls and puts prior to option expiration. Without dividends, it will never be optimal to exercise a call prior to expiration. In the absence of dividends, an American-style call option can be priced using the Black-Scholes Model for European-style options.[15]

OPTIONS ON STOCK INDEXES

In addition to options on individual stocks, option contracts are also written on stock indexes. The volume leader in index options trading is the CBOE followed closely by the AMEX (American Stock Exchange). Both exchanges list options on broad market indexes, sector-specific indexes, country-specific indexes, and index-linked exchange-traded funds (ETFs). In recent years, options on ETFs have become extremely popular, so popular in fact that they now dominate options written on the indexes themselves. ETFs are depository receipts linked to recognizable indexes such as the Dow Jones Industrial Average (under the product name Diamonds), the Nasdaq 100 (under the product name "The Cube" from its QQQ ticker symbol) or the S&P 500 (under the product name SPDRs, or "spiders").[16] The depositor receipts represent a claim on a unit trust holding a portfolio that tracks the specified index. These depository receipts trade like shares of stock, and options written on these depository receipts trade like stock options. The option contracts are cash-settled. The cash payment is based on the index value at the end of the day on which exercise instructions are issued.

European-style options on stock indexes can be valued using the continuous dividend version of the Black-Scholes Model. Consider a 60-day European call option written on the S&P 500. The current index value is 1200 index points, the annualized dividend yield is 1.5 percent, the option's strike price is $1,000, the annualized volatility rate is 20 percent, and the risk-free rate is 3 percent. Applying equation 4.11, we find that the value of the call is 108.78 index points. At $250 per index point, this call contract would be worth $27,195.

OPTIONS ON FUTURES

To this point, we have considered options that are written directly on a substantial underlying good. We now consider options on futures. A trader who buys a call option on a futures contract pays the option price. In return, the call owner receives the right to exercise the option and assume a long position in the futures contract. For the call owner, exercise makes sense only if the futures price exceeds the exercise price on the option. In that case, the call owner receives a long position in the futures contract and a cash payment that equals the difference between the futures price at the time of exercise and the exercise price. The seller of a call is subject to the exercise decision of the call holder. If the call holder exercises, the call seller receives a short position in the futures contract and pays the call holder the difference between the current futures price and the exercise price. As an example, assume that a trader buys a call option on the December wheat futures with an exercise price of $3.50 per bushel. Later, the futures price is $3.75, and the call holder exercises. On exercise, the call holder receives a long position in the futures contract with a contract price of $3.75. In addition, the call holder receives $.25 per bushel, or $1,250 on one 5,000 bushel contract. The seller of this call must pay the call holder $1,250 and accept a short position in the futures contract with a contract price of $3.75 per bushel.

At this point, both traders have completed all transactions related to the futures option. However, the call owner holds a long position in the futures and the call seller holds a short position in the futures, both at a contract price of $3.75 per bushel. At this point, neither trader has a profit or loss on the futures. They can both offset their futures position to avoid any future obligations. Alternatively, they can maintain their positions and hope to profit from subsequent movements in the futures price.

For a put, the buyer of a futures option acquires the right to force the seller of the put to assume a long futures position and to pay the long put trader the difference between the exercise price of the option and the futures price at the time of exercise. Assume a trader buys a put option with an exercise price of $3.80 per bushel. Later, when the futures price is $3.75, the put owner can exercise. The seller then receives a long position in the futures at the current futures price of $3.75. In addition, the seller pays the put owner the difference between the futures price and the exercise price. In this example, the futures price is $3.75 and the exercise price is $3.80. Therefore, the put seller pays the put buyer $.05 per bushel, or $250 on a 5,000-bushel contract. The buyer of a put receives a short position in the futures. Both the long and short put traders now hold futures positions at $3.75, the market price prevailing at the time of exercise. They may offset their futures positions to avoid further profits or losses, or they may maintain the futures position in pursuit of profit.[17]

The pricing of options on futures relies on the Cost-of-Carry Model discussed in Chapter 2. The Cost-of-Carry Model can be written in continuous discounting form as:

$$F_{0,t} = S_0 e^{ct} \tag{4.14}$$

Because we continue to assume perfect markets, our analysis of options on futures focuses on futures contracts that can be adequately described by the Cost-of-Carry Model. For the most part, this model works well for precious metal and financial futures. By contrast, the model does not describe agricultural and energy futures very well. The Cost-of-Carry Model implies that the futures price must always exceed the cash price, a condition that is often violated by agricultural goods and by energy products such as oil.[18]

When the Cost-of-Carry Model applies, we can treat a futures contract as an asset that pays a continuous dividend at the risk-free rate.[19] In terms of our notation for the continuous dividend version of the Black-Scholes Model, $\sigma = r$. In this case, the futures rate equals the cost-of-carry. The only cost of carrying the commodity is the interest cost. Applying the continuous dividend version of the Black-Scholes Model to a futures contract, F, gives the value of a call option on a futures, C^f:

$$\begin{aligned} C^f &= e^{-rt}\left[FN\left(d_1^f\right) - EN\left(d_2^f\right)\right] \\ d_1^f &= \frac{\ln\left(\frac{F}{E}\right) + \left(.5\sigma^2\right)t}{\sigma\sqrt{t}} \\ d_2^f &= d_1^f - \sigma\sqrt{t} \end{aligned} \tag{4.15}$$

To see why this model is correct, assume for the moment that there is no risk. From our previous discussions, we know that the $N(.)$ terms drop out from the Black-Scholes Model under conditions of certainty. In that case, the payoff at expiration is just the difference between the futures price at the time the call is purchased and the exercise price of the call:

$$C^f = \left[S - E\right]e^{-rt}$$

The value of the call at time zero must be the present value of the certain payoff at expiration.

Now we introduce risk and consider a European futures option. Assume a wheat futures contract trades at \$3.50 per bushel and the corresponding futures options have a striking price of \$3.45 and expire in 47 days. The standard deviation of the futures price is .23. The interest rate is 9 percent, which equals the cost-of-carry. With these values, the call price is \$.1395 and the put price is \$.0901 per bushel. With a contract size of 5,000 bushels, the two contracts cost \$697.50 and \$450.50, respectively.

Thus far, we have considered only European futures options. Because futures options are generally American, they can be exercised any time. This makes them extremely difficult to price. Essentially, an American futures option consists of an infinite series of European options. In essence, the American futures option has an exercise price equal to the explicit amount that must be paid, plus the sacrifice of the remaining European options in the series. Because the American futures option must be analyzed as an infinite series of European options, there is no closed-form solution for its value. Instead, we must approximate the value of the American futures option. The value of an American futures option is given by:

$$C = FW_1 - EW_2 \tag{4.16}$$

where W_1, W_2 = "weighting factors comprised of infinite sums of the products of discount factors and conditional probability terms that reflect, at each instant, the present value of the exercise value conditioned on the probability that exercise did not occur at a previous instant."[20]

Because W_1 and W_2 are sums of an infinite series, the value of the call must be estimated instead of being computed exactly. However, we can compute the estimate to a high degree of accuracy. In every instance, the value of the American futures option should equal or exceed the value of the corresponding European futures option.

With futures options, there is always the prospect of early exercise, whether the good on which the futures contract is written pays dividends or not. For a call option on a non-dividend-paying stock, early exercise never makes sense. If the owner exercises, he or she receives the intrinsic value, $S - E$. In effect, this means that the owner throws away the time value of the option. If the underlying stock pays a dividend, early exercise is sometimes reasonable. When a stock pays a dividend, the value of the share drops by the amount of the dividend. In this case, value is "leaking out" of the underlying good, so it may be wise to exercise before expiration.

Similarly, it may be wise to exercise futures options before expiration. Consider a call option with an exercise price $E = \$50$ and a futures contract with a price $F = \$100$. Assume for the moment that the futures price will not change anymore, and consider whether the call owner should exercise early or wait until expiration. With these data, the owner should exercise early. By doing so, the owner receives \$50 immediately. After exercising, the trader can earn interest on \$50 until the expiration date. The benefit of early exercise on a futures option is that exercise provides an immediate payment of $F - E$. The value of the early exercise is the interest that can be earned between the time of exercise and the expiration date:

$$[F - E]e^{rt}$$

In this example, we have assumed that the futures price does not change. In that circumstance, it would be wise to exercise early to capture the interest on the mark-to-market payment $F - E$. Usually, however, the futures price fluctuates. Therefore, early exercise discards the option's value over and above the intrinsic value $F - E$. As a consequence, early exercise of a futures option has a benefit and a cost:

Benefit: Use of the funds $F - E$ until expiration
Cost: Sacrifice of option value over and above intrinsic value $F - E$

Figure 4.9 illustrates the differences between the pricing of American and European futures call options. Consider a futures call with exercise price E. Figure 4.9 shows how the prices of otherwise identical American and European options vary as a function of the futures price, F. We have observed that the minimum price for a European futures call option is $(F - E)e^{-rt}$. In Figure 4.9, the European option attains this minimum when its price touches the line designated as $(F - E)e^{-rt}$. This happens when the futures price reaches F', which is equivalent to $N(d_1^f)$ and $N(d_2^f)$ both equaling

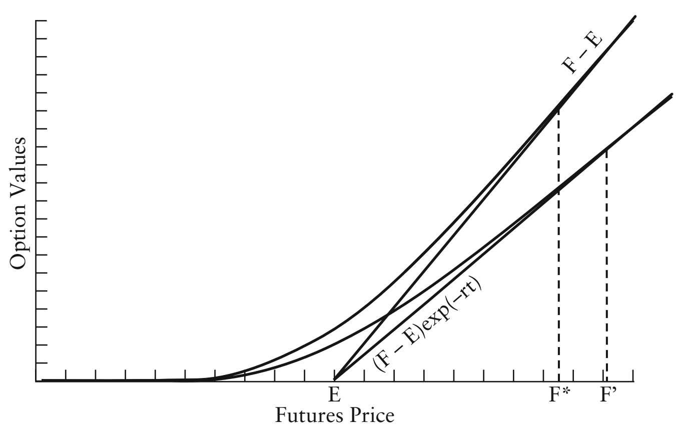

FIGURE 4.9 European and American opions on futures.

1.0 in equation 4.15. In economic terms, this situation arises when it is certain that the option will remain in-the-money. In that case, the option will pay $F - E$ at maturity. Before maturity, its price must equal the present value of $F - E$, or $(F - E)e^{-rt}$.

The American futures option price must equal or exceed the corresponding European futures call price shown in Figure 4.9.[21] The difference between the American and European futures option prices is the early exercise premium. The American futures option has extra value because it can be exercised before expiration. In the figure, the American futures option attains an important level of F*, at which the option has no excess value above its intrinsic value. When the futures price is F*, the intrinsic value of the American futures option is $F^* - E$ and its market value should be the same.

Following Whaley, we call F* the critical futures price.[22] This is the point at which the option price reaches a price that justifies immediate exercise. We can compute the critical price using various sophisticated methods. For any futures price above F*, the value of the American futures option equals its intrinsic value. Therefore, the owner of the call option should exercise the option at any futures price above F* and invest the proceeds to earn interest. For any futures price below F*, the option should not be exercised because the option will still have value above its intrinsic value. At F*, the owner is indifferent about exercising.[23]

OVER-THE-COUNTER (OTC) OPTIONS

Not all options are traded on exchanges. OTC options markets, where financial institutions and corporations trade directly with one another in "principal-to-principal" transactions are becoming increasingly popular. Trading in OTC options is particularly active on foreign exchange and interest rates.

The main advantage of an OTC option is that it can be tailored by a financial institution to meet the needs of corporate clients. Nonstandard features can be incorporated into the design of the option. The option might specify that it can be exercised only on specific days during the option's life. OTC options containing nonstandard features are referred to as *exotic options*. At one time, exotic options seemed fanciful and gained attention only as academic curiosities. Although the name persists, corporate treasury departments and other end users now commonly trade many exotic options. Much of the state-of-the-art work on Wall Street in recent years has been to develop models to value these exotic options.

Path-dependent options make up a broad group of exotic options. The value of a path-dependent option depends on the path that the price of the underlying asset follows during the life of the option. In contrast, the value of a standard European-style option depends only on the spot price of the underlying asset at the option's expiration date. Within the group of path-dependent options are Asian options, barrier options, lookback options, and contingent-premium options.

Asian options are also known as average-price or average-rate options. There appears to be no geographic origin to the term—it is just one of those clever names invented on Wall Street. For an Asian option, the payoff at expiration is based on the difference between the strike and the average spot value of the underlying asset observed during at least some part of the option's life. The averaging period is specified as part of the option contract. It need not include the entire life of the option nor does the average have to be an arithmetic average. Observations may be weighted in favor of prices observed on designated dates. Average price options are less expensive than regular options because the volatility of an average price is always less than the volatility of the price series that makes up the average.

Barrier options are options where the payoff depends on whether the underlying asset reaches a designated level during a designated period of time. In addition to the strike price, a barrier option also specifies a "trigger price," or barrier. When the trigger price is hit, the option will either appear ("knocked in") or disappear ("knocked out").

Knockout options are the most common type of barrier option. If the barrier is above the spot price at origination, the option is referred to as "up-and-out." If the barrier is below the spot price at origination, the option is referred to as "down-and-out." Because in some situations the option would disappear, the premium of the knockout option is generally less than that of the standard option. The term "disappear" seems to mean different things to different people. To some, it means that exercise is forced if the spot price hits the barrier. Other contracts are written so that the contract is simply ripped up with zero payout if the barrier is hit. In the real world, the designation of the barrier is subject to continual negotiation. As the spot price approaches the barrier, the holder of the knockout option may pay the writer to have the barrier moved further away from the current price. It is not uncommon to see the barrier moved five or six times a month in a volatile market. This is part of what the customized OTC market is all about—you can get anything you want for a price.

A *knock-in option* comes into existence only when the barrier is reached. Up-and-in calls and down-and-in puts are activated when the barrier is reached, producing the standard contract.

An important issue for barrier options is the frequency of asset price observations for determining whether the barrier has been reached. Often the terms of a contract state that the asset price is to be observed once a day at the close of trading.

The payoff from a *lookback option* depends on the maximum or minimum price reached during the life of the option. This path-dependent option allows the holder to exercise the option using the most favorable price that has occurred during the contract's life. For example, a lookback call on the S&P 500 would allow the holder to receive, at expiration, the difference between the strike price and the highest price the index has reached during the contract's life. The value of a lookback option will be at least as great as the value of a standard American option. The difference between the two is the value of the lookback feature. The higher the underlying volatility, the more valuable the lookback feature and hence the greater the difference in value between the lookback and the standard contract. As with barrier options, the value of a lookback is sensitive to the frequency with which the asset price is observed for computing the maximum or minimum.

A *contingent premium option* is a path-dependent option where the premium is paid only if the contract expires in-the-money. If the option is out-of the-money at expiration, the seller receives nothing. Since there is a possibility that the seller will receive no premium, the contingent premium is substantially higher than for a standard option. The benefit to the holder is that there is no up-front premium.

Binary options compose another broad group of exotic options. These options have discontinuous payoffs. Like a standard option, the payoff from a binary option is nothing if the stock price ends up out-of-the-money. Unlike a standard option, the payoff from a binary option is a predetermined fixed amount if it ends up in-the-money (i.e., the payoff amount is unrelated to the degree to which the option finishes in-the-money). The payoff is all or nothing, unlike that of a standard option where the payoff rises continuously depending on the degree to which the option is in-the-money. A binary option can be either a call or a put.

The payoffs from the binary options that have been described here are determined at expiration. However, binary options can also be path-dependent. For example, a "one-touch" option pays off if the price of the underlying reaches a specified trigger price during the life of the option.

The price of a binary option depends on two factors: (1) the likelihood that the underlying will touch the trigger (in the case of a one-touch option) or close above or below the a certain price (in the case of an all-or-nothing option) and (2) the present value of the payoff amount.

Another common exotic option is the *Bermudan option,* also referred to as a limited exercise option. Bermudan options can be exercised only at specific dates. These options are often embedded in callable or putable corporate bonds. For example, a corporation may issue a five-year callable bond where the issue may be called at any coupon date. By limiting exercise to coupon dates, a callable bond fits the Bermudan classification. The price of a Bermudan option lies between the prices of American and European options. Because an American option may be exercised at any time, it is more expensive than a Bermudan option. A Bermudan option is more expensive than a European option because it has fewer constraints on exercise than a European option.

Exotic options also include *quanto options* (options that eliminate the exchange rate risk), *rainbow options* (options on two or more risky assets—sometimes referred to as "better of options"), *compound options* (options whose underlying assets are themselves options), and *chooser options* (options that are paid for in the present but are chosen to be either a call or a put at some prespecified future date).

OTC INTEREST RATE PRODUCTS

Over-the-counter interest rate options masquerade under several names, including caps, caplets, floors, floorlets, and collars. Most people are familiar with caps in the context of interest rates because many mortgage contracts offer borrowers the opportunity to protect against the rate of interest on a floating-rate loan going above some level over a specified time period. If the rate of interest on the loan does go above the cap rate, the seller of the cap (i.e., the lender) is responsible for the difference.

A *cap* is simply a call option on interest rates. A cap on interest rates guarantees that the borrower (the buyer of the cap) will pay the lesser of the cap rate and the prevailing rate. Suppose that the rate on a loan is reset every three months equal to the three-month LIBOR and that the borrower has capped the loan at 10 percent. Since the loan is written for more than three months with several reset dates, the cap can be viewed as a portfolio of options. Practitioners refer to the individual options as *caplets*.

A *floor* guarantees the lender (the buyer of the floor) will receive the greater of the floor rate and the prevailing rate. Therefore, a floor can be thought of as a put on rate. For example, the loan contract considered previously may require a minimum payment of 5 percent. Since the loan is written with several reset dates, the floor, like the cap, can be viewed as a portfolio of options. The individual options are called *floorlets*.

Caps are often used in conjunction with floors to create *collars*. The combination of a purchased cap and a written floor results in a collar for a borrower. For example, the borrower may purchase a cap at 12 percent and sell a floor at 8 percent. Purchasing a collar is sometimes viewed as a way for a borrower to reduce the cost of a cap by accepting the obligation to make payments if rates fall below the floor rate. A lender can also obtain a collar by purchasing a floor and simultaneously selling a cap (e.g., by purchasing a floor at 8% and selling a cap at 12%). Like the collared borrower, the collared lender views a collar as a way of reducing the cost of purchasing a floor. Collars can be bought and sold as one transaction from a collar dealer.

A popular collar strategy is the *zero-cost collar.* For the borrower, this strategy is implemented by first selecting the cap level (i.e., the strike price of the call) and purchasing the appropriate caps in the market. At the same time, the borrower sells the floor that generates the premium to pay for the cap. The floor in this case is a residual: It is not a matter of independent choice but falls out of the strategy once the cap rate has been determined.

Likewise, a lender can generate a zero-cost collar by first selecting the floor rate (the strike price of the put) and purchasing the appropriate number of floors to cover the loan exposure. Simultaneously, the lender sells the cap that generates the premium to pay for the floor. Zero-cost collars are popular because they require no cash outlay.

FOREIGN CURRENCY OPTIONS

The many different kinds of futures and options come together in the foreign currency market, because it is only in foreign exchange that all four kinds of speculative contracts discussed in the past two chapters are traded. In this section, we focus on the euro FX, where we find:

- An option on the euro itself.
- A forward contract on euro FX.
- A euro FX futures contract.
- An option on the euro FX futures contract.

By now, the reader will suspect that there are likely to be law-like relationships among the prices of these diverse instruments. In fact, we observed the relationships between futures and forward prices. For the sake of convenience, assume that the futures and options contracts are all written for one million euros and that all prices are quoted in U.S. dollars per euro. We also assume that both options and the futures contract expire in four months, and that the risk-free interest rate is 1 percent per month.

To illustrate the pricing relationships between futures and options, consider a portfolio constructed by buying one call option and selling one put option, each with an exercise price of $.31 and the same expiration date in four months. The value of this option portfolio at expiration depends on the value of the euro at the time of expiration. If the euro is worth $.31 or less at expiration, the call expires worthless. For values of the euro above $.31, the call option increases in value. The short put position has no value unless the euro is worth less than \$.31 at expiration. However, for each cent the euro falls below \$.31 at expiration, there is a one-cent contribution to profit on the short put position. Considering the entire portfolio, there will be a positive value to the long call/short put portfolio if the euro lies above \$.31 when the options expire. However, if the spot price of the euro is worth less than \$.31 at expiration, the portfolio will have a negative value. So far, we have focused on the terminal value of the portfolio and have ignored the cost of the combined call and put portfolio.

Now we need to compare the terminal value of this two-option portfolio with the futures contract. In Figure 4.10, we assume that a futures contract is available at a price of \$.33. There is no cost for entering a futures contract, except for the transaction costs that we ignore. Therefore, the line for the futures contract in Figure 4.10 shows the profit or loss that will be realized on the futures at its expiration, which we assume is the same expiration date that prevailed for the options.

As the graph shows, no matter what the value of the euro might be at expiration, the option portfolio will have a value that exceeds the profit or loss for the futures. For example, if the euro is at \$.33 at expiration, the futures contract will have no value, but the option portfolio will be worth \$.02 per euro. If the euro is worth \$.31 at the expiration date in March, the options portfolio will be worthless, but the futures contract will show a loss of \$.02 per euro. No matter what the euro is worth at expiration, the options portfolio will perform \$.02 better than the futures contract.

Since the options portfolio will always perform better, it must be priced higher than the futures contract. Otherwise, all traders would prefer the options portfolio. In fact, since we know exactly how much better the options

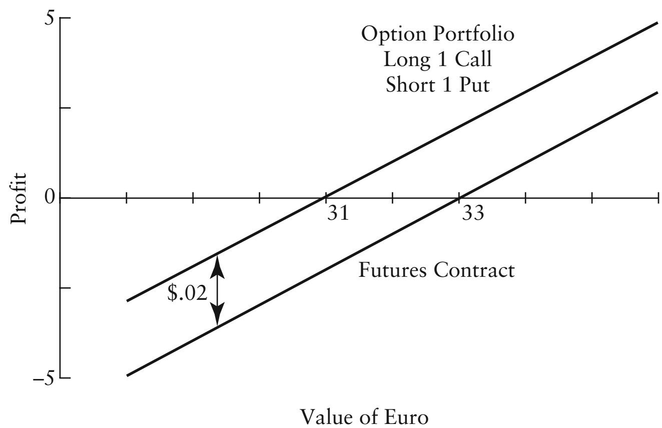

FIGURE 4.10 Currency futures and option portfolios profits and losses.

portfolio will perform at expiration, we can calculate how much more the options portfolio will be worth than the futures contract. If the current price of the futures contract is \$.33, it costs nothing to enter the futures contract at that price. The options portfolio is certain to pay off \$.02 per euro better than the futures contract at expiration, so it has a positive value. This must be the case since the futures contract costs nothing to enter.

How much will the options portfolio be worth? Using our assumption of the contract size being one million euros, what will be the difference in the dollar payoffs between the futures position and the options portfolio at maturity? With a payoff difference of \$.02 per euro, the difference in the payoffs between the futures and options portfolio must be a total of \$20,000, because we are assuming a contract size of one million euros. In mathematical terms, the difference in the payoffs can be represented as:

$$F - E = \$20{,}000$$

where E = \$310,000, the exercise price for a contract on 1,000,000 euros
F = \$330,000, the total futures price for 1,000,000 euros

By acquiring the options portfolio rather than the futures contract, the investor is certain to receive \$20,000 more at expiration than the futures contract holder will receive. Because this \$20,000 incremental payoff is certain, and because the price of the futures contract is zero, the holder of the options portfolio must be willing to pay the present value of that future payoff. The options portfolio must be worth the present value of the \$20,000 that will be received in June. That means the price of the options portfolio must equal the present value of the difference between the futures price and the exercise price on the options:

$$C - P = \frac{F - E}{\left(1 + R_f\right)^T} \tag{4.17}$$

In our example, the difference $F - E$ is \$20,000, so the cost of the options portfolio must be the present value of \$20,000 discounted at the risk-free rate. Assuming that the interest rate is 1 percent per month and that the expiration is four months away, the total cost of the options portfolio should be $\$20{,}000/(1.01)^4$ or \$19,220. This gives us a technique for establishing a relationship between futures and option prices.[24]

Another way to see the same point is to assume that a trader makes the following transactions:

Buy 1 call.

Sell 1 put.

Sell 1 futures.

We now consider the payoffs on this three-asset portfolio for different values of the euro at the expiration of the three instruments. First, if the euro is worth \$.31, the two options are both worthless and a short futures position is worth \$.02, so the total portfolio is worth \$.02. If the euro is worth \$.33 at expiration, the futures profit is zero, the short put expires worthless, and the long call is worth \$.02, for a total portfolio value of \$.02. In fact, for any value of the euro, the portfolio is worth \$.02. Therefore, a long call/short put/short futures simulates a risk-free pure discount instrument that pays the futures contract price less the common option exercise price. It may seem surprising that options and futures, instruments known for their riskiness, can be combined to create a risk-free investment. Chapter 5, however, explores combinations of instruments that can be created to tailor risk positions in an amazing variety of ways.

SUMMARY

This chapter presented an overview of the options market in the United States. Option trading on organized exchanges began in 1973 with the introduction of options on individual stocks. Since that time, option markets in the United States have expanded greatly with options on metals, stock and other indexes, foreign currencies, and futures contracts.

Options can be classified as put or call options, each of which may be bought or sold. Ownership of a call option confers the right to buy a given good at a specified price for a specified period of time. Selling a call option confers those same rights to the owner of a call option in exchange for a payment from the call option purchaser. Ownership of a put option permits the sale of a good at a specified price for a specified period of time. Selling a put option gives those rights to the buyer in exchange for a payment from the buyer.

The theory of option pricing is well developed. Starting merely from the assumption that options should be priced in a way that allows no arbitrage opportunities, it is possible to bound option prices very closely. Using the no-arbitrage condition, it can be shown that call option prices are a function of the stock price, the exercise price of the option, the time to expiration, the interest rate, and the risk level of the good underlying the option. Additionally, Black and Scholes developed an option pricing model that gives an exact price for a call option as a function of the same five variables.

Although this is a theoretical model, it accords very well with option prices that are actually observed in the market.

QUESTIONS AND PROBLEMS

1. Respond to the following claim: "Buying a call option is very dangerous because it commits the owner to purchasing a stock at a later date. At that time, the stock may be undesirable. Therefore, owning a call option is a risky position."

2. "I bought a call option with an exercise price of $110 on IBM when IBM was at $108 and I paid $6 per share for the option. Now the option is about to expire and IBM is trading at $112. There's no point in exercising the option because I will wind up paying a total of $116 for the shares—$6 I already spent for the option plus the $110 exercise price." Is this line of reasoning correct? Explain.

3. What is the value of a call option on a share of stock if the exercise price of the call is $0 and its expiration date is infinite? Explain.

4. Why is the value of a call option at expiration equal to the maximum of zero or the stock price minus the exercise price?

5. Two call options on the same stock have the following features: The first has an exercise price of $60, a time to expriation of three months, and a premium of $5. The second has an exercise price of $60, a time to expiration of six months, and a premium of $4. What should you do in this situation? Explain exactly, assuming that you transact for just one option. What is your profit of loss at the expiration of the nearby option if the stock is at $55, $60, or $65?

6. Two call options are identical except that they are written on two different stocks with different risk levels. Which will be worth more? Why?

7. Explain why owning a bond is like taking a short position in a put option.

8. Why does ownership of a convertible bond have features of a call option?

9. Assume tht following: A stock is selling for $100, a call option with an exercise price of $90 is trading for $6 and matures in one month, and

the interest rate is 1 percent per month. What should you do? Explain your transactions.

10. Consider a euro futures contract with a current price of $.35 per euro. There are also put and call options on the euro with the same expiration date in three months that happen to have a striking price of $.35. You buy a call and sell a put. How much should your combined option position cost? Explain. What if the interest rate were 1 percent per month and the striking prices were $.40? How much should the option position be worth then?

11. Two call options on the same stock expire in two months. One has an exercise price of $55 and a price of $5. The other has an exercise price of $50 and a price of $4. What transactions would you make to exploit this situation?

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CHAPTER 5

Risk Management with Options Contracts

In Chapter 4, we used the option pricing model (OPM) to value options. In this chapter, we show how the OPM can also guide portfolio management decisions. The OPM permits precise estimates of meaningful portfolio risk measures that can be used to characterize the portfolio's exposure to underlying risk factors. These measures can also be helpful in analytically determining the impact of portfolio management decisions on the portfolio's risk characteristics. This chapter provides examples of option contracts being used to hedge portfolio risks. Speculation strategies to take advantage of perceived profit opportunities are also considered.

THE OPTION PRICING MODEL AND THE MEASUREMENT OF PORTFOLIO RISK EXPOSURE

The option pricing model (OPM) shows how option prices are related to the key underlying variables S, E, T, σ, and R_f. In Chapter 4, we plugged values for these key variables into the model to determine the option's price. Although valuing options is an important application of the OPM, an equally important application is measuring portfolio risk exposures. The OPM isolates the independent effect of each underlying variable on the option's value. These isolated, independent effects measure the sensitivity of the option's value to changes in the key underlying variables. In the jargon of the derivatives industry, these sensitivity measures are called *the Greeks,* because Greek letters are used to denote them. We derive the measures by applying basic calculus tools to the OPM. In particular, we take the first derivative of the OPM with respect to each of the underlying variables while holding the value of the other variables constant. These measures are brought to life by evaluating each derivative using particular values of interest.

Table 5.1 describes each of the Greeks in terms of the OPM. *Delta* describes how sensitive the option value is to changes in the underlying stock price. For calls, delta is equal to $N(d_1)$ from equation 4.11 and takes on values between 0 and 1. For puts, delta is equal to $N(d_1) - 1$ and takes on values between 0 and –1. For deep-in-the-money calls, the call value moves one for one with the stock price implying that delta equals one. For deep-out-of-the-money calls, the call value hardly changes at all as the stock price moves implying a delta of zero. For options that are near the money, the delta value is approximately one-half. For deep-in-the-money puts, the put price increases one dollar for each one dollar decline in the stock price implying a delta of minus one. For a deep out-of-the-money put, the put price does not change at all when the stock price changes implying a delta of zero. And for near-the-money puts, the delta value is approximately minus one-half.

Suppose that the delta on a call is .8944. What does this mean? It means two things. First, when the underlying asset is a stock, it means that if the stock price rises by $1, then the call price rises by $.8944. Second, it implies a hedge ratio between stock and stock options that will leave a portfolio consisting of stock and options invariant to (small) changes in the underlying stock price. For example, a delta of .8944 means that one short call option for every .8944 shares of stock will hedge a portfolio consisting of this pair against small changes in the underlying stock price.

Vega, sometimes called *kappa*, measures the change in an option's value due to changes in volatility. Vega is the first derivative of an option's price

TABLE 5.1 Option Sensitivities (i.e, The Greeks) for the OPM

$\frac{\text{Change in option price}}{\text{Change in stock price}}$	Delta
$\frac{\text{Change in option price}}{\text{Change in volatilty}}$	Vega
$\frac{\text{Change in option price}}{\text{Change in time to expiration}}$	Theta
$\frac{\text{Change in option price}}{\text{Change in interest rate}}$	Rho
$\frac{\text{Change in Delta}}{\text{Change in stock price}}$	Gamma

with respect to the volatility of the underlying stock. Vega provides an analytical measure of what we already know intuitively—that option prices, either calls or puts, are more valuable the greater the volatility of the underlying stock. Vega is always positive. In general, option prices are extremely sensitive to changes in volatility. This means that vega is an important determinant of option prices.

Theta, sometimes called *tau,* is the term used to measure the *time decay* of an option, or a portfolio that includes options. One characteristic of options is that their value changes merely with the passage of time. Moreover, this change in value is not linear with respect to time. Option values decline at an accelerating rate as they approach expiration. Theta provides an exact analytical tool for estimating this effect on option value. Theta is almost always negative meaning that as the option's expiration draws near, the option price falls. This is true for both puts and calls. If time is measured in years, and value in dollars, then a theta value of –10 means that as time to option expiration declines by .1 years, option value falls by $1.

Rho measures the sensitivity of an option's value to changes in the interest rates. It is the first derivative of an option's price with respect to the interest rate. Rho for calls is always positive, whereas rho for puts is always negative. A rho of 25 means that a 1 percent increase in the interest rate would increase the value of a call by $.25 and decrease the value of a put by $.25. In general, option prices are not very sensitive to changes in interest rates.

Another exposure measure, *gamma,* measures the changes to delta resulting from changes in the stock price (i.e., the sensitivity of delta to changes in the stock price). Unlike the other sensitivity measures we have considered, gamma does not measure the sensitivity of an option to one of the underlying variables. In terms of calculus, gamma is the first derivative of delta with respect to the stock price, or equivalently, it is the second derivative, evaluated for particular values, of the OPM with respect to the stock price. Gamma can be either positive or negative depending on whether delta is increasing or decreasing for given changes in the underlying stock price.

HEDGING WITH OPTIONS

As we have seen with futures, risky financial instruments can be used to control risk. One of the most important applications of options is as a hedging vehicle. The option sensitivity measures provide important insights into constructing portfolio hedges with options. The option sensitivity measures not only characterize an individual option, but can also characterize the risk exposure of a portfolio that includes options and other assets. Because we know how individual options contribute to a portfolio's overall risk, we can determine analytically what adjustments are necessary to hedge the risk exposure of the portfolio.

For example, suppose we want our portfolio to be *delta neutral,* that is, to be hedged against changes in the value of the underlying asset. Another way to say this is that we want the delta of the portfolio to be zero. Consider a portfolio consisting of a long position in a nondividend stock and a short position in a European call written on this stock. The sensitivity measures for our call option from Chapter 4 are shown in Table 5.2. We use these sensitivity measures to choose the weights for the stock and call that will leave the portfolio delta neutral. With a current stock price of \$100, the call price is \$11.84 as determined by the OPM. Of the sensitivity measures displayed in Table 5.2, notice in particular that delta is evaluated to be .8944 for this call. This tells us that we must buy .8944 shares of stock for each option sold to produce a delta-neutral portfolio.

The value of such a portfolio, V_{port}, is as follows:

$$V_{\text{port}} = -\$11.84 + .8944(\$100.00) = \$77.60$$

Now suppose the stock price moves up from \$100 to \$100.10. The value of the portfolio is now:

$$V_{\text{port}} = -\$11.93 + .8944(\$100.10) = \$77.60$$

The value of the portfolio does not change. So for at least a small price change of \$.10 per share, the portfolio appears to be insensitive to changes in the underlying stock price. For larger stock price changes, the hedge still

TABLE 5.2 Option Sensitivities for Example

Option price	\$11.84
Delta	.8944
Vega	18.2649
Theta	−10.2251
Rho	77.5992
Gamma	.0183

Note: S = 100; E = 100; T = 365 days; $\sigma = .10$; $R_f = .12$; $\delta = 0$.

works but not quite as well as for smaller changes. For example, a \$10 per share increase in the stock price produces the following portfolio value:

$$V_{\text{port}} = -\$21.36 + .8944(\$110.00) = \$77.02$$

The \$10 change in the price of the stock produces a change of \$.58 in the value of the portfolio.

This portfolio is delta neutral because small changes in the price of the stock do not affect the value of the portfolio. Put another way, the value of the portfolio is insensitive to changes in the value of the stock. For larger changes in the stock price, the portfolio value changes, as we have just seen. This happens because the value of delta itself changes as the stock price changes. Even if we adjust the portfolio so that it is delta neutral according to the formula, if the stock price changes, delta can change, foiling our efforts. This property is captured by the sensitivity measure, gamma. Moreover, the value of delta changes when other factors change, such as volatility and time to option expiration. To maintain a delta-neutral portfolio, the portfolio weights must be continually adjusted as delta changes. By continuously rebalancing our portfolio as delta changes, we can create a risk-free portfolio. Of course, if the portfolio is risk free, we should expect it to earn the risk-free rate of return.

Our example really does not demonstrate how to set up a delta-neutral portfolio. It merely demonstrates the results of a delta-neutral portfolio after it has been set up. To set up the portfolio, we must first set up an expression involving the portfolio elements. This expression allows us to solve explicitly for the weights of these elements. Consider the following setup:

$$N_S(\text{delta}_S) + N_c(\text{delta}_c) = 0$$

where N_S is the number of shares of stock, delta_S is the delta of the stock, N_c is the number of calls bought or sold, and delta_c is the delta of the call. The left-hand side of the expression is set equal to zero, reflecting our desire to construct a delta-neutral portfolio. We can use this setup to solve explicitly for the number of shares to be held if we sell one call option. Being short one call is equivalent to saying that $N_c = -1$. We also know that, by construction, the delta of a share of stock must be one; that is, the change in the price of a stock due to its own price change must be one. We know from Table 5.2 that delta_c is .8944. Now we have an expression consisting of one equation and one unknown:

$$N_S(1) + (-1)(.8944) = 0$$

Solving for N_S we get .8944, which is consistent with the information used in our example of a delta-neutral portfolio.

The concept of a delta-neutral portfolio can be extended to other option sensitivity measures. Just as we created delta-neutral portfolios, we can create portfolios that are insensitive (i.e., hedged) with respect to any option variable. Table 5.3 provides information on two calls, denoted C_1 and C_2, that are identical in every respect except that the exercise price is \$100 for C_1 and \$110 for C_2. We can use these options to create a portfolio that is both delta neutral and gamma neutral. This example shows how the approach can be extended to creating portfolios that are customized with respect to their exposure to the risks resulting from changes in any underlying variable.

Creating a neutral portfolio involves a little bit of algebra, but other than that it is straightforward. In this example, the objective is to create a portfolio that is simultaneously delta neutral and gamma neutral. First, recall that the delta of a stock is, by construction, always equal to one. This implies that the gamma of a stock is zero since delta never varies from one no matter what happens to the stock price. To extend the setup expression used earlier so that it includes gamma neutrality, we need to add another option to the mix. This option provides an extra degree of freedom to play with in solving for the portfolio weights. Now we will have a stock and two options, whereas we had only a stock and a single option to construct a delta-neutral portfolio:

$$N_S\left(\text{delta}_S\right) + N_1\left(\text{delta}_{c1}\right) + N_2\left(\text{delta}_{c2}\right) = 0$$

TABLE 5.3 Sample Options

	C_1 E = \$100	C_2 E = \$110
Price	10.30	6.06
Delta	.6151	.4365
Vega	26.8416	27.6602
Theta	−12.2607	−11.4208
Rho	25.2515	27.6602
Gamma	.0181	.0187

S = \$100; T = 180 days; $R_f = .08$; $\sigma = .3$; $\delta = 0$.

To this expression, we will add another that gives the gamma neutral condition:

$$N_S(\text{gamma}_S) + N_1(\text{gamma}_{c1}) + N_2(\text{gamma}_{c2}) = 0$$

Assuming that we are holding one share of stock long (i.e., $N_S = 1$), we now need to determine the positions we must make in options C_1 and C_2 to satisfy the conditions previously stated. This will be easy because we now have two equations and two unknowns; thus, we know even before we start that there will be only one correct answer. The next step is to plug in the values for our share of stock and our two options. Referring to Table 5.3, we see that the deltas for the two options are .6151 and .4365, respectively. We also see that the corresponding value for the gammas are .0181 and .0187. We have already determined that the delta on a share of stock, delta_S is 1 and that gamma_S is 0. The resulting two-equation system looks like:

$$1(1) + N_1(.6151) + N_2(.4365) = 0$$
$$1(0) + N_1(.0181) + N_2(.0187) = 0$$

By premultiplying the bottom expression by (.6151/.0181), the resulting system looks like:

$$1 + N_1(.6151) + N_2(.4365) = 0$$
$$N_1(.6151) + N_2(.63549) = 0$$

Since both expressions are equal to zero, then they must be equal to each other. We can now write our system of two equations with two unknowns as one equation with one unknown:

$$1 + N_2(.4365) = N_2(.63549)$$

which means that $N_2 = (1/.19899) = 5.025377$. In other words, buy 5.025377 calls with an exercise price of \$110. Now that we have solved for N_2, we can easily solve for N_1 by going back to the original system of equations and plugging in N_2 as 5.025377 and doing the algebra to determine N_1:

$$1 + N_1(.6151) + (5.025377)(.4365) = 0$$
$$N_1(.0181) + (5.025377)(.0187) = 0$$

Using either the top or bottom expression (or both) we find that $N_1 = -5.191964$. The negative sign denotes a short position, indicating that we should sell 5.191964 calls with an exercise price of \$100. Alternatively, to hedge a long position of one share in a stock, sell a number of options equal to $1/N(d_1)$. This hedge will hold for small changes in the stock price. This general approach can be used to construct portfolios with virtually any conceivable risk exposure characteristics.

PORTFOLIO INSURANCE WITH OPTIONS

In portfolio insurance, a trader transacts to ensure that the value of a portfolio cannot fall below a given amount. By combining a long position in a put option with a long position in the underlying asset, a trader ensures that the value of the combined portfolio cannot fall below a given level. An outright investment in a portfolio can be transformed into an investment with a completely different risk profile through this technique. This section begins by explaining the rationale and strategy of portfolio insurance through a detailed case analysis using options. As described in Chapter 8, portfolio insurance can also be implemented synthetically by using futures contracts in a strategy called dynamic hedging.

Case Analysis

In this section, we consider how to use options to tailor the risk of an investment. For convenience, we focus on payoffs at option expirations, so we can ignore the difference between American and European options. Because the analysis focuses on European options, the conclusions apply to both futures options and options on assets.

Although we present the case analysis for a stock index, the conclusions apply to many different instruments. Consider a stock index that is currently at \$100. Stocks in the index pay no dividends, and the expected return on the index is 10 percent, with a standard deviation of 20 percent. A put option on the index with an exercise price of \$100 is available and costs \$4. We consider three investment strategies:

Portfolio A: Buy the index; total investment \$100.
Portfolio B: Buy the index and one half of a put; total investment \$102.
Portfolio C: Buy the index and one put; total investment \$104.

At expiration in one year, the profits and losses associated with these three portfolios depend entirely on the value of the index because the value of the put at expiration also depends strictly on the index value. For the put, the value at expiration equals the maximum of either zero or the exercise price minus the index value.

At expiration, the three portfolios will have profits and losses computed according to the following equations:

$$\text{Portfolio A: Index Value} - \$100$$

$$\text{Portfolio B: Index Value} + .5\text{MAX}\{0, \$100 - \text{Index Value}\} - \$102$$

$$\text{Portfolio C: Index Value} + \text{MAX}\{0, \$100 - \text{Index Value}\} - \$104$$

The value of Portfolio A at expiration is just the index value, and the profit or loss is the value of the portfolio at expiration less the investment of $100. The terminal value of Portfolio C is the index value plus the value of the put. The profit or loss is the terminal value less the investment of $104. Portfolio B consists of the index plus one half of a put. This gives a total investment of $102, and the terminal value of Portfolio B consists of the index value plus the value of the half put. Figure 5.1 graphs the profits and losses of these three portfolios for different terminal index values.

Of particular interest in Figure 5.1 is the profit-and-loss graph for Portfolio C, consisting of the index plus a put on the index. The worst possible loss on Portfolio C is $4. This loss occurs if the terminal index value is $100

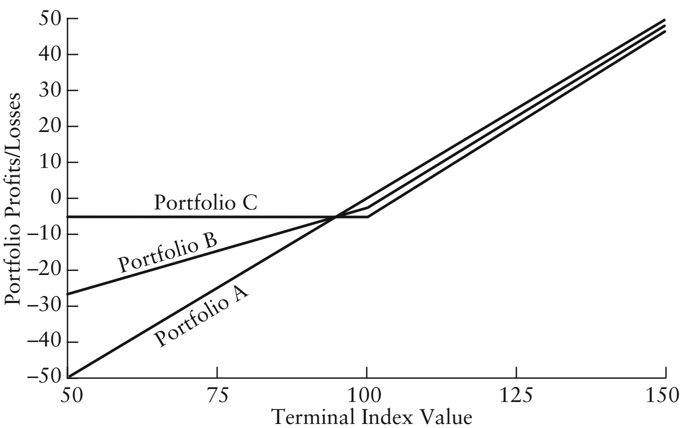

FIGURE 5.1 Profits and losses on three portfolios.

or below. With a terminal index value of $100, the portfolio is worth $100 because the put expires worthless. This is the worst possible loss, however. For example, if the terminal index value is $95, the put is worth $5 and the index investment is worth $95, for a total of $100. Portfolio C must always be worth at least $100.

Portfolio C is an insured portfolio, whose value cannot fall below $100. Further, this example is the classic case of portfolio insurance: buying a good at a given price and buying a put on the same good with an exercise price equal to the purchase price of the good. To create Portfolio C, a trader bought the index at $100 and bought an index put with an exercise price of $100.

Portfolio Insurance and Put-Call Parity

Figure 5.1 shows that the insured portfolio's profit-and-loss profile is exactly the profile for a call option on the stock index. This should not be surprising. Earlier we saw that a long position in the underlying good plus a long put would have the same profits and losses as a call. Applying the put-call parity equation to our index example, we have:

$$C = \text{INDEX} + P - \frac{E}{\left(1 + R_f\right)^T} \tag{5.1}$$

The put-call parity equation shows that an instrument with the same value and profits and losses as a call can be created by holding a long put and a long index, and borrowing the present value of the exercise price. The portfolio on the right-hand side of equation 5.1 will have the same value and same profits and losses as the call. By contrast, the long index plus long put merely have the same profits and losses as the call. At expiration, the value of the put plus the index will exceed the value of the call.

Therefore, an insured portfolio is the long put/long index position that has the same profits and losses as a call. From put-call parity, there is another way to create a portfolio that exactly mimics the insured portfolio's value at expiration. We can hold a long call plus invest the present value of the exercise price in the risk-free asset. From the put-call parity relationship, we see:

$$C + \frac{E}{\left(1 + R_f\right)^T} = P + \text{INDEX} \tag{5.2}$$

The long call plus investment in the risk-free asset creates the same insured portfolio as the long index plus long put. Both positions have the same value and the same profits and losses at expiration.

This also shows why the insured portfolio that has the same profits and losses as a call does not have the same value as the call. The insured portfolio requires considerable investment to purchase the underlying index.

TAILORING RISK AND RETURN CHARACTERISTICS WITH FUTURES AND OPTIONS

To this point, we have not explicitly considered Portfolio B as previously defined. Portfolio B consists of buying the index and buying a half put. In essence, Portfolio B is half insured. Expressed differently, Portfolio B consists of two equal portions: $50 in an insured portfolio, plus $50 in an outright position in the index. As Figure 5.1 shows, Portfolio B has profits and losses that fall between the totally insured and completely uninsured portfolios.

The partially insured Portfolio B has less risk than the uninsured Portfolio A, but it has more risk than the fully insured Portfolio C. Figure 5.1 shows this intermediate risk position: The losses for the half-insured Portfolio B are less than the losses for the uninsured Portfolio A, but more than the losses for the fully insured Portfolio C. This example suggests that traders can use futures and options to tailor the risk characteristics of the portfolio to individual taste. With the available futures and option instruments, the financial engineer can create almost any feasible combination of risk and return.

One of the dominant lessons of modern finance concerns the risk/expected return trade-off. In well-functioning markets, finding the chance for higher returns always means accepting higher risk. Comparing the fully and partially insured portfolios with the uninsured portfolio shows that portfolio insurance reduces risk. However, a reduction in expected return accompanies the reduction in risk.

RISK AND RETURN IN INSURED PORTFOLIOS

We now explore the risk and expected return characteristics for Portfolios A–C. The portfolios have different probabilities of achieving given terminal values that depend on the price of the index at expiration. Likewise, the probability of achieving a given return on the portfolios depends on the index value at expiration. We explore these issues by assuming that returns on the index follow a normal distribution with a mean of 10 percent and a standard deviation of 20 percent.

Terminal Values for Portfolios A–C

The portfolio values at expiration depend on the price of the index at expiration. For each, the terminal value is:

$$\text{Portfolio A} = \text{Index}$$

$$\text{Portfolio B} = \text{Index} + \text{MAX}\{0, .5(100.00 - \text{Index})\}$$

$$\text{Portfolio C} = \text{Index} + \text{MAX}\{0, 100.00 - \text{Index}\}$$

We can now answer such questions as, What is the probability that Portfolio C will have a terminal value equal to or less than \$100? Portfolio C will have a terminal value of at least \$100 no matter what the value of the underlying index. In fact, there is a 30.85 percent probability that Portfolio C will have a terminal value of exactly \$100. Portfolio C is worth \$100 at expiration if the index is \$100 or less at expiration, and there is a 30.85 percent chance that the index value will be \$100 or less. What is the probability that Portfolio A will have a terminal value less than \$90? The probability that the terminal value of Portfolio A will lie below \$90 is the probability that the terminal index value will fall more than 1.0 standard deviation below its expected value. Because we assume the returns on the index are normally distributed, there is a 15.87 percent chance that Portfolio A's value will be less than \$90 at expiration. Table 5.4 shows some portfolio values and the probabilities

TABLE 5.4 Probability that the Terminal Portfolio Value Will Be Equal to or Less than a Specified Value

	Probabilities		
Terminal Portfolio Value	Uninsured Portfolio A	Half Insured Portfolio B	Fully Insured Portfolio C
50.00	0.0014	0.0000	0.0000
60.00	0.0062	0.0000	0.0000
70.00	0.0228	0.0002	0.0000
80.00	0.0668	0.0062	0.0000
90.00	0.1587	0.0668	0.0000
100.00	0.3085	0.3085	0.3085
110.00	0.5000	0.5000	0.5000
120.00	0.6915	0.6915	0.6915
130.00	0.8413	0.8413	0.8413
140.00	0.9332	0.9332	0.9332
150.00	0.9773	0.9773	0.9773
160.00	0.9938	0.9938	0.9938
170.00	0.9987	0.9987	0.9987

that each portfolio will be equal to or less than the given terminal value at the expiration date.

In Table 5.4, the uninsured Portfolio A has the largest chance of an extremely low terminal value; the chance that Portfolio A will be worth $80 or less is 6.68 percent. For Portfolio B, the chance of such an unhappy outcome is less than 1 percent, and there is no chance that Portfolio C could be worth $80 or less. (We already know that Portfolio C has to be worth at least $100.) It is interesting to note in Table 5.4 that the chance of each portfolio being worth $100 or less is the same—30.85 percent. Likewise, there is a 50 percent chance for each portfolio that the portfolio's value will be $110 or less. In fact, for terminal portfolio values at or above $100, the three portfolios have exactly the same probabilities. This makes sense, because if the terminal index value is $100 or more, the put option has zero value, and the remaining portion of each portfolio is the same.

Figure 5.2 graphs terminal portfolio values from $50 to $170 and shows the probability for each portfolio that the terminal portfolio value will be below or equal to the given amount. The three probability graphs differ for terminal portfolio values below $100. However, for all terminal portfolio values at or above $100, the graphs are identical. This matches the values in Table 5.4.

Concentrating only on terminal values, and neglecting the different investments required to obtain each portfolio, Figure 5.2 shows that the fully insured portfolio is the most desirable, followed by the half-insured portfolio, and then the uninsured portfolio. If we could choose one of these three

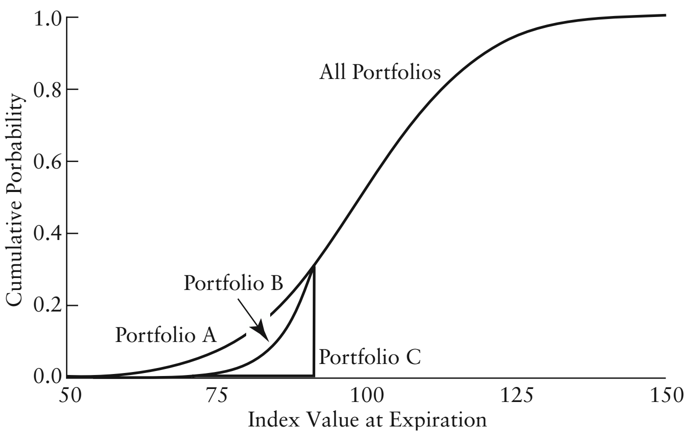

FIGURE 5.2 Probabilities that terminal values of portfolios A–C will be equal to or less than a given amount.

portfolios as a gift, the fully insured portfolio is the clear choice. No matter what the terminal index value is, the fully insured Portfolio C will pay at least as much as either Portfolio A or B. If the terminal index value is less than $100, the insured portfolio still pays $100, which is more than either Portfolio A or B. However, this conclusion neglects the different investment costs. Portfolio A costs only $100, whereas Portfolio B costs $102 and Portfolio C costs $104.

Returns on Portfolios A–C

Because Portfolios A–C have different costs, we need to compare the returns on each portfolio to make them more directly comparable. Portfolio C is preferable to Portfolios A or B if we neglect cost, but once we consider it, the answer is much less clear. Instead of having a clear choice, the investor faces the risk/expected return trade-off in portfolio insurance.

For each portfolio, we can evaluate the chance of a given return. For example, the lowest possible terminal value for the fully insured portfolio is $100, which implies a return of (100/|104) – 1 = –0.0385. The chance of a return on Portfolio C below –0.0385 is zero. However, the chance of Portfolio C having a return of exactly –0.0385 is 30.85 percent, the chance that Portfolio C is worth $100 at expiration.

Table 5.5 shows the probability that each portfolio will achieve a return greater than a specified return. For example, there is an 84.13 percent

TABLE 5.5 Probability of Achieving a Return Equal to or Greater than a Specified Return

	Probabilities		
Portfolio Return	Uninsured Portfolio A	Half Insured Portfolio B	Fully Insured Portfolio C
–0.5000	0.9987	1.0000	1.0000
–0.4000	0.9938	1.0000	1.0000
–0.3000	0.9773	0.9996	1.0000
–0.2000	0.9332	0.9904	1.0000
–0.1000	0.8413	0.9066	1.0000
0.0000	0.6915	0.6554	0.6179
0.1000	0.5000	0.4562	0.4129
0.2000	0.3085	0.2676	0.2297
0.3000	0.1587	0.1292	0.1038
0.4000	0.0668	0.0505	0.0375
0.5000	0.0228	0.0158	0.0107

probability that the uninsured Portfolio A will do better than –10 percent. The half-insured Portfolio B has a 90.66 percent chance of returning at least –10 percent. For fully insured Portfolio C, there is no chance the return could be as bad as –10 percent.

So far, everything still looks good for the insured portfolios. It appears as if the greater the level of insurance, the better the portfolio performs. However, we must now consider other possible returns. What is the probability of no gain or a loss? For the uninsured Portfolio A, there is a 30.85 percent chance of a loss. The fully insured Portfolio C, however, stands a 38.21 percent chance of a zero gain or a loss. Similarly, let us consider the chances of gaining more than 10 percent. The uninsured portfolio has a 50 percent chance because there is a 50 percent chance the terminal index value will exceed the expected value of $110. The insured portfolio has only a 41.29 percent chance of beating a 10 percent return.

Now we can see the risk/expected return trade-off implied by portfolio insurance strategies. Portfolio insurance protects against large losses by sacrificing the chance for large gains. Thus, portfolio insurance is aptly named. With any insurance contract, the insured pays the insurance premium to insure against some unpleasant event. By paying the insurance, the insured knows that the expected return on the portfolio will be less than it would be without insurance, but the insured hopes to avoid the extreme loss.

Figure 5.3 graphs the probabilities for each portfolio for the range of returns from –50 percent to 50 percent. Each point in the graph shows the

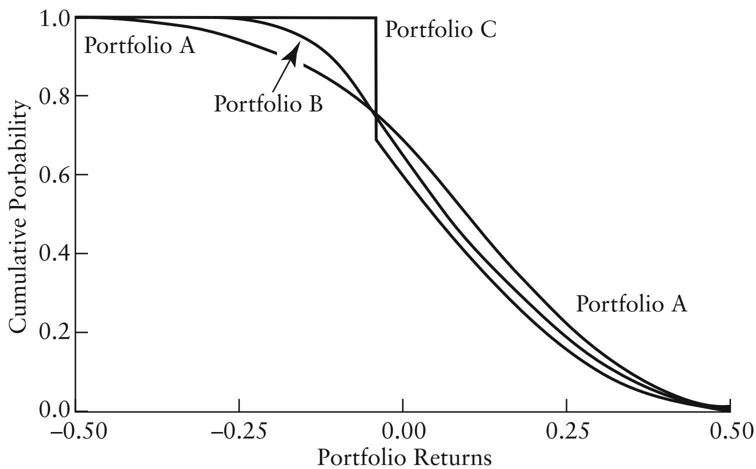

FIGURE 5.3 Cumulative probability of returns for portfolios A–C exceeding a given value.

probability that a portfolio will have returns greater than the return specified on the X axis. For example, consider the returns in the range of –15 percent. There is a 100 percent chance that the fully insured Portfolio C will beat a –15 percent return. Also, Portfolio C has a 100 percent chance of beating any return up to –3.846 percent. The chance of doing better than –3.846 percent, however, is only 61.15 percent. Similarly, the half-insured Portfolio B has a very good chance of beating –15 percent. Portfolio A has the lowest chance of beating –15 percent.

As noted from Table 5.5, however, the fortunes of the portfolios turn when we consider the probability of particularly favorable outcomes. The probability of exceeding a 20 percent return is 30.85 percent for Portfolio A, but only 26.76 percent for Portfolio B and only 22.97 percent for Portfolio C. Thus, the uninsured Portfolio A has the biggest chance of big gains and big losses. By comparison, the fully insured Portfolio C gives up the chance for big gains to avoid the chance of large losses. The half-insured Portfolio B occupies the middle ground.

SPECULATING WITH OPTIONS

For a potential speculator in options, pricing relationships between options are of the greatest importance. As in the futures market, much option speculation relies on spreading techniques that involve trading two or more related options to create a single position. This section examines some of these speculative strategies.

Many option traders are attracted to the market by the exciting speculative opportunities that options offer. Relative to stocks, options offer a great deal of leverage. This leverage means that trading options can give investors much more price action for a given investment than simply holding the stock. It also means that options can be much riskier than holding stock. Although options can be risky as investments, they need not be. By using options in combinations, traders can take low-risk speculative positions. We explore techniques for combining options to generate profit profiles that are not available with positions in single options. These include strategies with rather colorful names: straddles, strangles, bull and bear spreads, and butterfly spreads.

Option Combinations

The reader of the popular financial press almost surely would gather that options are risky instruments, and this is partially correct. However, traders can combine them in certain ways to create positions with almost any desired level of risk exposure. In this section, we explore techniques for combining options to create new payoff profiles.

Straddles

A straddle is an option position involving a put and a call option on the same stock. To buy a straddle, an investor will buy both a put and a call that have the same expiration and the same striking price. To sell a straddle, a trader sells both the call and the put. Consider a put and a call option and assume that both have an exercise price of $100. Assume further that the call sells for $10 and that the put trades at $7. Table 5.6 shows the profits and losses for the call, the put, and the straddle as a function of the stock price at expiration. If the stock price equals the exercise price at expiration, both the put and the call expire worthless, and the loss on the straddle is $17, the entire premium paid for the position.

Any movement in the stock price away from $100 at expiration gives a better result. In fact, the value of the straddle increases $1 for every $1 movement in the stock price at expiration away from $100. The straddle position breaks even if the stock price either rises to $117 or falls to $83 (i.e., a $17 price movement away from the exercise price at expiration will cover the initial investment of $17). If the price of the stock differs greatly

TABLE 5.6 Profits and Losses for a Call, Put, and Straddle

	Elements of a Straddle		
Stock Price at Expiration	Call E = $100 P = $10	Put E = $100 P = $7	Straddle P = $17
$ 50	$–10	$43	$33
80	–10	13	3
83	–10	10	0
85	–10	8	–2
90	–10	3	–7
95	–10	–2	–12
100	–10	–7	–17
105	–5	–7	–12
110	0	–7	–7
115	5	–7	–2
117	7	–7	0
120	10	–7	3
150	40	–7	33

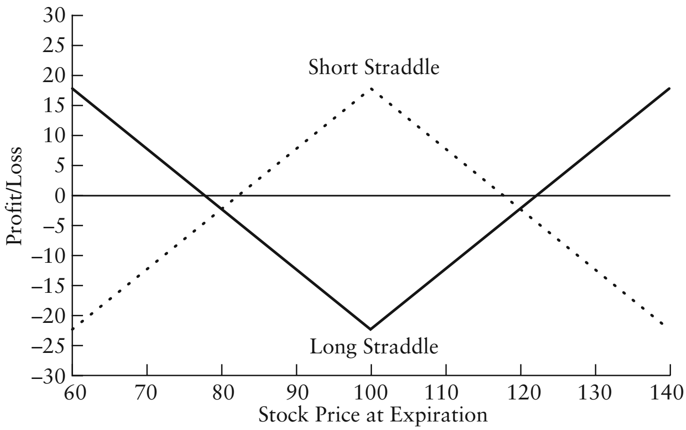

FIGURE 5.4 Profits and losses on a straddle.

from the exercise price, there is an opportunity for substantial profit. Figure 5.4 shows graphically the possible results for the long and short straddle positions.

The graph shows the profits and losses for buying the straddle position with a solid line. As this graph makes clear, the purchaser of a straddle is betting that the price of the stock will move dramatically away from the exercise price of $100. The owner of the straddle will profit if the stock price goes above $117 or below $83. Figure 5.4 shows the profit-and-loss position for the seller of a straddle with the dotted lines. The seller of the straddle will profit if the stock price at expiration lies between $83 and $117. The purchaser of this straddle would be making a bet on a large movement in the stock price in some direction, whereas the seller would be betting that the stock price remains reasonably close to the exercise price of $100.

STRANGLES

A strangle is similar to a straddle, which involves buying a call and buying a put option with the same striking price and the same term to expiration. A long position in a strangle consists of a long position in a call and a long position in a put on the same underlying good with the same term to expiration, with the call having a higher exercise price than the put. Consider the same put option of the previous example, which had an exercise price of $100 and a premium of $7. A call option on the same good with the same term to expiration has a striking price of $110 and sells for $3.

To buy a strangle with these options, a trader buys both the put and the call, for a total outlay of $10. Table 5.7 shows the profits and losses at expiration for the call and put individually, and on the strangle position as well. Figure 5.5 shows the profit profile for the long and short strangle. As the table and figure show, the put and the call cannot both have value at expiration. If the stock price rises above $110, the call has a value, whereas a stock price below $100 allows the put to finish in the money. For the long strangle to show a profit, the call or the put must be worth more than the $10 total cost of the strangle. This means that the stock price must exceed $120 or fall below $90 for the strangle to show a net profit.

The figure shows that a wide range of stock prices will give a loss, even a total loss of the $10 investment for some prices. For example, if the stock price is between $100 and $110 at expiration, both the put and the call will expire worthless, giving a net loss of $10.

Bull and Bear Spreads

A bull spread in the options market is a combination of call options designed to profit if the price of the underlying good rises.[1] Both calls in a bull spread have the same expiration, but they have different exercise prices. The buyer of a bull spread buys a call with an exercise price below the stock

TABLE 5.7 Profits and Losses for a Call, Put, and Strangle

	Elements of a Strangle		
Stock Price at Expiration	**Call E = $110 P = $3**	**Put E = $100 P = $7**	**Strangle P = $10**
$ 50	$–3	$43	$40
80	–3	13	10
83	–3	10	7
85	–3	8	5
90	–3	3	0
95	–3	–2	–5
100	–3	–7	–10
105	–3	–7	–10
110	–3	–7	–10
115	2	–7	–5
117	5	–7	–2
120	7	–7	0
125	12	–7	5
150	37	–7	30

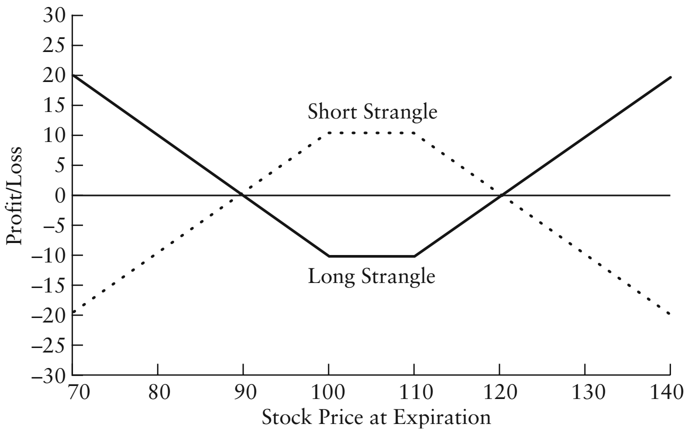

FIGURE 5.5 Profits and losses on a strangle.

price and sells a call option with an exercise price above the stock price. The spread is a "bull" spread because the trader hopes to profit from a price rise in the stock. The trade is a "spread" because it involves buying one option and selling a related option. Compared with buying the stock itself, the bull spread with call options limits the trader's risk. However, it also limits the profit potential compared with that of the stock.

To illustrate this spread, assume that the stock trades at $100. One call option has an exercise price of $95 and costs $7. The other call has an exercise price of $105 and costs $3. To buy the bull spread, the trader buys the call with the $95 exercise price and sells the other. The total outlay for the bull spread is $4. Figure 5.6 graphs the profits and losses for the two call positions individually. The long position profits if the stock price moves above $102. The short position profits if the stock price does not exceed $108. As the graph shows, low stock prices result in an overall loss on the position, because the cost of the long position outweighs the amount received from the short position. It is also interesting to consider prices at $105 and above. For every dollar by which the stock price exceeds $105, the long position has an extra dollar of profit. However, at prices above $105, the short position starts to lose money. Thus, for stock prices above $105, the additional gains on the long position match the losses on the short position. Therefore, no matter how high the stock price goes, the bull spread can never give a greater profit than it does for a stock price of $105.

Figure 5.7 graphs the bull spread as the solid line. For any stock price at expiration of $95 or below, the bull spread loses $4. This $4 is the difference between the cash inflow for selling one call and buying the other. The

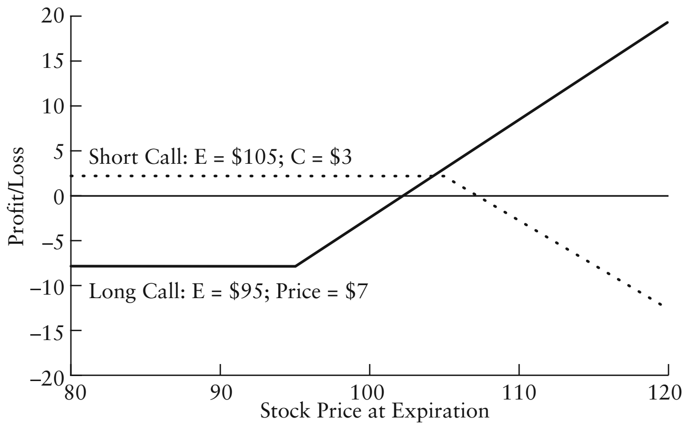

FIGURE 5.6 The two call options for a bull spread.

bull spread breaks even for a stock price of $99. The highest possible profit on the bull spread comes when the stock sells for $105. Then the bull spread gives a $6 profit. For any stock price above $105, the profit on the bull spread remains at $6. Therefore, the trader of a bull spread bets that the stock price goes up, but hedges the bet. We can see that the bull spread protects the trader from losing any more than $4. However, the trader cannot make more than a $6 profit. We can compare the bull spread with a position in the stock itself in Figure 5.7. Comparing the bull spread and the

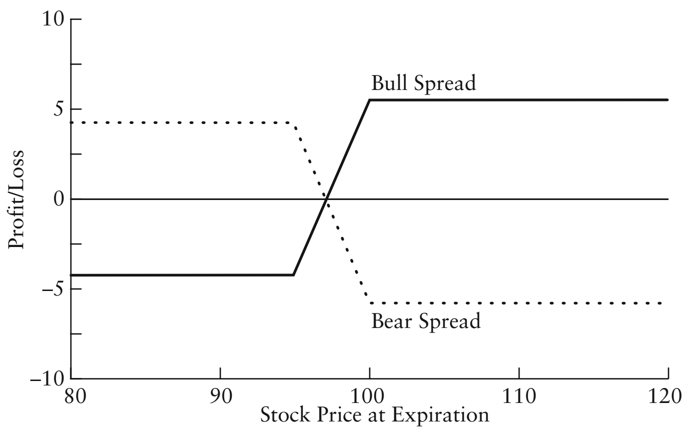

FIGURE 5.7 Profits and losses on bull and bear spreads.

stock, we find that the stock offers the chance for bigger profits, but it also has greater risk of a serious loss.

Figure 5.7 also shows the profit and loss profile for a bear spread with the same options. A bear spread is a combination of options designed to profit from a drop in the stock price. In our example, the bear spread is just the short positions that match the bull spread. In other words, the short position in the bull spread is a bear spread. The dotted line shows how profit and losses vary if a trader sells the call with the $95 strike price and buys the call with the $105 strike price. This position exactly mirrors the bull spread previously discussed. In a bear spread, the trader bets that the stock price will fall. However, the bear spread also limits the profit opportunity and the risk of loss compared with a short position in the stock itself. We can compare the profit and loss profiles of the bear spread in Figure 5.7 with the short position in the stock.

Butterfly Spreads

To buy a butterfly spread, a trader buys one call with a low exercise price and buys one call with a high exercise price, while selling two calls with a medium exercise price. The spread profits most when the stock price is near the medium exercise price at expiration. In essence, the butterfly spread gives a payoff pattern similar to a straddle. Compared with a straddle, however, a butterfly spread offers lower risk at the expense of reduced profit potential.

Assume that a stock trades at $100 and a trader buys a spread by trading options using the prices listed in Table 5.8. As the table shows, the buyer of a butterfly spread sells two calls with a striking price near the stock price and buys one each of the calls above and below the stock price.

Figure 5.8 graphs the profits and losses from these three option positions. (This is the most complicated option position we consider.) To understand the profits and losses from the butterfly spread, we need to combine these profits and losses, remembering that the spread involves selling two options and buying two.

Let us consider a few critical stock prices to see how the butterfly spread profits respond. The critical stock prices always include the exercise prices

TABLE 5.8 Three Option Positions

	Exercise Price ($)	Option Premium ($)
Long 1 Call	105	3
Short 2 Calls	100	4
Long 1 Call	95	7

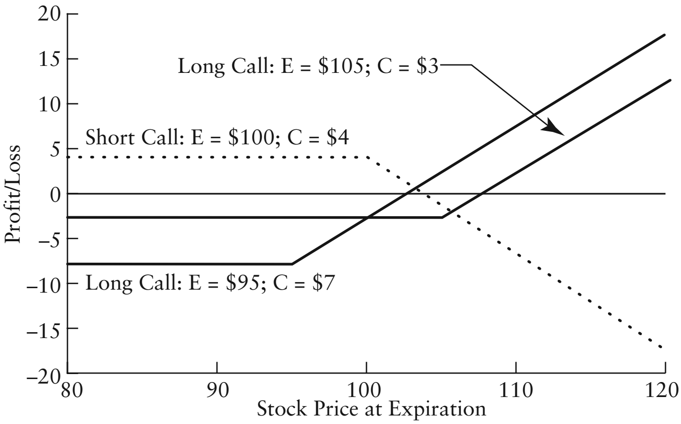

FIGURE 5.8 Individual options for a butterfly spread.

for the options. First, if the stock price is $95, the call with an exercise price of $95 is worth zero and a long position in this call loses $7. The long call with the $105 exercise price also cannot be exercised, so it is worthless, giving a loss of the $3 purchase price. The short call position gives a profit of $4 per option and the spread trader sold two of these options, for an $8 profit. Adding these values gives a net loss on the spread of $2, if the stock price is $95. Second, if the stock price is $100, the long call with a striking price of $95 loses $2 (the $5 stock profit minus the $7 purchase price). The long call with an exercise price of $105 loses its full purchase price of $3. Together, the long calls lose $5. The short call still shows a profit of $4 per option, for a profit of $8 on the two options. This gives a net profit of $3 if the stock price is $100. Third, if the stock price is $105 at expiration, the long call with an exercise price of $95 has a profit of $3. The long call with an exercise price of $105 loses $3. Also, the short call position loses $1 per option for a loss on two positions of $2. This gives a net loss on the butterfly spread of $2. In summary, we have a $2 loss for a $95 stock price, a $3 profit for a $100 stock price, and a $2 loss for a $105 stock price.

Figure 5.9 shows the entire profit-and-loss graph for the butterfly spread. At a stock price of $100, we noted a profit of $3. This is the highest profit available from the spread. At stock prices of $95 and $105, the spread loses $2. For stock prices below $95 or above $105, the loss is still $2. As the graph shows, the butterfly spread has a zero profit for stock prices of $97 and $103. The buyer of the butterfly spread essentially bets that stock prices will hover near $100. Any large move away from $100 gives a loss on the butterfly spread. However, the loss can never exceed $2. Comparing the butterfly spread with the straddle in Figure 5.6, we see that the butterfly spread

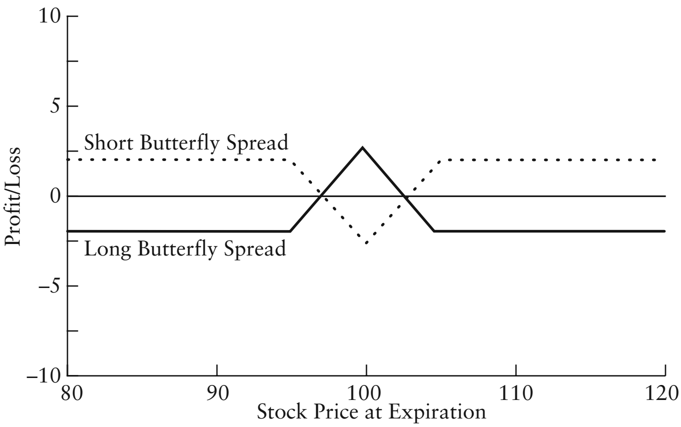

FIGURE 5.9 Profits and losses on a butterfly spread.

resembles a short position in the straddle. Compared with the straddle, the butterfly spread reduces the risk of a very large loss. However, the reduction in risk necessarily comes at the expense of a chance for a big profit.

SUMMARY

In this chapter, we applied the option pricing model to portfolio management decisions. The option pricing model is useful in risk management because it permits precise estimates of meaningful portfolio risk measures. These measures can be used to characterize the portfolio's exposure to underlying risk factors and can help analytically determine the impact of portfolio management decisions on the portfolio's risk characteristics. This chapter provided examples of how to use option contracts to speculate and to hedge portfolio risks . We also saw how to combine options to create new payoff profiles. Although options are typically regarded as very risky instruments, it is possible to create option positions that have substantially lower risk than an outright position in an option.

QUESTIONS AND PROBLEMS

1. Explain the difference between a straddle and a strangle.

2. Consider the following information about a stock and two call options, C_1 and C_2 written on the stock. The current stock price: \$100. C_1 current

price: 6.0581, delta = .4365, gamma = .0187. C_2 current price: 16.3328, delta = .7860, gamma = .0138. What combination of stock and the two options will produce a simultaneously delta-neutral and gamma-neutral portfolio? Assume you are long the stock.

3. Your largest and most important client's portfolio includes option positions. After several conversations, it becomes clear that your client is willing to accept the risk associated with exposure to changes in volatility and stock price. However, your client is not willing to accept a change in the value of her portfolio resulting from the passage of time. Explain how you can protect her portfolio against changes in value due to the passage of time.

4. Your newest client believes that the Asian currency crisis is going to increase the volatility of earnings for firms involved in exporting, and that this earnings volatility will be translated into large stock price changes for the affected firms. Your client wants to create speculative positions using options to increase his exposure to the expected changes in the riskiness of exporting firms. That is, your client wants to prosper from changes in the volatility of the firm's stock returns. Discuss which Greek your client should focus on when developing his option positions.

5. Your brother-in-law has invested heavily in stocks with a strong Asian exposure, and he tells you that his portfolio has a positive delta. Give an intuitive explanation of what this means. Suppose the value of the stocks that your brother-in-law holds increases significantly. Explain what will happen to the value of his portfolio.

6. Your mother-in-law has invested heavily in the stocks of financial firms, and she tells you that her portfolio has a negative rho. Give an intuitive explanation of what this means.

7. Your brother, Daryl, has retired. With the free time necessary to follow the market closely, Daryl has established large option positions as a stock investor. He tells you that his portfolio has a positive theta. Give an intuitive explanation of what this means.

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CHAPTER 6

The Swaps Market

A *swap* is an agreement between two parties, usually an end user and a swaps dealer, to exchange sets of cash flows over a period in the future. For example, Party A might agree to pay a fixed rate of interest on $1 million each year for five years to Party B. In return, Party B might pay a floating rate of interest on $1 million each year for five years. The parties that agree to the swap are known as *counterparties*. There are five basic kinds of swaps: *interest rate swaps, currency swaps, equity swaps, commodity swaps,* and *credit swaps*. Swaps can also be classified as *plain vanilla* or *flavored*. An example of a plain vanilla swap is the fixed-for-floating swap previously described. Some plain vanilla swaps are highly standardized, not unlike the standardized contracts found on organized exchanges. Flavored swaps can contain numerous nonstandard features, customized to meet the particular needs of the swap's counterparties.

This chapter provides a basic introduction to the swaps market. This market has grown rapidly in the past few years because it provides firms with a flexible way to manage financial risk. We explore in detail the risk management motivation that has led to this phenomenal growth. We also examine swap pricing.

Swaps are privately negotiated derivatives. They trade in an off-exchange, over-the-counter environment. A significant industry has arisen to facilitate swap transactions. This chapter considers the role of *swap dealers* who stand ready to accept either side of a transaction (e.g., pay-fixed or receive-fixed) depending on the customer's demand at the time.

SPECIAL FEATURES OF THE SWAPS MARKET

In this section, we consider the special features of the swaps market. For comparison, we first summarize key features of futures and options markets. Against this background, we focus on the most important features of the swap product. Finally, we briefly describe the development of the swaps market.

Futures and Options Market Features

In Chapters 2 and 4, we explored the key features of the futures and options markets. In Chapter 2, we observed that futures contracts generally trade in markets operated by futures exchanges and regulated by the Commodity Futures Trading Commission. In Chapter 4, we observed that exchange-traded options are highly formalized and regulated by the Securities Exchange Commission (SEC).

Futures and option contracts are highly standardized, with specific contract terms that cannot be altered. For example, the S&P 500 futures contract is based on a particular set of stocks, for a particular dollar amount, with four fixed maturity dates per year. In addition, futures and exchange-traded options generally have a fairly brief time until expiration. In many cases, futures contracts are listed only about one to two years before they expire. Even when it is possible to trade futures with expiration dates of three years or more, the markets do not become liquid until the contract comes much closer to expiration. For exchange-traded stock options, the longest time to maturity is generally less than one year. These futures and options cannot provide a means of dealing with risks that extend farther into the future than the expiration of the contracts that are traded.

The futures and options markets that we have explored are regulated markets and are dominated by the exchanges where trading takes place. The futures and options contracts are highly standardized, are limited to relatively few goods, and have a few fixed expirations per year. In addition, the horizon over which they trade is often much shorter than the risk horizon that businesses face.

Characteristics of the Swaps Market

In large part, the swaps market has emerged because swaps escape many of the limitations inherent in futures and exchange-traded options markets. Swaps, however, have their own limitations.

Swaps can be custom-tailored to the needs of the counterparties. If they wish, the potential counterparties can start with a blank sheet of paper and develop a contract that is completely dedicated to fulfilling their particular needs. Thus, swap agreements are more likely to meet the specific requirements of the participants than exchange-traded instruments. The counterparties can select the dollar amount that they want to swap, without regard to the fixed contract terms that prevail in exchange-traded instruments. Similarly, the swap counterparties choose the exact maturity that they need (i.e., the swap's "tenor") instead of having to fit their needs to the offerings available on an exchange. This flexibility allows the counterparties to deal with much longer horizons than can be addressed through exchange-traded instruments.

In addition, many other swap terms can be customized, including (1) whether the notional amount is subject to an amortization schedule; (2) the index to which the floating rate resets (e.g., six-month LIBOR); (3) the spread (if any) to be added to the floating-rate index, reflecting considerations such as credit risk; (4) the frequency and timing of the floating-rate reset; and (5) any terms affecting the credit risk of the settlements.

Even though swaps can be tailored to meet the individual needs of the counterparties, some degree of standardization has evolved in the market. As a consequence, participants can typically expect tighter bid-ask spreads when they use more standardized structures and wider spreads for customized deals.

In the swaps market, contracts are privately negotiated. Therefore, only the counterparties know that the swap takes place. In contrast, on futures and options exchanges, major financial institutions worry that rival traders will be able to glean confidential information about their trading activity. In a futures pit, traders will be able to discern the activity of particular firms because traders know who represents which firm. Thus, exchange trading necessarily involves a certain loss of privacy that can be avoided in the swaps market.[1]

Swaps markets are subject to a regulatory scheme separate from that used for futures and options markets. By law, participation in the swaps market is limited to firms, institutions, or individuals with high net worth. Because swap market participants are wealthy and (presumably) sophisticated, the regulatory scheme allows these participants to fend for themselves. Thus, the regulation of swaps relies less on direct government regulation and more on generic contract law and bankruptcy law. However, some swap market participants are subject to direct government regulation as a result of the industry in which they conduct business. For example, the use of swaps in the banking industry is subject to banking regulations. In general, the swaps market is characterized by limited direct government regulation relative to the futures and options markets. The passage of the Commodity Futures Modernization Act (CFMA) of 2000 addresses concerns about these side-by-side regulatory schemes. When fully implemented, the CFMA should put a significant portion of exchange-traded derivatives market on a more equal regulatory footing with swaps markets.

Although the swaps market evolved to avoid limitations in futures and options markets, the swaps market also has inherent limitations. Because a swap agreement is a contract between two counterparties, the swap cannot be altered or terminated early without the agreement of both parties. In addition, parties to the swap must be certain of their counterparty's credit-worthiness. The clearinghouses of futures and options exchanges effectively guarantee performance on the contracts for all parties. By its very nature, the swaps market has no such guarantor.

Later in this chapter, we discuss the mechanisms that the swaps market has developed to deal with these limitations. Potential default is perhaps the most important problem. The simplest and most prevalent way to manage credit risk is to establish a cutoff level for credit quality below which the swap dealer will not do business. For example, a dealer may not consider deals with firms rated below AA. Therefore, participation in the swaps market is effectively limited to creditworthy firms, institutions, and individuals with high net worth.

PLAIN VANILLA SWAPS

In this section, we analyze the kinds of swaps that are available and show how swaps can help corporations manage risk exposure. A *plain vanilla swap,* the simplest kind, can be an interest rate swap, a foreign currency swap, an equity swap, a commodity swap, or a credit swap.

Interest Rate Swaps

In a plain vanilla interest rate swap, one counterparty has an initial position in a fixed-rate debt instrument, while the other counterparty has a floating-rate obligation. In this initial position, the party with the floating-rate obligation is exposed to changes in interest rates. By swapping, this counterparty eliminates exposure to changing interest rates. For the party with a fixed-rate obligation, the interest rate swap increases the interest rate sensitivity. (Later, we explore the motivation these counterparties might have for taking their respective positions. First, however, we need to understand the transactions.)

To show the features of the plain vanilla interest rate swap, we use an example. We assume that the swap covers a five-year period and involves annual payments on a $1 million principal amount. Assume that Party A agrees to pay a fixed rate of 12 percent to Party B. In return, Party B agrees to pay a floating rate of LIBOR (London Interbank Offered Rate) + 3 percent to Party A. The LIBOR is the base rate at which large international banks lend funds to each other.[2] Floating rates in the swaps market are most often set as equaling LIBOR plus some additional amount. Figure 6.1 shows the basic features of this transaction. Party A pays 12 percent of $1 million, or $120,000 each year to Party B. Party B makes a payment to Party A in return, but the actual amount of the payments depends on movement in LIBOR.

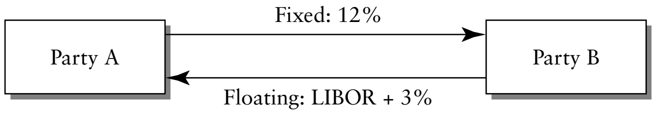

FIGURE 6.1 A plain vanilla interest rate swap.

Conceptually, the two parties also exchange the principal amount of $1 million. However, actually sending each other $1 million would not make practical sense. As a consequence, principal amounts are generally not exchanged. Instead, the principal plays a conceptual role in determining the amount of the interest payments to be swapped. Because the principal is not actually exchanged, it is called *notional principal,* an amount that is used as a base for computations, but is not actually transferred from one party to another. In our example, the notional principal is $1 million, and knowing that amount lets us compute the actual dollar amount of the cash flows that the two parties make to each other each year.

Assume that LIBOR is 10 percent at the time of the first payment. This means that Party A will be obligated to pay $120,000 to Party B. Party B will owe $130,000 to Party A. Offsetting the two mutual obligations, Party B owes $10,000 to Party A. Generally, only the *net payment,* the difference between the two obligations, actually takes place. Again, this practice avoids unnecessary payments.[3]

Foreign Currency Swaps

In a currency swap, one party holds a currency that it desires to replace with a different currency. The swap arises when one party provides a certain principal in one currency to its counterparty in exchange for an equivalent amount of a different currency. For example, Party C may have euros and be anxious to swap those euros for U.S. dollars. Similarly, Party D may hold U.S. dollars and be willing to exchange those dollars for euros. With these needs, Parties C and D may be able to engage in a currency swap.

A plain vanilla currency swap involves three sets of cash flows. First, at the initiation of the swap, the two parties actually do exchange cash. The entire motivation for the currency swap is the actual need for funds denominated in a different currency. This differs from the interest rate swap in which both parties deal in dollars and can pay the net amount. Second, the parties make periodic interest payments to each other during the life of the swap agreement. Third, at the termination of the swap, the parties again exchange the principal.

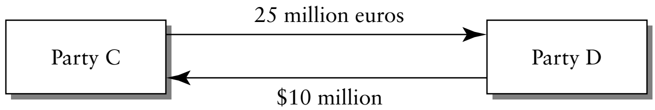

FIGURE 6.2 A plain vanilla currency swap (initial cash flow).

Assume that the current spot exchange rate between euros and U.S. dollars is 2.5 euros per dollar. Thus, the euro is worth $.40. We assume that the U.S. interest rate is 10 percent and the European Union interest rate is 8 percent. Party C holds 25 million euros and wishes to exchange those euros for dollars. In return for the euros, Party D will pay $10 million to Party C at the initiation of the swap. We also assume that the term of the swap is seven years and the parties will make annual interest payments. With the interest rates in our example, Party D will pay 8 percent interest on the 25 million euros it received, so the annual payment from Party D to Party C will be 2 million euros. Party C received $10 million and will pay interest at 10 percent, so Party C will pay $1 million each year to Party D.

In actual practice, the parties will make only net payments. For example, assume that at year 1 the spot exchange rate between the dollar and euro is 2.2222 euros per dollar, so the euro is worth $.45. Valuing the obligations in dollars at this exchange rate, Party C owes $1 million and Party D owes $900,000 (2 million euros times $.45). Thus, Party C would pay the $100,000 difference. At other times, the exchange rate could be different, and the net payment would reflect that rate.

At the end of seven years, the two parties again exchange principal. In our example, Party C would pay $10 million and Party D would pay 25 million euros. This final payment terminates the currency swap. Figure 6.2 shows the first element of the swap, which is the initial exchange of principal. Figure 6.3 represents the payment of interest; in our example there would be seven of these payments, one for each year of the swap. Finally, Figure 6.4 shows the second exchange of principal that completes the swap.

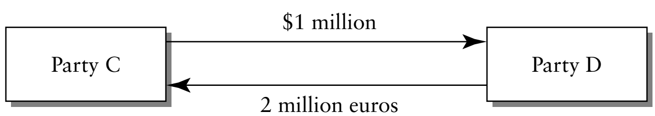

FIGURE 6.3 A plain vanilla currency swap (annual interest payment).

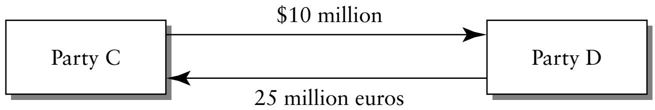

FIGURE 6.4 A plain vanilla currency swap (repayment of principal).

Equity Swaps

An *equity swap* is similar to an interest rate swap in that there is an underlying notional principal, a fixed tenor, and one party that pays a fixed rate while the other pays a floating rate. The difference is that the floating rate is determined by the rate of return on a stock index.

Consider an institutional investor with a $100 million portfolio of stocks invested in an index fund that tracks the S&P 500. If the manager of this portfolio becomes bearish, she has several choices for avoiding the risk of a stock market decline. She could sell the stocks in the portfolio, hedge the stock market risk in the futures market, or hedge the risk by using index options. She could also use an equity swap.

For her situation, the portfolio manager could enter an equity swap agreement in which she pays the S&P 500 return each period and receives a fixed percentage payment, with both payments being based on the $100 million notional principal of her portfolio. Each quarter, the portfolio manager might pay the total return earned by the S&P 500 and receive a quarterly payment of 2.5 percent, both payments being based on the $100 million notional principal. Figure 6.5 illustrates these cash flows. This arrangement would insulate the value of the portfolio against any drop in the stock market and guarantee the portfolio manager a quarterly return of 2.5 percent.

If the S&P 500 enjoyed a return of 3 percent in a given quarter, her undisturbed portfolio would rise in value by 3 percent since the portfolio tracks the index. The floating rate payment would be 3 percent of the notional principal, and she would pay this amount to the counterparty, leaving the portfolio value unchanged. However, the portfolio manager would also receive a payment of 2.5 percent. If the S&P 500 had a return of –5 percent,

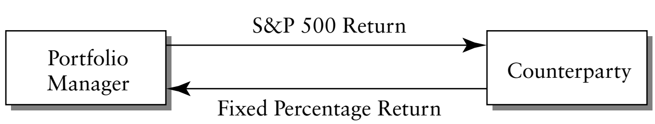

FIGURE 6.5 An equity swap.

the portfolio manager would make no payment and would receive a payment of 7.5 percent. This inflow of 7.5 percent, combined with the drop in the value of the portfolio of 5 percent, would still give a net return on the portfolio of 2.5 percent.

As with the other swap structures considered in this chapter, the equity swap can be elaborated by allowing variations in the notional principal or the periodic payment. One variant of the equity swap might be for one counterparty to pay the S&P 500 return and receive the Russell 2000 return, creating a swap agreement to speculate on the differential return between large capitalization and small capitalization stocks.

Commodity Swaps

In a commodity swap, the counterparties make payments based on the price of a specified amount of a commodity; one party pays a fixed price for the good over the tenor of the swap, while the second party pays a floating price. In general, the commodity is not actually exchanged, and the parties make only net payments.

Consider a rice farmer producing 200 tons of rice annually. He is anxious to avoid the price fluctuations of the spot rice market, particularly as import restrictions in Japan and Korea wax and wane. However, the farmer does not want to use the futures market in rice because of its low liquidity and uncertain future. Therefore, he seeks a swap arrangement in which he takes the receive-fixed side of the deal. The farmer agrees to receive a fixed payment per ton for each of the next five years and promises to pay the actual market price of rice each year. Each year, the farmer pays the actual price of rice based on the nominal amount of 200 tons, while the counterparty pays a fixed price negotiated when the swap agreement was established. With this arrangement, the farmer knows that he will receive a certain price for the rice for each of the next five years. Figure 6.6 illustrates these cash flows.

If we consider a single annual crop, this swap agreement has a structure that is similar to the classic hedging example with agricultural futures. In the classic short hedge, the farmer anticipates a harvest and sells futures to establish a fixed price for the crop.

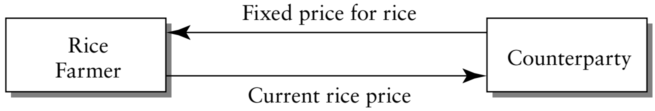

FIGURE 6.6 A commodity swap for rice.

Credit Swaps

A credit swap is a privately negotiated, over-the-counter derivative to transfer credit risk from one counterparty to another. The payoff of a credit swap is linked to the credit characteristics of an underlying reference asset, also called a reference credit. Credits swaps enable financial institutions and corporations to manage credit risks. The market for credit swaps is small relative to other types of swaps. Surveys of the market by several sources show the size of the market, in terms of outstanding notional principal, to be less than one trillion dollars as of December 2001.[4]

Credit swaps take many forms. In a *credit default swap,* two parties enter into a contract where company A makes a fixed periodic payment to company B for the life of the agreement. Company B makes no payments unless a specified credit event occurs. Credit events are typically defined to include a failure to make payments when due, bankruptcy, debt restructuring, change in external credit rating, or a rescheduling of payments for a specified reference asset.[5] If such an event occurs, the party makes a payment to the first party, and the swap then terminates. The size of the payment is usually linked to the decline in the reference asset's market value following the credit event.

In a *total return swap,* two companies enter an agreement whereby they swap periodic payments over the life of the agreement. Company C (called the protection buyer) makes payments based on the total return—coupons plus capital gains or losses—of a specified reference asset or group of assets. Company C (the protection seller) makes fixed or floating payments as with a plain vanilla interest rate swap. Both companies' payments are based on the same notional amount. The reference asset can be almost any asset, index, or group of assets. Among the underlying assets of a total return swap are loans and bonds.

Total return swaps have numerous applications. For example, total return swaps enable banks to manage the credit exposure resulting from their lending activities. Consider a Milwaukee bank that lends $10 million to a local brewery at a fixed interest rate of 7 percent. This interest rate charged by the bank includes a built-in risk premium to account for expected credit risk over the life of the loan. However, the bank still faces exposure to an unexpected increase in the brewery's credit risk over the life of the loan. If credit risk unexpectedly increases, the market value of the loan (an asset to the bank) will fall. To hedge this credit risk, the bank can enter into a total return swap. Assume the life of the swap is one year with a single exchange of cash flows at maturity and a notional principal of $10 million. Assume also that the swap is structured so that the bank pays the swap dealer a fixed rate of 9 percent plus the change in the loan's market value. In return, the bank receives one-year U.S. dollar LIBOR. Over the following year, an increase in credit risk causes the market value of the loan to fall so that on the swap's maturity date the loan is worth only 95 percent of its initial value. Under the terms of the swap, the bank owes the swap dealer the fixed rate of 9 percent minus the 5 percent capital loss on the market value of the loan, for a net total of 4 percent. In return, the bank receives a floating payment of one-year LIBOR, assumed to be 8 percent, from the swap dealer. Thus, the net inflow to the bank is 4 percent (8%minus 4%) multiplied by the swap's notional principal. This gain can be used to offset the loss of market value on the loan over the period.

Total return swaps provide protection against loss in value of the underlying asset irrespective of cause. If the interest rate changes, then the net cash flows of the total return swap will also change even though the credit risk of the underlying loans has not necessarily changed. In other words, the swap's cash flows are influenced by market risk as well as credit risk. The credit default swap enables the bank to avoid the interest-sensitive element of total return swaps. A key difference between a credit default swap and a total return swap is that the credit default swap provides protection against specific credit events, whereas the total return swap provides protection against loss due to market risk and credit risk. Finally, either credit default swaps or total return swaps entail two sources of credit exposure: one from the underlying reference asset and another from possible default by the counterparty to the transaction.[6]

MOTIVATIONS FOR SWAPS

In the preceding sections, we considered the transactions involved in plain vanilla interest rate, currency, equity, commodity, and credit swaps. For interest rate, equity, and commodity swaps, the essential feature is the transformation of a fixed-rate obligation to a floating-rate obligation for one party, and a complementary transformation of a floating-rate obligation to a fixed-rate obligation for the other party. In a currency swap, the two parties exchange currencies to obtain access to a foreign currency that better meets their business needs. In a credit swap, the payoff is linked to the credit characteristics of an underlying reference asset, such as a loan. Credit swaps are designed to transfer credit risk from one counterparty to another. In this section, we consider the motivations that lead to swap agreements.

In our example of a plain vanilla interest rate swap, we saw that one party begins with a fixed-rate obligation and seeks a floating rate obligation. The second party exchanges a floating rate for a fixed-rate obligation.

For this swap to occur, the two parties have to be seeking exactly the opposite goals.

We consider two basic motivations in this section. First, the normal commercial operations of some firms naturally lead to interest rate and currency risk positions of a certain type. Second, some firms may have certain advantages in acquiring specific types of financing. Firms can borrow in the form that is cheapest and use swaps to change the characteristics of the borrowing to one that meets the firm's specific needs.

Commercial Needs

As an example of a prime candidate for an interest rate swap, consider a typical savings and loan association. Savings and loan associations accept deposits and lend those funds for long-term mortgages. Because depositors can withdraw their funds on short notice, deposit rates must adjust to changing interest rate conditions. Most mortgagors want to borrow at a fixed rate for a long time. As a result, the savings and loan association can be left with floating-rate liabilities and fixed-rate assets. This means that it is vulnerable to rising rates. If rates rise, the savings and loan will be forced to increase the rate it pays on deposits, but it cannot increase the interest rate on the mortgages it has already issued.

To escape this risk, the savings and loan might use the swap market to transform its fixed-rate assets into floating-rate assets or transform its floating-rate liabilities into fixed-rate liabilities. Assume that the savings and loan wishes to transform a fixed-rate mortgage into an asset that pays a floating rate of interest. In terms of our interest rate swap example, the savings and loan association is like Party A—in exchange for the fixed-rate mortgage that it holds, it wants to pay a fixed rate of interest and receive a floating rate of interest. Engaging in a swap as Party A did will help the association to resolve its interest rate risk.

Assume that the savings and loan association has just loaned $1 million for five years at 12 percent with annual payments and that it pays a deposit rate that equals LIBOR plus 1 percent. With these rates, the association will lose money if LIBOR exceeds 11 percent, and this danger prompts the association to consider an interest rate swap.

Figure 6.7 shows our original plain vanilla interest rate swap with the additional information about the savings and loan. In the figure, Party A is the savings and loan association, and it receives payments at a fixed rate of 12 percent on the mortgage. After it enters the swap, the association also pays 12 percent on a notional principal of $1 million. In effect, it receives mortgage payments and passes them through to Party B under the swap agreement. Under this agreement, Party A receives a floating rate of LIBOR

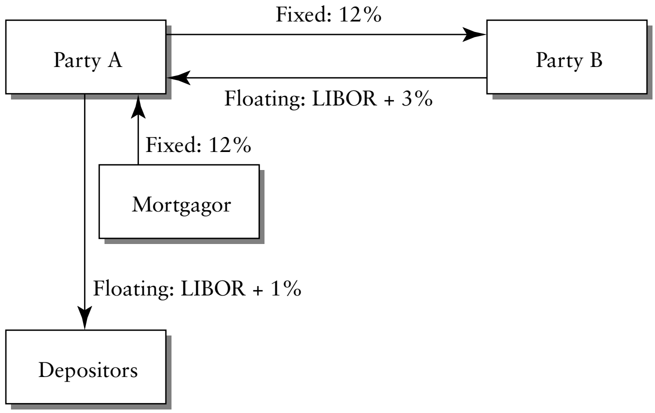

FIGURE 6.7 Motivation for the plain vanilla interest rate swap.

plus 3 percent. From this cash inflow, the association pays its depositors LIBOR plus 1 percent. This leaves a periodic inflow to the association of 2 percent, which is the spread that it makes on the loan.

In our example, the association now has a fixed rate inflow of 2 percent and has succeeded in avoiding its exposure to interest rate risk. No matter what happens to the level of interest rates, the association will enjoy a net cash inflow of 2 percent on $1 million. This example clarifies how the savings and loan association has a strong motivation to enter the swaps market. The very nature of the savings and loan industry exposes the association to rising interest rates. However, by engaging in an interest rate swap, the association can secure a fixed rate.

Comparative Advantage

In many situations, one firm may have better access to the capital market than another firm.[7] A U.S. firm may be able to borrow easily in the United States, but it might not have favorable access to the capital market in Europe. Similarly, a European firm may have good borrowing opportunities domestically but poor opportunities in the United States.

Table 6.1 presents borrowing rates for Parties C and D, the firms of our plain vanilla currency swap example. In the plain vanilla example, we assumed that, for each currency, both parties faced the same rate. We now assume that Party C is a European firm with access to euros at a rate of 7 percent, while the U.S. firm, Party D, must pay 8 percent to borrow euros. On the other hand, Party D can borrow dollars at 9 percent, while the European Party C must pay 10 percent for its dollar borrowings.

TABLE 6.1 Borrowing Rates for Two Firms in Two Currencies

Firm	U.S. Dollar Rate (%)	Euro Rate (%)
Party C	10	7
Party D	9	8

As the table shows, Party C enjoys a comparative advantage in borrowing euros and Party D has a comparative advantage in borrowing dollars. These rates suggest that each firm can exploit its comparative advantage and share the gains by reducing overall borrowing costs. This possibility is shown in Figures 6.8 to 6.10, which parallel Figures 6.2 to 6.4.

Figure 6.8 resembles Figure 6.2, but it provides more information. In Figure 6.8, Party C borrows 25 million euros from a third party lender at its borrowing rate of 7 percent, while Party D borrows $10 million from a fourth party at 9 percent. After these borrowings, both parties have the funds to engage in the plain vanilla currency swap already analyzed. To initiate the swap, Party C forwards the 25 million euros it has just borrowed to Party D, which reciprocates with the $10 million it has borrowed. In effect, the two parties have made independent borrowings and then exchanged the proceeds. For this reason, currency swaps are also known as an *exchange of borrowings*.

Figure 6.9 shows the same swap terms. Party C pays interest payments at a rate of 10 percent on the $10 million it received from Party D, and Party D pays interest of 2 million euros per year on the 25 million euros it received from Party C. The two firms could obtain the same rates from other sources. However, Figure 6.9 also shows the interest payments that

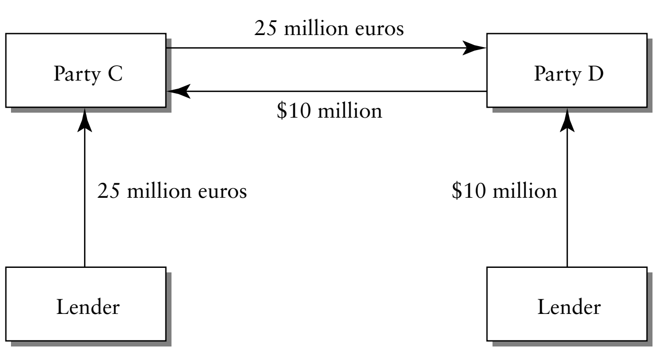

FIGURE 6.8 A plain vanilla currency swap (initial cash flow with lenders).

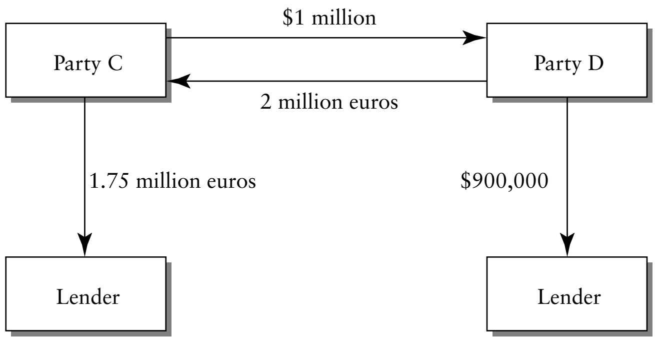

FIGURE 6.9 A plain vanilla currency swap (interest payments with lenders).

Parties C and D must make on their borrowings. Party C pays interest of 1.75 million euros annually, but it receives 2 million euros from Party D. For its part, Party D receives $1 million from Party C, from which it pays interest of $900,000.

This swap benefits both parties. Party C gets the use of $10 million and pays out 1.75 million euros. Had it borrowed dollars on its own, it would have paid a full 10 percent, or $1 million per year. At current exchange rates of 2.5 euros per dollar, Party C is effectively paying $700,000 annual interest on the use of $10 million. This is an effective rate of 7 percent. Party D pays $900,000 interest each year and receives the use of 25 million euros. This is equivalent to paying 2,250,000 euros annual interest ($900,000 times 2.5 euros per dollar) for the use of 25 million euros, or a rate of 9 percent. By using the swap, both parties achieve an effective borrowing rate

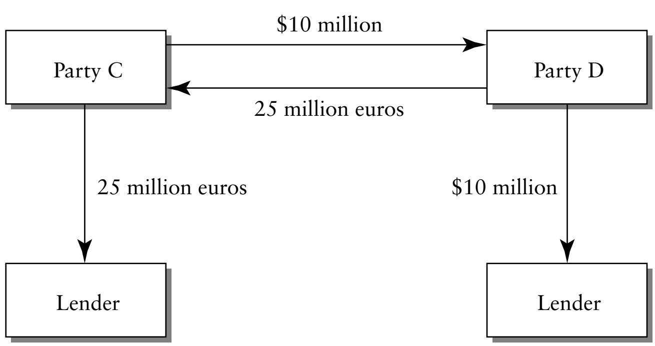

FIGURE 6.10 A plain vanilla currency swap (repayment of principal with lenders).

that is much lower than they could have obtained by borrowing the currency they needed directly. By engaging in the swap, both firms can use the comparative advantage of the other to reduce their borrowing costs. Figure 6.10 shows the termination cash flows for the swap, when both parties repay the principal.

We have explored two motivations for engaging in swaps: commercial needs and comparative borrowing advantages. The first led to an interest rate swap; the second motivated a currency swap. They are both plain vanilla swaps. Although swaps can become much more complex, they are generally motivated by the considerations that were discussed in this section.

SWAP DEALERS

Swap transactions are facilitated by dealers who stand ready to accept either side of a transaction (e.g., pay-fixed or receive-fixed) depending on the customer's demand at the time. These dealers generally run a *matched book,* in which the cash flows on numerous transactions on both sides of a market net to a relatively small risk exposure on one side of the market. Many of these matched transactions are termed *customer facilitations,* meaning that the dealer serves as a facilitating agent, simultaneously providing a swap to a customer and hedging the associated risk with an offsetting swap position or a futures position. Dealers collect a fee for the service and, if they perform it properly, incur little risk. When exact matching is not feasible for offsetting a position, dealers incur *mismatch risk*—the risk resulting from unmatched positions to which the dealer is a counterparty. Interest rate swap dealers, for example, rely heavily on CME Eurodollar futures to manage the mismatch risk of an interest rate swap-dealing portfolio. Dealers may also choose to bear some amount of mismatch risk as part of a proprietary trading position.

Because dealers act as financial intermediaries in swap transactions, they typically must have a relatively strong credit standing, large relative capitalization, good access to information about end users, and relatively low costs of managing the residual risks of an unmatched portfolio of customer transactions. Firms already active as financial intermediaries are natural candidates for being swap dealers. Most dealers, in fact, are commercial banks, investment banks, and other financial enterprises such as insurance company affiliates.

To explore the functions served by the swap dealer, assume that the dealer begins with an optimal set of investments. This swap dealer has financial assets, but they are invested in a way that the dealer finds optimal. Therefore, if she takes part in a swap transaction that alters her financial position, the change represents an unwanted risk that the dealer accepted only to help complete the swap transaction and to earn profits thereby. Against this background, we return to our example of a plain vanilla interest rate swap to explore the additional role performed by the swap dealer.

In the plain vanilla interest rate swap example, Party A was a savings and loan association that paid a floating rate of LIBOR + 1 percent to its depositors and made a five-year fixed-rate mortgage loan at 12 percent. This initial business position left Party A exposed to rising interest rates, and Party A wanted to avoid this risk by converting the fixed rate it received on its mortgage loan to a floating rate. Party A's ability to complete this swap depended on finding a suitable counterparty with a matching need, such as Party B.

To complete the swap transaction for Party A, the swap dealer may act as a counterparty. Figure 6.11 shows the plain vanilla interest rate swap example as before, except the swap dealer acts as the counterparty to Party A. As a result, the swap gives the swap dealer the same cash flows that Party B had in Figure 6.7.

As a result of this transaction, the swap dealer now has an undesired risk position. Over the next five years, the dealer is obligated to pay a floating rate of LIBOR + 3 percent and to receive a fixed rate of 12 percent on a notional amount of $10 million. The swap dealer must believe that she can make money by acting as a counterparty to Party A. To do so, the swap dealer wants to offset the risk on better terms than she undertook as a counterparty to Party A.

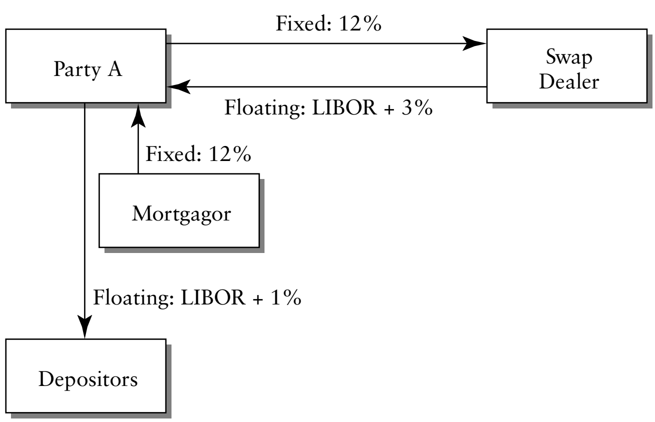

FIGURE 6.11 A plain vanilla interest rate swap with a swap dealer.

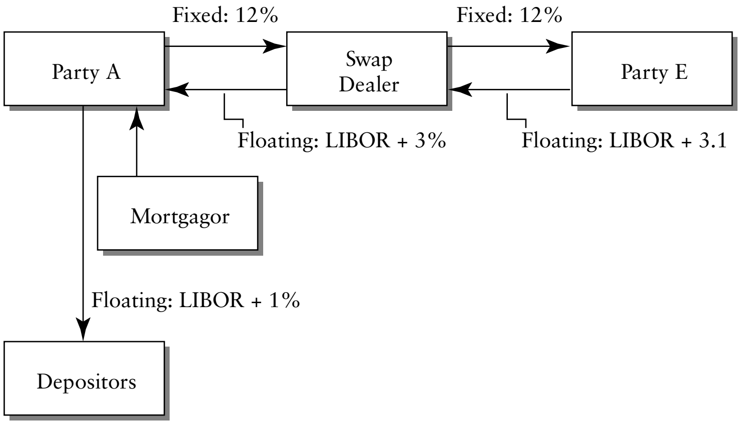

FIGURE 6.12 The swap dealer as intermediary in a plain vanilla interest rate swap.

Assume that the dealer knew of a potential party in the swap market, Party E, that was willing to pay a floating rate of LIBOR + 3.1 percent in exchange for a fixed rate of 12 percent on a notional amount of $10 million. However, Party E is willing to accept a term of only three years, not the five years that Party A desires. The swap dealer decides to act as a counterparty to Party A. By also transacting with Party E, the swap dealer offsets a substantial portion of the risk in transacting with Party A. Figure 6.12 shows the transactions involving Parties A and E, along with those of the swap dealer. After completing these transactions, the swap dealer has some profits to show for her efforts. She is making 10 basis points on the floating rate side of the transaction because she receives LIBOR + 3.1 percent and pays LIBOR + 3 percent. However, the swap dealer still has considerable risk as a result of the transaction.

Table 6.2 shows the swap dealer's cash flows resulting from the swap. The first two columns of the table show the cash flows from the swap dealer's transactions with Party A. To serve the needs of Party A, the swap dealer has agreed to receive a 12 percent fixed-rate payment in exchange for paying LIBOR + 3 percent on a $10 million notional amount. Based on the portion of the transaction with Party A, the swap dealer will receive $1.2 million each year and pay LIBOR + 3 percent on $10 million each year. Which is the better set of cash flows is uncertain because the future course of interest rates is unknown. For example, if LIBOR stays constant at 8 percent over the five years, the swap dealer will profit handsomely, making 1 percent per year for five years on $10 million. However, if LIBOR jumps to 11 percent

TABLE 6.2 The Swap Dealer's Cash Flows

Year	From Party A ($)	To Party A (%)	From Party E (%)	To Party E ($)
1	1,200,000	LIBOR + 3	LIBOR + 3.1	1,200,000
2	1,200,000	LIBOR + 3	LIBOR + 3.1	1,200,000
3	1,200,000	LIBOR + 3	LIBOR + 3.1	1,200,000
4	1,200,000	LIBOR + 3	0	0
5	1,200,000	LIBOR + 3	0	0

and remains constant, the swap dealer will be paying 14 percent on $10 million each year. As a result, the swap dealer will receive $1.2 million but must pay $1.4 million each year, for an annual net loss of $200,000. Thus, the riskiness of acting as a counterparty to Party A is clear.

Table 6.2 also shows the swap dealer's cash flows that result from transacting with Party E. For each of the first three years, the dealer will pay a fixed interest rate of 12 percent on $10,000,000, or $1,200,000. In addition, the dealer will receive a rate of LIBOR + 3.1 percent on a notional amount of $10,000,000.

The swap dealer's net cash flows are shown in Table 6.3. For the first three years, the swap dealer has achieved a perfect match in cash flows, receiving $1.2 million from Party A and paying it to Party E. The dealer has a net zero cash flow on this part of the transaction. During the first three years, the dealer also receives LIBOR + 3.1 percent from Party E and pays LIBOR + 3 percent to Party A, both on notional amounts of $10 million. On this portion of the transaction, the dealer receives a net spread of 10 basis points on a $10 million notional amount. Taking all of the dealer's cash flows during the first three years into account, the dealer has a net cash inflow of $10,000 per year.

Even after transacting with both Parties A and E, the swap dealer has a residual risk that is evident from the total cash flows. In years four and

TABLE 6.3 Dealer's Net Cash Flows

Year	Dealer's Net Cash Flow
1	$ 10,000
2	$ 10,000
3	$ 10,000
4	$1,200,000 – LIBOR + 3%
5	$1,200,000 – LIBOR + 3%

five, the dealer will receive $1.2 million from Party A, but she must pay LIBOR + 3 percent. Whether this will create a profit or loss for the dealer depends on future interest rates. In Table 6.2, however, we can see that the dealer has substantially reduced this risk position by trading with Party E.

Swap Dealers as Financial Intermediaries

Table 6.2 also shows that the swap dealer is making a profit as a financial intermediary. Because of her superior knowledge of the market, the dealer was able to find Party E. By transacting with Party E, instead of just transacting with Party A, the swap dealer secures a spread of 10 basis points on the notional amount for three years. In addition to earning a profit on the spread, the dealer's transaction with Party E offsets a substantial portion of the risk inherent in acting as a counterparty to Party A in the initial swap transaction.

In this example, we assumed that the swap dealer had an initial portfolio of assets that met her needs in terms of risk and diversification. By acting as a counterparty to Party A, the swap dealer assumed a risk in pursuit of profit. The dealer could have taken this position as a speculation on interest rates, but preferred to act as a financial intermediary, making a profit by providing informational services. The swap dealer was able to capture a spread of 10 basis points and reduce risk by transacting with Party C. Ideally as a financial intermediary, the swap dealer would also like to avoid the remaining risk exposure in years 4 and 5. To do so, the dealer would have to find another swap counterparty. In Chapter 7, we explore how swap dealers manage the risks associated with acting as a counterparty.

Swap transactions are facilitated by dealers who stand ready to accept either side of a transaction as a swap counterparty (either pay-fixed or pay-floating) depending on customer demand. The swap dealer is a financial intermediary, who earns profits by helping to complete swap transactions. If swap dealers must take a risk position to complete a swap, they try to manage that risk with offsetting swap transactions or hedging transactions in the futures market.

PRICING OF SWAPS

In this section, we explore the principles that underlie swap pricing. To simplify the discussion, we focus on plain vanilla interest rate swaps, and we assume that the swap dealer wants to act as a purely financial intermediary (i.e., does not want to assume a risk position with respect to interest rates). The principles apply, however, to swaps of all types.

Factors That Affect Swap Pricing

The swap dealer must price swaps to reflect the creditworthiness of the potential swap partner, the availability of other swap opportunities that can offset the risk of an initial swap, and the term structure of interest rates.[8] We discuss each of these in turn.

Creditworthiness The swap dealer must appraise the creditworthiness of the swap partner. As explained earlier in this chapter, there is no clearinghouse in the swap market to guarantee performance on a contract if one of the counterparties defaults. A swap dealer who suffers a default by a counterparty must either absorb the loss or institute a lawsuit to seek recovery on the defaulted obligation.

In most swaps, the timings of cash flows between the counterparties are matched fairly closely. In the plain vanilla interest rate swap of Figure 6.12, the fixed and floating cash flows occur at similar times, and only the net amount is actually exchanged. Thus, default on a swap seldom could involve failure to pay the notional amount or even an entire periodic payment. In this sense, default on a swap is not as critical as default on a corporate bond, in which an investor might lose the entire principal. Instead, a swap default would generally imply a loss of the change in value due to shifting interest rates. Although this amount might be significant, such a default would not be as catastrophic as a bond default in which the entire principal could be lost.

As shown in Figure 6.12 and as explored in detail later in this chapter, the swap dealer seeks to build a swap portfolio in which the risks of individual swaps offset each other. In Figure 6.12, the risks in the swap with Party A are largely offset by the risks in the swap with Party E. When a swap dealer suffers a default, it can upset the elaborate structure of offsetting risks. This is a riskier position, and the dealer must struggle to reestablish the risk control that was upset by the default.

Because of the potential costs associated with default, the swap dealer will adjust the pricing on swaps to reflect the risk of default. Parties that have a high risk of default are likely to be excluded from the market. For example, airlines under bankruptcy protection probably have very limited access to the swap market. As noted earlier, default considerations and the need for one party to confirm the creditworthiness of a prospective counterparty make the swap market mainly a market for financial institutions and corporations.

Availability of Additional Counterparties A swap dealer who wants to act only as a financial intermediary will be eager to offset the risk involved in a prospective swap by participating in other swaps. In the dealer's swap of Figure 6.12, the willingness of the swap dealer to enter the transaction with Party A may well depend on the dealer's knowing about Party E. Without such knowledge, the dealer may require more favorable terms to transact with Party A. However, if the dealer knows about Parties A and E from the outset, she may accept less favorable terms because some of the risk of acting as Party A's counterparty can be offset in a second swap with Party E.

As noted, the swap dealer faces the net cash flows in the last column of Table 6.2 after engaging in the two interest rate swaps with Parties A and E. Assume now that another potential swap participant, Party F, is available to swap the cash flows in years 4 and 5. Party F would be willing to pay a floating rate on a $10 million notional amount for years 4 and 5 and to receive a fixed rate of 12 percent. The swap dealer would find Party F to be an attractive counterparty. The dealer might be willing to swap with Party F on even terms ($1,200,000 vs. LIBOR + 3%) just to offset the risk that remained after swapping with Parties A and E. In sum, the swap dealer will be pleased to create a structure of swaps that leaves no interest rate risk and still provides a decent profit.

The Term Structure of Interest Rates The term structure of interest rates is an important feature in bond pricing. Not surprisingly, the market for interest rate swaps must reflect the term structure that prevails in the bond market. If the swaps market did not reflect the term structure, traders would find ready arbitrage opportunities, and they could quickly discipline swap traders to pay attention to the term structure. For example, if the term structure is rising, the swap dealer must charge a higher yield on swaps of longer maturity. The next section illustrates these considerations.

The Indication Swap Pricing Schedule

In the early 1980s to the middle of that decade, swap banks were often able to charge a front-end fee for arranging a swap. Competition has eliminated that possibility as the market has matured. (For highly complicated swaps that require substantial analysis, front-end fees are still charged, however.) Today swap dealers' total compensation comes from charging a *bid-ask spread* between the rates they are willing to pay and the rate they demand on swap transactions. With a maturing market, this bid-ask spread has also narrowed. Whereas in the mid-1980s, bid-ask spreads might have been 50 basis points, a spread of 2 to 4 basis points is much more common today, depending on the swap tenor. This tightening spread reflects the increasing liquidity, sophistication, and pricing efficiency of a maturing financial market.

Table 6.4 shows a sample indication pricing schedule for an interest rate swap. The table assumes that the customer of the swap dealer will offer LIBOR flat, that is, a rate exactly equal to LIBOR without any yield adjustment. Prices are quoted as a spread to Treasury issues. This spread is called the *swap spread* (a spread separate from the bid-ask spread). Table 6.4 has two important features. First, the rate the dealer pays or receives increases with the maturity in question. This increase reflects the upward sloping term structure revealed by the column of current Treasury yields. Second, the swap dealer makes a gross profit that equals the spread between what the bank pays and what it receives. Consequently, the spread ranges from 2 basis points for a two-year horizon to 4 basis points for a 20-year horizon. This increasing spread for more distant maturities reflects the lower liquidity of longer term instruments.

To see how the pricing schedule in Table 6.4 functions, assume that the customer wants to pay a floating rate and receive a fixed rate for seven years. Based on the pricing schedule of Table 6.4, the customer would pay the LIBOR rate on the notional amount in each period and would receive a fixed rate from the swap dealer that equals the seven-year T-note rate of 4.51 percent plus 80 basis points for a total rate of 5.31 percent. By contrast, if the customer wants to pay a fixed rate for a seven-year horizon, the customer would pay the seven-year T-note rate of 4.51 percent plus 82 basis

TABLE 6.4 Sample Swap Indication Pricing

	Dealer's Fixed Rates: (U.S. Treasury Rate Plus Indicated Basis Point Spread)		
Maturity (Years)	**Dealer Pays**	**Dealer Receives**	**T-Note Yields**
2	43.50	45.50	2.95
3	74.50	76.50	3.38
4	79.50	81.50	3.80
5	69.00	71.00	4.23
6	76.50	78.50	4.39
7	80.00	82.00	4.51
8	78.50	80.50	4.66
9	75.00	77.00	4.81
10	70.50	72.50	4.94
15	89.75	91.75	5.05
20	88.00	90.00	5.17
30	66.50	68.50	5.39

Source: Compiled from various market sources. Reflective of market prices for mid-January, 2002.

points for a rate of 5.33 percent. In return, the bank would pay the customer the LIBOR rate in each period.[9]

It is customary for the dealer's pricing quotations to assume no risk of default. In actual practice, the bid-ask spread must be adjusted to reflect the credit risk of the counterparty. The bid-ask spread must be wide enough to compensate the dealer for potential defaults. We discuss counterparty credit risk later in this chapter.

The Zero Curve

The zero-coupon yield curve, or *zero curve,* captures the relationship between zero-coupon spot yields and term to maturity as of a particular date. The zero curve, unlike other yield curves, represents a set of yields unencumbered by complicating assumptions about reinvestment rates for coupons received before bond maturity. As a result, practitioners can be confident that the discount factors and expected forward rates derived from the zero curve do not depend on any reinvestment rate assumptions. Because of its desirable properties, the zero curve has become a key ingredient for the valuation of many financial instruments, including swaps.

Because zero curves can be based on any class of interest rates and denominated in any type of currency, there are many different types. However, we focus on the U.S. dollar LIBOR zero curve. The zero curve must be constructed—as opposed to observed—because directly observable, and reliable, zero rates are available for only a limited number of maturities. For other maturities, zero rates must be constructed using a combination of "bootstrapping" and interpolation techniques.[10]

One of the main uses of the zero curve is to determine the discount factors for finding the present value of swap cash flows. We denote a zero-coupon discount factor for a cash flow received at time y and measured at time x as $Z_{x,y}$. We denote x and y in months. For example, $Z_{0,.3}$ would be the

TABLE 6.5 Three-Month LIBOR Zero-Coupon Forward Rates

Futures Expiration Date	Rate (% Annual)
Spot Rate	3.00
3 Months	3.20
6 Months	3.40
9 Months	3.60

discount factor applied to a cash flow expected in three months in order to determine the present value today.

Consider the Eurodollar spot and futures yields displayed in Table 6.5. These yields are zero-coupon forward rates representing the structure of three-month Eurodollar time deposit rates over time. Only the spot rate is in a form that can immediately be turned into a zero-coupon discount factor. This three-month rate can be used to find $Z_{0,.3}$ as follows:

$$Z_{0,.3} = 1 + .25 \times .03 = 1.0075$$

where .25 reflects the three-month (quarter-year) period and .03 is the current three-month LIBOR spot rate. Given our initial zero-coupon factor and Eurodollar futures yields, we can use a bootstrapping technique to find successive factors. We illustrate the technique using the yields displayed in Table 6.5. We can compute the value of $Z_{0,.3}$ as follows:

$$Z_{0,.6} = Z_{0,.3} \times (1 + .25 \times .032) = 1.01556$$

Values for $Z_{0,.9}$ and $Z_{0,.12}$ follow similarly:

$$Z_{0,9} = Z_{0,.6} \times (1 + .25 \times .034) = 1.024192$$

$$Z_{0,12} = Z_{0,.9} \times (1 + .25 \times .036) = 1.03341$$

Pricing a Plain Vanilla Interest Rate Swap

In a plain vanilla fixed-for-floating interest rate swap, the receive-fixed party will pay a floating rate equal to LIBOR flat in each period, but the fixed rate must be analytically determined at the time the swap is initiated. This rate, called the par value swap rate (PVSR), is defined as the fixed rate equating the net present value of the fixed swap coupons over the life of the swap with the net present value of the expected floating-rate interest payments. The PVSR is the fixed rate that makes the value of the swap equal to zero, or par.

Determining the PVSR requires information about the expected floating-rate payments when the swap is initiated. We can assume that both swap counterparties form floating LIBOR rate expectations that are consistent with the term structure of interest rates. One way of determining the

PVSR is to use Eurodollar futures yield quotations as representative of the implied forward LIBOR rates from the term structure. Based on these Eurodollar futures yield quotations, we equate the present value of floating-rate payments with the present value of the fixed payments to find the PVSR.

We begin by using a plain vanilla interest rate swap with a notional of \$20 million, a tenor of one year, and payments every three months. One feature of this swap is standard in most swaps: Floating-rate payments are "determined in advance, paid in arrears." This means that when a payment date arrives, the prevailing spot LIBOR rate determines the amount of the floating-rate payment on the next payment date. This feature is important in determining the present value of the swap's cash flows. We use the yield strip of Eurodollar futures from Table 6.5 to determine the expected floating payments:

Today	$.030 \times .25 \times \$20{,}000{,}000 = \$150{,}000$
3 months	$.032 \times .25 \times \$20{,}000{,}000 = \$160{,}000$
6 months	$.034 \times .25 \times \$20{,}000{,}000 = \$170{,}000$
9 months	$.036 \times .25 \times \$20{,}000{,}000 = \$180{,}000$

Each payment on the fixed side would be the unknown par value swap rate, PVSW, multiplied by .25 (representing the quarter year between payment dates) times the notional principal of \$20,000,000. Therefore, each fixed payment would be:

$$\text{PVSR} \times .25 \times \$20{,}000{,}000 = \text{PVSR} \times \$5{,}000{,}000$$

Using the zero-coupon factors previously determined, we can set up an equation that, when solved, will determine the value of PVSR. On the left-hand side of this equation is the present value of the expected floating-rate payments, and on the right-hand side is the present value of the (yet-to-be-determined) fixed-rate payments:

$$\frac{\$150{,}000}{1.0075} + \frac{\$160{,}000}{1.01556} + \frac{\$170{,}000}{1.024192} + \frac{\$180{,}000}{1.03341} = \frac{(\text{PVSR} \times \$5{,}000{,}000)}{1.0075}$$
$$+ \frac{(\text{PVSR} \times \$5{,}000{,}000)}{1.01556} + \frac{(\text{PVSR} \times \$5{,}000{,}000)}{1.024192}$$
$$+ \frac{(\text{PVSR} \times \$5{,}000{,}000)}{1.03341}$$

This equation reduces to:

$$\$646{,}597.01 = \text{PVSR} \times \$19{,}606{,}417.83$$

This means that PVSR equals .03298, or 3.298 percent. This result is consistent with our intuition that the fixed rate should be some kind of average of the floating rates. This is the fixed rate that makes the value of the swap equal to zero, or par at initiation.

In actual practice, swap dealers do not calculate a single par value swap rate. Instead, bid and ask par value swap rates are calculated, the difference being the bid-ask spread that is part of the dealer's compensation for completing the swap transaction and managing the associated risks. As a result of this spread, swaps will have a positive value from the dealer's perspective at initiation. Dealers refer to this initial positive value as an *inception gain.* Various sources quote bid and ask par value swap rates for plain vanilla fixed-for-floating swaps. We have already seen how bid and ask par value swap rates are quoted in Table 6.4.

The pricing approach described here can also be used to determine the swap value after initiation. Suppose after three months, rates rise so that the expected floating-rate payments for the three remaining payment dates are a spot rate of 3.5 percent, a rate of 3.7 percent three months hence, and 3.9 percent six months hence. As a result, the expected floating-rate payments will be:

Today	$.035 \times .25 \times \$20{,}000{,}000 = \$175{,}000$
3 months	$.037 \times .25 \times \$20{,}000{,}000 = \$185{,}000$
6 months	$.039 \times .25 \times \$20{,}000{,}000 = \$195{,}000$

The corresponding zero-coupon discount factors will now be: $Z_{0,.3} = 1.00875$, $Z_{0,.6} = 1.018081$, $Z_{0,.9} = 1.028007$.

As interest rates move up over the life of the swap, the swap becomes more valuable to the fixed-rate payer/floating-rate receiver. The present value of the expected floating-rate payments is now:

$$\frac{\$175{,}000}{1.00875} + \frac{\$185{,}000}{1.018081} + \frac{\$195{,}000}{1.028007} = \$544{,}884.85$$

Given the PVSR determined at the swap's initiation, each fixed payment will be \$164,993.8, that is, $.03298 \times .25 \times \$20{,}000{,}000$. Now, three months into the life of the swap, the present value of the stream of fixed payments is:

$$\frac{\$164{,}993.8}{1.00875}+\frac{\$164{,}993.8}{1.018081}+\frac{\$164{,}993.8}{1.028007}=\$486{,}124.87$$

This means the swap is now in the money to the pay-fixed/receive-float counterparty by $58,758.98 (i.e., $544,884.85 – $486,124.87). Equivalently, the swap is out-of-the-money to the pay-float/receive-fixed counterparty by the same amount.

Pricing a Plain Vanilla Foreign Currency Swap

As we have seen for interest rate swap pricing, the par value swap rate is the rate (PVSR) that gives a zero net present value to both counterparties to the swap. The same method can be applied to currency swap pricing at initiation. In a plain vanilla fixed-for-fixed currency swap, each party will pay the PVSR for the currency it receives. The PVSR for each party is the same as the PVSR on a plain vanilla interest rate swap in the home currency for the same tenor.

Par value swaps are of limited interest for illustrating swap pricing since they are, by construction, equal to zero. The best way to illustrate the pricing of a currency swap is with a non-par swap. Consider a financial institution that has entered into a currency swap where it receives 5.6 percent in euros and pays 9.3 percent in dollars once a year. The principal amounts for the two currencies are $1,000,000 and 1,200,000 euros. The swap will last another two years.

On each payment date, the financial institution has agreed to exchange an inflow of 67,000 euros (i.e., 5.6% of 1,200,000 euros) for an outflow of $93,000 (i.e., 9.3% of $1,000,000). The principal amounts are exchanged at the start of the swap and exchanged again at swap maturity. At the start of the swap, the principal amounts in the two currencies are by construction exactly equivalent, meaning that the initial exchange contributes zero value.

TABLE 6.6 Zero-Coupon Discount Factors and FX Forward Rates

Maturity (Years)	U.S. Dollar Zero-Coupon Rates	U.S. Dollar Zero-Coupon Discount Factors	Forward FX Rate (Euros per Dollar)	Forward FX Rate (Dollar per Euro)
0	N/A	N/A	1.20	.833333
1	.080	1.08000	1.23	.813008
2	.085	1.77688	1.25	.800000

Pricing a currency swap can be accomplished by decomposing the swap into a series of forward contracts. The logic of the approach is to use the forward foreign exchange rates to determine the expected net dollar cash flows on each payment date. The expected net dollar cash flows are then discounted to the present using zero-coupon discount factors. The value of the swap is the sum of these discounted net cash flows.

Zero-coupon discount factors and forward euro/dollar exchange rates are given in Table 6.6. Using these values, the present value of the forward contracts underlying the swap will be:

$$\frac{(6720 \times .8333 - \$9,300)}{1.08} = -\$34,259.28$$

$$\frac{(6720 \times .813008 - \$9,300)}{1.177688} = -\$32,577.27$$

The final exchange of principal involves receiving 1,200,000 euros and paying $1,000,000. The value of the forward contract corresponding with this is:

$$\frac{(1,200,000 \times .80000 - \$1,000,000)}{1.177688} = -\$33,964.85$$

The total value of the swap is –$34,259.28 – 32,577.27 – 33,964.85 = –$100,801.41. This means that the swap is out-of-the-money to the financial institution since it is paying dollars and receiving euros. If the financial institution had been on the other side of the swap (i.e., paying euros and receiving dollars), the value of the swap would have been $100,801.41.

Our examples cover the essential elements of pricing a plain vanilla swaps. In actual practice, there are many variations on this basic template. Actual market pricing of swaps is influenced by technicalities such as day counting conventions, holidays, and yield quoting methods. We abstract from these technicalities to focus on the main elements of swap pricing.[11]

SWAP COUNTERPARTY CREDIT RISK

Counterparty credit risk is a significant concern of end users and dealers in the swaps market. Credit risk arises from the possibility of a default by the swap counterparty when the value of the swap is positive (i.e., in the money).

Current credit exposure is measured by the swap's current *replacement cost,* that is, the amount required to replace the swap in the event of a counterparty default today. Only positive swap values are of interest in determining swap credit exposure. This is because with negative- or zero-value swaps (i.e., out-of-the-money or at-the-money swaps), the counterparty owes nothing in the event of a default. The only time that money is at risk is when the default occurs with a counterparty owing money.

Current replacement cost represents current credit exposure. However, current replacement cost alone does not accurately portray the potential credit risk over the life of the swap. A counterparty might default at some future date with swap values significantly different than current swap values. The potential loss is greater because the replacement cost can potentially become larger over the life of the swap.

As a first step to managing counterparty credit risk, end users and swap dealers estimate potential replacement cost for each of the swaps they hold with individual swap counterparties. The estimated potential replacement cost is essential for monitoring, managing, and allocating counterparty credit line usage relative to the counterparty credit limits. Dealers and end users are particularly interested in monitoring the incremental effect on credit line usage resulting from an additional transaction with a counterparty.

Measuring potential replacement cost relies on simulation techniques. Simulation techniques rely on probability distributions to project the range of potential values of the swap over the swap's remaining life. Only the positive values the swap can potentially take on are of interest. For simulations far out into the future, uncertainty about potential swap value can result in a wide range of estimates.

It is common for swap participants to characterize counterparty credit risk with a single number. However, projecting potential exposures over the life of the swap will produce many numbers. For example, a swap with five years of remaining life will have credit exposures measured for potential default dates (e.g., one year in the future, two years in the future). When the series of credit exposure measures is plotted against tenor, a time-varying *exposure profile* for the counterparty is produced. To reduce credit exposure to a single number, swap market participants frequently choose the maximum exposure (i.e., the greatest amount that would potentially be lost in the event of a counterparty default). The maximum exposure is a probability estimate of the true maximum exposure.

A contentious issue in the evaluation of credit risk concerns what is known as "netting." Most swap contracts, called *master agreements,* state that if a counterparty defaults on one swap it must default on all swaps. Therefore, swaps with a negative replacement cost will offset swaps with a positive replacement cost when the defaulting counterparty is the same in the two cases. Under such a netting arrangement, counterparty credit exposure is defined as the net positive replacement cost after netting.

To be effective, simulation methods must account for netting effects and bankruptcy rules. In addition, the method must account for what a bankruptcy court might do in the event of a default. In one legal scenario, a bankruptcy court might determine the replacement cost of each swap in the portfolio and simultaneously close out all positions. In an alternate legal scenario, a bankruptcy court might allow each swap to run until its settlement, maturity, or expiration date, and then close out only those swaps with positive replacement cost. Selectively closing out only those swaps with positive value is called *cherry picking*. Possible cherry picking is a scenario that must be accounted for in measuring potential counterparty credit exposure.

The same legal rules will not apply to all swaps between dealers and end users. Some swaps may be covered by a legally binding netting agreement and others may not. The simulation method must apply the appropriate legal rules to each contract.

SUMMARY

This chapter has provided a basic introduction to the swaps market. From origins in the late 1970s and early 1980s, the swaps market has grown to enormous proportions, with notional principal surpassing $100 trillion by some estimates. Most of the market is concentrated in interest rate swaps, but billions of dollars of foreign currency swaps are outstanding as well. Of all swaps worldwide, over one-third involve the U.S. dollar.

In contrast with futures and exchange-traded options, swap agreements are extremely flexible in amount, maturity, and other contract terms. As further points of differentiation between futures and exchange-traded options versus swaps, the swaps market does not use an exchange and is virtually free of governmental regulation.

In this chapter, we also analyzed plain vanilla interest rate, currency, equity, commodity swaps, and credit swaps. An interest rate swap essentially involves a commitment by two parties to exchange cash flows tied to some amount of notional principal. One party pays a fixed rate, while the second party pays a floating rate. In equity and commodity swaps, we saw that, like interest rate swaps, the essential feature is the transformation of a fixed-rate obligation into a floating-rate obligation for one party, and a complementary transformation of a floating-rate obligation to a fixed-rate obligation for the other party. In a foreign currency swap, both parties acquire funds in different currencies and exchange those principal amounts. Each party pays interest to the other in the currency that was acquired, with these interest payments taking place over the term of the swap agreement. To terminate the agreement, the parties again exchange foreign currency. In a credit swap, the payoff is linked to the credit characteristics of an underlying reference asset, such as a loan. Credit swaps are designed to transfer credit risk from one counterparty to another. Motivations for swaps arise from a desire to avoid financial risk or a chance to exploit some borrowing advantage.

Swap transactions are facilitated by dealers who stand ready to accept either side of a transaction as a swap counterparty depending on customer demand. By taking a position in a swap transaction, a swap dealer accepts a risk position. We considered the factors that influence swap pricing and provided an example to show how a swap dealer prices a fixed-for-floating plain vanilla interest rate swap. Finally, we examined swap counterparty credit risk, a significant concern of all swap market participants.

QUESTIONS AND PROBLEMS

1. Explain the differences between a plain vanilla interest rate swap and a plain vanilla currency swap.

2. What role does the swap dealer play?

3. Assume that you are a financial manager for a large commercial bank and that you expect short-term interest rates to rise more than the yield curve would suggest. Would you rather pay a fixed long-term rate and receive a floating short-term rate, or the other way around? Explain your reasoning.

4. Explain the role that the notional principal plays in understanding swap transactions. Why is this principal amount regarded as only notional? (Hint: What is the dictionary definition of "notional"?)

5. Consider a plain vanilla interest rate swap. Explain how the practice of net payments works.

6. Assume that the yield curve is flat, that the swap market is efficient, and that two equally creditworthy counterparties engage in an interest rate swap. Who should pay the higher rate, the party that pays a floating short-term rate or the party that pays a fixed long-term rate? Explain.

7. In a currency swap, counterparties exchange the same sums at the beginning and the end of the swap period. Explain how this practice

relates to the custom of making interest payments during the life of the swap agreement.

8. Explain why a currency swap is also called an "exchange of borrowings."

9. Assume that LIBOR stands today at 9 percent and the seven-year T-note rate is 10 percent. Establish an indication pricing schedule for a seven-year interest rate swap, assuming that the swap dealer must make a gross spread of 40 basis points.

10. Why are only positive swap values used to measure counterparty credit risk?

11. Suppose that the fixed-rate payer in the interest rate swap used in our pricing example agreed to pay a fixed coupon of 4 percent per year (as opposed to the 3.298 percent for the par value swap). What would the price of the swap be now?

12. What is "cherry picking"? Why does cherry picking complicate the scenario analysis of swap credit risk?

13. What is replacement cost? Why is potential replacement cost an important consideration for measuring swap counterparty credit risk?

14. What is the key difference between a credit default swap and a total return swap?

15. What features of swaps can be customized?

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CHAPTER 7

Risk Management with Swaps

In chapter 6, we described how plain vanilla swaps are used and priced. In this chapter, we further explore how to use swaps for managing portfolio risks. Like other financial derivatives, swaps can eliminate, decrease, or increase risk. Using examples, we first show how plain vanilla swaps can be used to manage portfolio risks. Next, we explore more sophisticated, so-called flavored swap structures.

As with other financial derivatives used to manage interest rate risks, duration is an important concept. In this chapter, we show how to use swaps to manage the duration gap between a firm's assets and liabilities. The duration gap summarizes the interest rate risk that firms face.

In addition to demonstrating how to use swaps for managing risks inherent in underlying business operations, we also explore how a swap dealer manages the risks resulting from holding a portfolio of swaps. By taking part in swap transactions, swap dealers expose themselves to financial risk. This risk can be serious because it is exactly the risk that the swap counterparties are trying to avoid.

MANAGING INTEREST RATE RISK WITH INTEREST RATE SWAPS

Interest rate swaps can be applied to managing the interest rate risk of a firm. In this application, we consider a financial services firm holding most of its assets in amortizing loans with a 10-year average maturity. On the liability side, the firm relies largely on money market obligations and a floating rate note (FRN) with a 5-year maturity. The firm has also issued a coupon bond with a remaining life of 10 years.

We can analyze the interest rate risk of this firm with the concept of duration. Table 7.1 summarizes the market values and durations of the balance sheet items. Based on this information, we can compute the duration of the assets and liabilities. The durations are weighted averages of the

TABLE 7.1 Summary of Durations for Balance Sheet Items

Assets			Liabilities		
Item	Market Value	Duration (Years)	Item	Market Value	Duration (Years)
A: Cash	\$7,000,000	0.00000	D: Money market obligations	\$75,000,000	0.500000
B: Marketable securities	\$18,000,000	0.50000	E: Floating rate notes	\$40,000,000	0.500000
C: Amortizing loans	\$130,467,133	4.604562	F: Coupon bond	\$24,111,725	7.453369
Weighted duration	[(\$7,000,000 × 0.00) + (\$18,000,000 × .5) + (\$130,467,133 × 4.604562)] ÷ \$155,467,133 = 3.922013	3.922013		[(\$75,000,000 × 0.50) + (\$40,000,000 × .5) + (\$24,111,725 × 7.453369)] ÷ \$129,111,725 = 1.705202	1.705202

duration of the individual items; they are weighted by their percentage contribution to the market value of the assets and liabilities. Letting D_A denote the duration of the assets and D_L denote the duration of the liabilities, we can see from Table 7.1 that $D_A = 3.922013$ years and $D_L = 1.705202$ years.

The difference in durations is 2.286999 years, but this measure fails to account for the fact that assets exceed liabilities by the net worth of the firm. What we need is an integrated measure of the duration difference in market value between the assets and liabilities. This measure is called the *duration gap*, D_G, which is defined as:

$$D_G = D_A - \left(\frac{\text{Total liabilities}}{\text{Total assets}}\right) \times D_L$$

The ratio of total liabilities to total assets acts as a scale factor to reflect the difference in market value between the assets and liabilities. For our financial services firm, the duration gap is:

$$D_G = 3.922013 - \left(\frac{\$139,111,725}{\$155,467,133}\right) \times 1.705202 = 2.396201 \text{ years}$$

The duration gap is greater than the difference in durations because the market value of the assets exceeds the market value of the liabilities. The duration gap summarizes the interest sensitivity of the entire firm. We can interpret the duration gap from our example as saying that the firm has an interest rate risk behaving like a long position in a bond with a duration of 2.396201 years.

The firm could use a pay-fixed swap to set the duration gap of the hedged firm so that it equals zero. Consider an interest rate swap with fixed payments of 7 percent, a tenor of seven years, and semiannual payments. To use this swap for hedging purposes, we need to know the swap's interest rate sensitivity as measured by duration. The duration of a swap can be measured by decomposing the swap into its fixed-rate side and its floating-rate side. Viewed in this way, we can think of this pay-fixed swap as consisting of a short position in a seven-year, 7 percent coupon bond and a long position in a floating-rate instrument. Therefore, an interest rate swap has a duration that equals the combined duration of the fixed-rate and floating-rate sides of the swap. The duration of the floating-rate side equals the time between reset dates for the interest rates. Thus, if the floating rate on the swap is reset every six months (like the swap in our example) then the duration of the swap's floating side is equal to six months (i.e., .5 years). Since finding the duration of the floating side of the swap is trivial, the calculation of a swap's duration really depends on finding the duration of the fixed-rate side of the swap. For the swap in our example, we take as given that the duration of the fixed side is 5.651369 years. Therefore, for this pay-fixed swap the duration, D_S is:

$$D_S = .5 - 5.651369 = -5.151369 \text{ years}$$

Had this been a receive-fixed swap, the duration would have been +5.151369 years. The signs remind us that a receive-fixed swap lengthens the duration of an existing position, whereas pay-fixed swaps can effectively decrease the duration of an existing position.

We now need to determine the notional principal amount that will allow us to construct the risk-minimizing hedge, that is, the notional principal of the swap that will produce a duration gap for the hedged firm equal to zero. This is determined in a straightforward way by equating the duration gap of the firm (weighted by the value of the firm's assets) to the duration of the swap (weighted by the swap's notional principal) and then solving for the notional principal:

$$2.396201 \times \$155{,}467{,}133 - 5.151369 \times \text{Notional Principal of Swap} = 0$$

This equation yields a notional value of \$72,316,800 for the pay-fixed swap.

Now suppose that our firm wants to reduce, but not eliminate, the interest rate risk inherent in the firm's operations. Management decides to make the firm behave like a bond with a duration of one year, instead of behaving like a bond with a duration of 2.396201 years.

In general, we can use swaps to set the duration gap of the firm to any desired level as follows:

$$D_G^* = D_G + D_S\left(\frac{\text{Notional principal of swap}}{\text{Total assets}}\right) \tag{7.1}$$

where D_G^* = the desired duration gap
D_S = the duration of the swap

To set the duration gap of the firm to one year, we can determine the required notional principal of the swap using equation 7.1:

$$D_G^* = 1 = 2.396201 + 5.151369\left(\frac{\text{Notional principal of swap}}{\$155{,}467{,}133}\right)$$

This equation yields a notional principal of –\$42,137,025. The negative sign for notional principal indicates that a pay-fixed swap is required. If one were sure that a pay-fixed swap would be required, the sign for D_S could be input as a negative reflecting the duration of the pay-fixed position.

With a duration gap greater than zero, the firm's balance sheet is exposed to the danger of rising interest rates. If the firm expects rates to rise and wants to speculate on this view, it could set its duration gap to be less than zero. A modest speculative position could be achieved with a duration gap of –.5 years. The swap position required to achieve this exposure would be:

$$D_G^* = -.5 = 2.396201 + 5.151369\left(\frac{\text{Notional principal of swap}}{\$155{,}467{,}133}\right)$$

This equation yields a notional principal of –\$87,406,681 for our swap with a tenor of seven years and a 7 percent fixed swap rate.

So, a pay-fixed swap with a notional principal of \$87,406,681 is required to change the duration gap to –.5 years. This procedure moves the duration gap of the firm from its original positive value of 2.396201 years. Now if rates rise, the firm will benefit. However, it is exposed to losses if rates fall.

MANAGING EQUITY RISK WITH EQUITY SWAPS

To see how an equity swap can be used to manage equity risk, consider an equity portfolio manager who is bearish for stocks over the near term, but expects current values to once again prevail within a year. The portfolio is well diversified and tracks the S&P 500. The current value of the portfolio is \$150,000. The manager decides to avoid the risk of short-term fluctuations by entering a swap to pay a floating rate on the equity side and to receive a fixed interest payment for each quarter over the next year.

Under the terms of the swap, the manager agrees to pay the return on a reference index each quarter (e.g., the S&P 500) times \$135,000 per index point, in return for a fixed interest payment.[1] With the index standing at 1107.10, this gives a notional principal on the swap of \$149,458,500, which is almost exactly the same as the current value of the portfolio. In return, the portfolio manager will receive a fixed rate of 2.3676 percent per quarter on the same notional principal, giving a quarterly inflow of \$3,538,579. The portfolio itself is undisturbed, so the manager continues to receive the dividends, which are expected to total \$5,850,000 over the next year.

Table 7.2 shows two possible outcomes for this swap. In the upper panel, the portfolio manager's expectations are essentially realized. Starting from an index value of 1107.10, the index first falls and then returns to its original level by the end of the year at a value of 1108.50. The table shows the four fixed interest receipts of \$3,538,579 for a total interest inflow of \$14,154,316. Because the manager is obligated to make payments based on the return index, the payments can be either positive or negative. The fifth column of Table 7.2 shows the cash flows for the equity side of the swap. A negative value in this column indicates that the equity side of the swap will receive a payment based on the index return in addition to the interest payment for the debt side of the swap. For example, in the first quarter, the index falls from 1107.10 to 1043.06 for a return of –0.057845. The corresponding payment on the equity side is really an inflow in this circumstance of \$8,645,400 = 0.057845(\$149,458,500). The final column of Table 7.2 shows the net cash flow from the swap in each period. In the upper panel, the total outflows on the equity side of the swap were \$650,090, and the swap generated a net cash inflow of \$13,504,226.

TABLE 7.2 Possible Outcomes for the Equity Swap

Quarter	Interest Received	Index Values	Index Return	Equity Side Cash Outflow	Net Cash Flow on Swap
		Manager's Expectations Realized			
0		1107.10			
1	$ 3,538,579	1043.06	−0.057845	$−8,645,400	$12,183,979
2	3,538,579	1056.03	0.012435	1,858,452	1,680,127
3	3,538,579	1110.10	0.051201	7,652,454	−4,113,875
4	3,538,579	1108.50	−0.001441	−215,416	3,753,995
Total:	$14,154,316			$ 650,090	$13,504,226
		Alternative Scenario			
0		1107.10			
1	$ 3,538,579	1123.45	0.014768	$ 2,207,250	$ 1,331,329
2	3,538,579	1157.09	0.029943	4,475,307	−936,728
3	3,538,579	1215.02	0.050065	7,482,677	−3,944,098
4	3,538,579	1232.00	0.013975	2,088,694	1,449,885
Total:	$14,154,316			$16,253,929	$−2,099,613

Without the swap, the terminal value of the portfolio would be the result of the change in the value of the stocks plus the dividends received:

$$\text{Terminal portfolio value without swap} = \$150,000,000 \times \left(\frac{1108.50}{1107.10}\right) + \$5,850,000 = \$156,039,685$$

The total return on the portfolio would have been 4.03 percent. With the swap, the terminal portfolio value equals the terminal value without the swap, plus the interest payments received, less the payments on the equity side of the swap. From the upper panel of Table 7.2:

$$\text{Terminal portfolio value including swap} = \$156,039,685 + \$14,154,316 - \$650,090 = \$169,543,911$$

With the terminal portfolio value $169,543,911, the total return on the portfolio was 13.03 percent, making the portfolio manager a hero.

Things might have gone differently, however. The bottom panel of Table 7.2 shows the results of an alternative scenario—the stock market enjoys a bull rally over the next year. In this scenario, the index rises from 1107.10 to 1232.00.

$$\text{Terminal portfolio value without swap} = \$150{,}000{,}000 \times \left(\frac{1232.00}{1107.10}\right) + \$5{,}850{,}000 = \$172{,}772{,}591$$

This gives a total return on the portfolio of 15.18 percent.

In this alternative market environment, the swap would still yield the quarterly interest receipts of \$3,538,579, for a total of \$14,154,316. However, with the market rising every period, the equity side of the swap has substantial outflows each quarter. These outflows total \$16,253,929. Therefore, the portfolio value with the swap would be:

$$\text{Terminal portfolio value including swap} = \$172{,}772{,}591 + \$14{,}154{,}316 - \$16{,}253{,}929 = \$170{,}672{,}978$$

With the swap, the terminal value is \$2,100,000 less than it would have been otherwise, and the return on the portfolio is only 13.78 percent instead of the 15.18 percent the undisturbed portfolio would have enjoyed.

In addition to serving as a useful hedging tool, an equity swap can be used for other purposes. The rising stock market of the 1990s produced many wealthy individuals with highly concentrated stock holdings. It was not uncommon to observe corporate CEOs who had acquired tens of millions of dollars of their company's stock through option grants. Their undiversified holdings can be a problem for these wealthy individuals. An attempt to diversify by selling shares means they will realize taxable capital gains if they have a low tax basis in the shares they own. In addition, by selling they will give up voting privileges attached to the stock. To diversify without triggering a taxable event or losing their voting rights, they can use equity swaps. With an equity swap, the wealthy individual can pay the dealer the dividends and capital gains on a set number of shares over the life of the swap in return for a payment from a swap dealer based on a well-diversified portfolio like the S&P 500. The end result is that the portfolio return moves up and down with the S&P 500, while the wealthy individual retains the voting rights of the company shares and does not trigger a tax event. In recent years, however, advisers have warned wealthy clients that the Internal Revenue Service may make such transactions taxable.

MORE SOPHISTICATED SWAP STRUCTURES

In this section, we consider more sophisticated swap structures, including instruments that combine features of options and swaps. We begin by analyzing so-called flavored swaps, that is, swaps with features that differ from the key features of plain vanilla swaps. We also consider forward swaps and extension swaps, which allow for the changing of the payment streams obligated in the swap agreements. We provide an example of how one type of flavored swap, a seasonal swap, can be used to manage interest rate risk for a firm with seasonal financing requirements. Finally, we consider swaptions, which are options on swaps.

FLAVORED SWAPS[2]

In addition to plain vanilla interest rate swaps, many other flavors of swaps can be found, most of which are distinguished by differences in their key underlying economic terms. Following are some of the more standardized structures. These swaps masquerade under many names because swap dealers frequently attach proprietary trademarked labels to generic structures. Whenever possible, we have attempted to use generic names.

A popular flavored swap structure is the *amortizing swap,* where the notional principal is reduced over time. This means that the fixed interest payment becomes smaller during the life of the swap, and the floating payment does as well, at least if interest rates are stable. Because mortgage principal is generally amortized, an amortizing swap provides a useful instrument for managing the interest rate risk associated with mortgages.

Although the notional principal diminishes in an amortizing swap, in other kinds of swaps it can be scheduled to increase. In an *accreting swap,* the notional principal becomes larger during the life of the swap. This kind of swap matches the cash flows often encountered in construction finance. For example, consider a construction project in which the builder will draw down $10 million of additional financing at the end of each of the seven years of the life of a particular building project as interim construction objectives are achieved. Typically, this kind of financing is committed at the outset of the project, and the additional loans are promised at a floating rate. These flows could be converted to a fixed rate through an accreting swap designed with a principal that increases $10 million each year and has a fixed interest rate.

By combining features of amortizing and accreting swaps, it is possible to create interest rate swaps with variable notional principals that increase and decrease over the life of the swap. In a *seasonal swap,* the notional principal varies according to a fixed plan. This kind of swap can be useful in matching the financing needs of retailers. For example, the swap could be structured on a seasonal basis to match the typically heavy fourth-quarter cash needs of retailing firms. When the notional principal on the swap first increases and then amortizes to zero over the life of the swap, the swap is called a *roller-coaster swap*. Thus, the notional principal can be structured to conform to any financing or risk management need.

In addition to allowing the notional principal of an interest rate swap to vary, swaps can be created with variations on coupon payments. In a plain vanilla swap, the fixed and floating payments are typically established at the prevailing rates when the swap is initiated. Consider a five-year swap with annual payments on a notional principal of $25 million initiated when the yield curve is flat at 8 percent. In this situation, one might create an *off-market swap* by setting the fixed payment at 9 percent and the floating payment equal to LIBOR. In this example, the pay-fixed party has agreed to pay a rate above the par value swap rate. To compensate for this series of extra payments, the pay-floating party must make an additional cash payment to the pay-fixed party. With a fixed rate of interest at 1 percent above the market on a notional principal of $25 million, the pay-fixed party will be paying an excess $250,000 per year. With the yield curve flat at 8 percent, the appreciated payment from the pay-floating party to the pay-fixed party would be the present value of those five payments of $250,000 discounted at 8 percent, which is $998,178.

In this example, the pay-fixed party receives $998,178 at the initiation of the swap and pays $2,250,000 annually. The pay-floating party pays on a notional principal of $25 million. Thus, this example includes features of a loan and an interest rate swap, with the pay-floating party providing approximately $1 million of financing to the pay-fixed side of the deal as part of the swap agreement.

In a *forward swap,* the parties agree that the cash flows will begin at a date in the future and that they can be interest rate, currency, commodity, or equity swaps. Two counterparties might agree to exchange LIBOR for a fixed rate beginning in two years, with the tenor of the swap being five years from that date. If the contractual rates are based on the forward rate for the two instruments at the planned initiation date, there should be no exchange of cash at the initiation of the agreement. If the rates specified do not conform to the forward rates for the planned inception of the cash flows, then the swap is an off-market swap and one party will be obligated to pay the other.

An *extension swap* is an agreement designed to extend the tenor of an existing swap. As such, an extension swap is a special type of forward swap. In the middle of the tenor of an interest rate swap in which one party pays LIBOR and receives a fixed rate, the parties might agree to extend the tenor of the swap by an additional three years. If the agreement is initiated based on the forward rates for the date of inception (i.e., at the termination of the current swap agreement), there should be no payment at the time the agreement is made.

In a plain vanilla interest rate swap, one party pays a fixed rate of interest, while the second pays a floating rate. In a *basis swap,* both parties pay a floating rate of interest, but the payments are computed on different reference rates. Assume that three-month LIBOR currently stands at 6 percent, while the three-month T-bill rate is 5.25 percent. In a basis swap, one party might pay the three-month T-bill rate plus 75 basis points, while the other party would pay LIBOR. Although the T-bill rate and LIBOR tend to move together, the spread between the two rates changes with the perceived differential in default risk between the bank obligations and U.S. Treasury issues.

If the rate spread widens, the party paying the T-bill rate plus 75 basis points will win; if the rate spread narrows, the party paying LIBOR will win. Assuming payments based on a notional principal of $100 million, at initiation both parties expect to pay $6 million annually, corresponding with a 5.25 percent T-bill rate and LIBOR of 6.00 percent. Thus, both parties expect to pay as much as they receive initially. Suppose that both rates rise, but that LIBOR rises more, say, to a LIBOR rate of 6.4 percent and a T-bill rate of 5.5 percent. With these new rates, the LIBOR payment would be $6.4 million; the T-bill payer would be obligated to pay $6.25 million, leaving a net payment of $150,000 per year from the LIBOR payer.

Another kind of basis swap is a yield curve swap. In a *yield curve swap,* both parties pay a floating rate, but the reference rates differ in term to maturity along the yield curve. One party might pay based on the three-month T-bill rate, while the other party might pay based on a five-year Treasury note. Assuming that long-term rates are initially above short-term rates, a flattening yield curve would benefit the party paying based on the five-year rate, while a steepening yield curve would benefit the payer basing payment on the short-term rate.

Yield curve swaps have particular appeal to financial institutions with long-term assets and short-term liabilities, such as a savings and loan association. These institutions are subject to losses if short-term rates rise relative to long-term rates. The savings and loan might enter into a yield curve swap in which it makes payments based on a long-term reference rate and receives payment based on a short-term reference rate. This swap arrangement could help the savings and loan manage the yield curve risk it faces as part of its core business.

A *rate differential swap* or *diff swap* has payments tied to reference rates in two different currencies, but all payments are made in a single currency. For example, a diff swap might be structured with all payments in

U.S. dollars, with one party paying based on three-month US Dollar LIBOR and the other party paying based on three-month Euribor, but with payments in dollars. Assume that both the U.S. dollar and euro yield curves are flat, that the dollar rate is 3.25 percent and the euro rate is 3.00 percent. The payment based on US Dollar LIBOR might be LIBOR flat, and the payment based on Euribor would be Euribor flat, but paid in dollars. This kind of swap would exploit changes in U.S. versus European Union interest rates over the tenor of the swap.

The interest rate swaps considered so far all focus on differences in levels of interest rates, whether those rates are of the same or different maturities, or in the same or different currencies. A *corridor swap* is structured so that payment obligations occur only when the reference rate is within some specified range or corridor. One might contract to receive a fixed rate and pay six-month LIBOR only when LIBOR is greater than 3 percent but less than 5 percent. In this situation, the fixed rate would be lower than on a comparable plain vanilla interest rate swap. The corridor swap is essentially a speculation on the volatility of LIBOR.

Foreign currency swaps can also be flavored. A *currency annuity swap* is similar to a plain vanilla currency swap without the exchange of principal at the initiation or the termination of the swap. It is also known as a *currency basis swap*. In a currency annuity swap, one party might make a sequence of payments based on 3-month European Union Euribor while the other makes a sequence of payments based on 3-month US Dollar LIBOR. The currency annuity swap generally requires one party to pay an additional spread to the other or to make an up-front payment at the time of the swap. Allowing one, or both, parties to pay at a fixed rate can create variations on this structure. In pricing these swaps, the key is to specify a spread or up-front payment that equates the present value of the cash flows incurred by each party.

A *CIRCUS swap* is a fixed-for-fixed currency swap created by combining a plain vanilla interest rate swap with a plain vanilla currency swap. (CIRCUS stands for "combined interest rate and currency swap.") Consider a firm that enters a plain vanilla currency swap. In this swap, the firm receives fixed-rate Japanese yen and pays US Dollar LIBOR. The firm also enters a U.S. dollar plain vanilla interest rate swap to pay-fixed/receive-floating. Assuming the two swaps have the same tenor and notional principal, the firm has combined two plain vanilla swaps to create a fixed-for-fixed currency swap.

A *macro swap* ties the floating rate of the swap to a macroeconomic variable or reference index such as the growth rate in GNP or the wholesale price index. Macro swaps differ from the swaps discussed so far, which exist primarily to manage interest rate or exchange rate risk. For companies whose sales and profits are highly correlated with the business cycle, macro swaps can be used to manage business cycle risk.

MANAGING SEASONAL FINANCING REQUIREMENTS WITH SEASONAL SWAPS

Many businesses face seasonal financing requirements. Consider a toy store chain facing a surge in financing requirements when stocking up for the Christmas season. The chain's financing needs are constant over the first three quarters of the year and then surge in the fourth quarter. To meet its financing requirements for the year, the chain might issue a one-year bond at a fixed rate. The proceeds of the bond would be sufficient to meet its cash needs for the year. Another possibility would be to issue short-term commercial notes each quarter at the prevailing rate each quarter. A third alternative would be for the chain to issue short-term commercial notes each quarter and hedge the interest rate risk with a pay-fixed seasonal swap.

Suppose the chain anticipates financing needs for the upcoming year of \$10 million for each of the first three quarters and \$50 million for the fourth quarter. Based on the current forward rates for three-month US Dollar LIBOR (as displayed in Table 6.4), the expected interest payments for the year are:

First Quarter	$.030 \times .25 \times \$10{,}000{,}000 = \$70{,}500$
Second Quarter	$.032 \times .25 \times \$10{,}000{,}000 = \$80{,}000$
Third Quarter	$.034 \times .25 \times \$10{,}000{,}000 = \$80{,}500$
Fourth Quarter	$.036 \times .25 \times \$50{,}000{,}000 = \$450{,}000$

As shown in Chapter 6, the zero-coupon factors consistent with these forward rates are $Z_{0,.3} = 1.0075$; $Z_{0,.6} = 1.01556$; $Z_{0,9} = 1.024192$; and $Z_{0,12} = 1.03341$. We can determine the fixed rate for this swap that will make the present value of the expected floating-rate payments equal to the present value of the (yet-to-be-determined) fixed-rate payments:

$$\frac{\$70{,}500}{1.0075} + \frac{\$80{,}000}{1.01556} + \frac{\$80{,}500}{1.024192} + \frac{\$450{,}000}{1.03341} = \frac{(\text{PVSR} \times .25 \times \$10{,}000{,}000)}{1.0075} + \frac{(\text{PVSR} \times .25 \times \$10{,}000{,}000)}{1.01556} + \frac{(\text{PVSR} \times .25 \times \$10{,}000{,}000)}{1.024192} + \frac{(\text{PVSR} \times .25 \times \$50{,}000{,}000)}{1.03341}$$

This equation reduces to:

$$\$667{,}193.30 = \text{PVSR} \times \$19{,}479{,}910.92$$

This means that PVSR equals .03425, or 3.426 percent. This is the fixed rate that makes the value of the seasonal swap equal to zero, or par, at initiation. This is the rate that the toy store chain pays to the swap dealer to turn its floating-rate obligation into a fixed-rate obligation.

SWAPTIONS

The holder of a *swaption* has the option to enter into a swap in the future. Swaptions can be European-style, American-style, or Bermudan-style. The holder of a European-style swaption can enter into the swap only on a specified date (the expiration date of the swaption). The holder of an American-style swaption can enter into the swap at any time prior to the expiration date of the swaption. The holder of a Bermudan-style swaption can enter into the swap only on specific dates prior to the expiration date of the swaption. In the jargon of the swaps market, a *payer swaption* gives the holder the right to enter into a swap as the fixed-rate payer. A *receiver swaption* gives the holder the right to enter into a swap as the fixed-rate receiver.

A payer swaption is the functional equivalent of a call option on interest rates. The holder of the payer swaption can exercise the right to enter into a pay-fixed swap if the floating rate rises. Conversely, a receiver swaption is the functional equivalent of a put option on interest rates. The holder of the receiver swaption can exercise the right to enter into a receive-fixed swap if the floating rate declines.

Consider a payer swaption. The purchaser of the payer swaption pays a premium to the seller at the inception of the transaction. Usually the premium is stated as a number of basis points on the notional principal of the swap underlying the swaption. The amount of the premium will depend on the exercise price (or rate), the time to expiration, and the volatility of the underlying rates. Assuming the swaption is European-style, the holder will exercise the payer swaption if the contractual fixed rate (the exercise price) is lower than the fixed rate prevailing in the open market for swaps with the same tenor as that underlying the payer swaption. When exercised by the holder of the payer swaption, the seller of the payer swaption is obligated to make the series of floating rate payments specified in the swap agreement.

Consider a European payer swaption on a five-year swap with annual payments and a notional principal of $10,000,000. Assume that the fixed rate specified in the swap agreement is 8 percent and the floating rate is LIBOR plus 50 basis points. For this payer swaption, the premium might be 30 basis points. With a principal of $10,000,000, the premium would then be $30,000. Six months after contracting, at the expiration date of the option, the owner of the payer swaption can either exercise or let the option expire worthless. If the owner exercises, he will pay a fixed rate of 8 percent and receive a floating rate of LIBOR for the five years of the swap. The owner of the payer swaption will exercise if the fixed rate on swaps of the type underlying the option is greater than 8 percent. Assume that at expiration the market for this type of swap calls for a fixed rate of 8.5 percent in exchange for LIBOR for a five-year tenor. The holder of the payer swaption will exercise because he can enter the agreement at terms more favorable than those prevailing in the market. Conversely, if the market fixed rate for such a swap is less than 8 percent, say 7.75 percent, the owner of the payer swaption will allow the swaption to expire. It is worthless because it gives the owner the right to enter a swap to pay 8 percent and receive LIBOR. However, the prevailing market rate allows anyone to enter a swap to pay 7.75 percent and receive LIBOR.

Consider now the choices facing the holder of a European receiver swaption. To acquire this swaption, the owner pays a premium to purchase the swaption. At the expiration date, the owner may exercise. If she exercises, she will enter a swap to receive-fixed and pay-floating. The owner should exercise if the fixed rate on the swap underlying the swaption exceeds the market fixed rate on swaps of the same type as that underlying the swaption. Assume that the swap underlying this receiver swaption has a seven-year tenor with semiannual payments, a notional principal of $50 million, and a fixed rate of 6.5 percent in exchange for LIBOR. At expiration, assume that the market fixed rate for this type of swap is 7 percent. In this circumstance, the holder will allow the receiver option to expire worthless because it allows the holder to enter a swap to receive 6.5 percent, but the market rate for this type of swap is a fixed rate of 7 percent. By contrast, if the prevailing fixed rate is 6 percent at the expiration of the swaption, the holder of the receiver swaption should exercise. The swaption entitles her to enter a swap to receive a 6.5 percent fixed rate when the prevailing market offers only a 6 percent fixed rate on this type of swap.

Swaptions offer the same speculative and hedging opportunities that other options do. Consider a firm that has issued a callable bond. We may analyze a bond with a call provision as consisting of a noncallable bond plus the purchase of a call option on the bond. From the issuing firm's point of view, the firm has paid for the option by promising a higher coupon rate for the callable bond than the rate necessary for a noncallable bond.

Now assume that the firm has determined that it will not call the bond because it anticipates a continuing need for the funds and does not want to incur the transaction cost of calling the bond and refinancing. Having made this determination, the call feature on the bond has no value to the firm, yet the option still has value in the marketplace. The firm's problem is to find a way to capture the value of the call feature without actually calling the bond. The bond-issuing firm could sell a call swaption (i.e., a payer swaption) with terms that match the call feature of the bond. In effect, this transaction would unwind the call feature embedded in the original bond. The firm bought a call from the bondholders with its original issuance, and by selling a call swaption, it recaptures the remaining value in the call feature that the firm no longer desires.

If rates do not fall sufficiently to trigger exercise of the call swaption against the firm, the firm merely keeps the option premium, and this cash inflow offsets the value of the call feature inherent in the callable bond. On the other hand, if the owner of the call swaption exercises against the firm, it will have done so because fixed rates are below the currently available fixed rate on a swap. If this exercise is reasonable, the issuer of the callable bond now has an incentive to exercise its call option against its own bondholders, and it should not lose on the exercise.

We have described flavored swaps, forward swaps, extension swaps, and swaptions. Flavored swaps can be distinguished from plain vanilla swaps by differences in key underlying economic terms. A payer swaption gives the holder the right to enter into a swap as the fixed-rate payer. A receiver swaption gives the holder the right to enter into a swap as the fixed-rate receiver.

SWAP PORTFOLIOS

In this section, we briefly consider the principal risks that a swap dealer faces in managing a swap portfolio. The swap dealer essentially has a portfolio of swaps from numerous transactions in the swaps market. The risks residing in the swap portfolio include credit risk, basis risk, mismatch risk, and interest rate risk. We illustrate how the swap dealer can manage some of these risks.

Risks in Managing a Swap Portfolio

In managing a portfolio of many swaps, the swap dealer faces several risks. First, one of its counterparties might default, as discussed in Chapter 6. Second, the dealer faces *basis risk*—the risk that the normal relationship between two prices might change. Assume that a dealer engages in an interest rate swap agreeing to receive the T-note rate plus some basis points and to pay LIBOR. After this agreement is reached, assume that market disturbances in Europe cause LIBOR to rise relative to the T-note rate. The swap dealer must still pay LIBOR, but this rate is now higher than the swap dealer anticipated when initiating the swap. Therefore, the swap dealer suffers a loss due to basis risk as the normal relationship between LIBOR and the T-note rate has changed.

The swap dealer also faces mismatch risk. When acting as a counterparty in a swap, the swap dealer accepts a risk position and is anxious to offset it by engaging in other swaps. *Mismatch risk* refers to the swap dealer's risk of being left in a position that he cannot offset easily through another swap. This arises if there is a mismatch in the needs between the swap dealer and other participants. In Chapter 6, we discussed a swap dealer with mismatch risk resulting from transactions with swap counterparties. The dealer's transactions with two separate counterparties left the swap dealer with a residual risk position due to the mismatch between the needs of the counterparties.

One of the most serious risks that the swap dealer faces is interest rate risk. The dealer may have promised to pay a floating rate and to receive a fixed rate. If the general level of interest rates rises, the dealer's cash outflows will rise as well. However, the dealer continues to receive the stipulated fixed rate. The swap dealer incurs a loss due to a shift in interest rates. In Table 6.2, the swap dealer is to receive \$1.2 million annually and to pay LIBOR + 3% on a notional principal of \$10 million in years four and five. If rates rise, the payments that the dealer must make will increase, while cash inflows will remain the same. Such a rise in interest rates would generate a loss, so the dealer faces interest rate risk.

Managing Mismatch and Interest Rate Risk in a Swap

Swap dealers can manage mismatch and interest rate risk by considering the swap dealer's transactions with Parties A and E, as shown in Table 6.2. As noted, the swap dealer accepts a risk position by acting as a counterparty in a swap. The swap dealer, who wishes to function strictly as a financial intermediary and not as a speculator, is eager to overcome any temporary risk that might have been incurred to complete the swap.

In our discussion of Table 6.2, we saw that the dealer participated in a swap with Party A and was able to offset part of the risk by engaging in another swap with Party E. However, some residual risk remains: The swap dealer is still committed to receiving \$1.2 million and paying LIBOR + 3% on a notional amount of \$10 million in years four and five.

This residual risk position reflects both mismatch risk and interest rate risk. The mismatch risk occurs because the dealer was unable to offset the risks associated with the swap with Party A. The transaction with Party E offset most of the risk arising from the swap with Party A, but some risk remains because of the mismatch between the needs of Parties A and E. The transactions of Table 6.2 also reflect a continuing interest rate risk. As noted, if rates rise, the dealer suffers a loss in paying the higher floating rates that result.

The swap dealer will be anxious to avoid these two risks remaining in periods four and five. Ideally, the dealer would arrange a third swap, in addition to those with Parties A and E, to offset the risk. For example, the dealer would like to swap a receive-floating and pay-fixed for years four and five. Such a transaction would avoid both the mismatch and the interest rate risk. However, such swaps are not always immediately available. As a consequence, the swap dealer will seek other means to control this risk.

A swap dealer who faces a risk such as that shown in Table 6.2 can use the futures market to temporarily offset the risk. For example, the swap dealer might sell Eurodollar futures with a distant expiration. Recall that Eurodollar futures are based on three-month US Dollar LIBOR. With this transaction, the swap dealer offsets a considerable portion of the remaining risk. By executing the futures transaction properly, the dealer will be left only with an obligation to pay a fixed amount.

Even after this transaction, however, some risk remains. Eurodollar futures may be a close substitute for the unavailable swap, but they are unlikely to provide a perfect substitute. In our example, the dealer will probably not be able to match the futures expiration with the four- and five-year cash flows and there is likely to be some imperfection in setting the quantity of futures to trade.

Because of these imperfections in substituting for the unavailable swap, the swap dealer will likely continue to seek a swap that meets the risk needs exactly. Until it is available, however, the Eurodollar futures position can act as an effective risk-reducing position.

THE SWAP AS A PORTFOLIO OF FORWARDS

In Chapter 6, we saw that a swap can be viewed as a portfolio of forward contracts. In this section, we show how Eurodollar futures can hedge LIBOR-based swaps. We present the analysis in terms of forwards to avoid the complications with margin cash flows on futures. However, if we ignore the daily settlement cash flows characteristic of futures, the analysis holds equally well for both futures and forwards. A swap agreement may be analyzed as a portfolio of forward contracts by focusing on an interest rate swap. However, the principle holds for any kind of swap. This equivalence means that we may regard a swap as a synthetic portfolio of forward contracts. Likewise, a portfolio of forward or futures positions can be used to synthetically create a swap agreement. In Chapter 6, we applied this logic to the pricing of swap contracts.

In an interest rate swap, two parties agree to make interest payments on the same underlying principal or notional amount over a specified period. One party agrees to pay a fixed interest rate, while the second party promises to pay a floating rate. On contracting, the fixed-rate payer knows exactly the cash flows that it is obligated to make, but the floating-rate payer's cash flows depend on the course of interest rates during the life of the agreement. An interest rate swap might have a notional amount of $1 million and the fixed-rate payer might promise to pay 10 percent annually for 10 years. Thus, the fixed-rate payer promises to make 10 annual payments of $100,000. For its part, the floating-rate payer might promise to pay LIBOR plus 2 percent. If LIBOR is 8 percent at the time a particular payment is made, the floating-rate payer will also pay 10 percent of $1 million or $100,000. If LIBOR is less than 8 percent, the floating-rate payer will pay less than it receives; if LIBOR exceeds 8 percent, the floating-rate payer will pay more than it receives. (Generally, only the difference is paid on any particular payment date.)

Let us consider just one of the 10 payments in this example of an interest rate swap. The fixed-rate payer has promised to pay $100,000 in return for a payment that depends on LIBOR. We may analyze this payment as a forward contract to pay $100,000 at a future date in return for a value that is to be determined by the value of LIBOR on that future date. In essence, this forward contract has the same structure as any interest rate forward or futures contract.

We can see this equivalence by considering a T-bill futures contract. The purchaser of a futures contract promises to pay a certain amount on a future date in return for a 90-day T-bill to be delivered at that time. At the time of contracting, the buyer of the futures knows what payment it will be required to make, but it does not know the value of the T-bill it will receive. The value of the bill depends on future interest rates. Similarly, in the swap agreement, the fixed-rate payer knows what payment it will make on a future date, but it does not know what payment it will receive.

An interest rate swap generally includes a series of payments. In our example, the swap had 10 annual payments. Each payment can be analyzed as an interest rate forward contract. Because the swap agreement includes a sequence of 10 such arrangements, the swap is a portfolio of forward contracts.[3] Consider a forward contract on an interest rate instrument that calls for the purchaser to receive delivery of a three-month money market instrument on the expiration date in exchange for a payment determined when the forward contract is negotiated. The purchaser of the forward contract will gain from the transaction when the yield implied by the forward contract is lower than the yield prevailing on three-month instruments at the expiration of the forward contract. In effect, the forward contract requires the exchange of a fixed payment for a floating payment, with the gain or loss being realized at the expiration date.

Assume that today is December 15, 2002, and a party buys a forward contract to mature on December 15, 2003, calling for the purchase of $1,000,000 face value of three-month Eurodollar deposits at a yield of 3 percent. This contract essentially establishes the payment of a fixed rate of 3 percent for these instruments. If the actual yield on these instruments at the expiration date is lower than 3 percent, say 2 percent, then the purchaser of the forward contract has a gain. For a three-month zero coupon instrument and typical money market yield conventions, this 1 percent interest rate differential would be worth $2,500, reflecting a $25 per basis point value. If rates had risen, the futures hedge would have offset the swap dealer's gains under the terms of the swap contract. By hedging, the swap dealer has deliberately chosen to avoid exposure to fluctuating interest rates. In the jargon of swap dealers, our example describes the *delta hedging* of a swap.

This forward contract is similar to one of the payments in an interest rate swap agreement. An interest rate swap agreement is a portfolio of such payments. Therefore, we may analyze an interest rate swap as a portfolio of forward contracts with successive expiration dates.

As discussed in Chapter 2, futures contracts are a type of forward contract with specific additional institutional features. A futures is distinguished from other forward contracts by the margining and daily resettlement feature along with the presence of exchange trading and clearinghouse guarantees. Aside from these institutional considerations, an interest rate futures contract is essentially like an interest rate forward contract. Therefore, an interest rate swap can be viewed as a portfolio of successively maturing interest rate futures contracts. In terms of our earlier example, an interest rate swap is similar to a strip of interest rate futures contracts. A *strip* is a sequence of futures contracts with successive expirations. Because interest rate swap agreements so often use LIBOR as the floating rate in the contract, a strip of Eurodollar futures contracts is highly analogous to an interest rate swap agreement.

Because a strip of futures contracts can be regarded as a substitute for an interest rate swap, the swap dealer can use Eurodollar strips to hedge unwanted interest rate risk that arises in the swap business. However, such a strategy requires an active market in distant Eurodollar maturities. In the early days of the swaps market, the Eurodollar futures contract certainly did not possess the depth or liquidity to allow Eurodollar strips to serve this role. At the end of 1986, total open interest in all Eurodollar contract expirations was 214,000 and the most distant listed Eurodollar futures was the December 1988 contract, only two years distant. Figure 7.1 shows recent quotations for the Eurodollar contract with a total open interest approaching five million contracts and with contract expirations extending for nearly 10 years. This extended maturity range and deep liquidity is unparalleled for any other futures contract of any type. The main reason for these special features is the interest of swap dealers in using Eurodollar strips to offset risk inherent in the interest rate swap positions that they undertake.

Managing Residual Risks in Swap Portfolios

After hedging, swap dealers often hold residual risk in their swap portfolios because the hedging is imprecise or too costly. In addition to residual risks, swap dealers often consciously take proprietary risk positions in hopes of profiting on these positions. To manage the risk in their portfolios, swap dealers must first be able to measure it. Swap dealers rely on two important tools to manage this market risk: value at risk (VaR) and stress testing.

Value at Risk (VaR) is a method of assessing market risk. It is a summary measure of potential loss from an unlikely event occurring in a normal, everyday market environment. For example, a swap dealer might estimate its one-day-ahead VaR to be $20 million at the 99 percent confidence level. There is estimated to be a 1 percent chance, under normal market conditions, that a daily loss greater than $20 million will occur. This single number summarizes the swap dealer's exposure to market risk as well as the probability of an adverse move. The VaR is a probabalistic statement. Therefore VaR is a statistical measure of risk exposure.[4]

There are several ways to measure VaR. The *variance-covariance approach* is a popular method. In this approach, individual risk factors are first identified. These factors may be interest rates, exchange rates, stock prices, or any other variables that contribute to the risk of the swap portfolio. The risk factors affecting the portfolio's risk will depend on the swaps the dealer holds. In the variance-covariance approach, the variance of each individual risk factor is measured using historical daily data. In addition, the covariance between factors is estimated using the same set of data. Using pricing models for each swap in the portfolio, the sensitivity of the portfolio to each factor is estimated. These factor sensitivities, when combined with the variance and covariance estimates, yield an estimate of the probability distribution of possible values for the swap portfolio one day ahead. From this estimated distribution, which is assumed to be a normal distribution, the value of the portfolio corresponding with the 99th percentile of the distribution can be estimated. This value would be the VaR at a 99 percent level of confidence.

[Image not available in this electronic edition.]

FIGURE 7.1 Quotations for Eurodollar futures.

Another popular method of measuring VaR is *Monte Carlo simulation*. This method is used to simulate different scenarios for portfolio value. The approach requires an assumption about the distribution of changes in market prices and rates. It is common to assume a normal distribution. Historical data on market prices and rates are then used to estimate the parameters of the distribution. Using this information, possible future values for changes in rates and prices are simulated. For simulation, the portfolio is revalued. The simulation procedure generates a set of portfolio values corresponding to possible changes in rates and prices. From this distribution of possible portfolio values, the 99th percentile loss would represent the VaR at a 99 percent level of confidence.

A third method of measuring VaR is *historical simulation*. This simulation technique skips the step of making assumptions about the distribution of daily changes in market prices and rates. Instead, it assumes that whatever changes in prices and rates were observed in the past can represent the distribution of changes over the forecast horizon. These historical changes in rates and prices are used to revalue the current portfolio. Historical simulation generates a set of portfolio values corresponding to possible changes in rates and prices. From that distribution of possible portfolio values, the 99th percentile loss will represent the VaR at a 99 percent level of confidence.

Value at Risk is a statistic about the distribution of possible future losses on a portfolio. The actual gain or loss will not be known until it happens. It is a useful statistic for swap portfolio risk managers because the measure can be tracked to determine whether market risk in the portfolio is approaching predetermined risk limits. The VaR measure serves as a benchmark for measuring risk. It can also help determine the amount of capital the swap dealer should hold to adequately cover the normal risks of the portfolio. In addition, dealers can use the measure to determine the impact of an additional position on portfolio risk. But the measure has shortcomings, too. For example, VaR does not completely characterize the risks in the swap portfolio; it only measures the risk of unlikely events in "normal" markets. But portfolio managers also want to know about risks in abnormal markets since these are the risks that can cause the most pain. To learn more about their portfolio's exposure to abnormal market risks, managers rely on a technique called *stress testing*.

Stress tests use scenario simulation to identify vulnerable positions within the portfolio. Generally, managers combine historical and hypothetical scenarios to simulate the portfolio value in different stress environments. Stress tests can identify positions contributing disproportionately to portfolio risk. Stress tests can also be used in conjunction with stress test limits to limit portfolio risks. The difficulty in implementing stress tests is simply identifying meaningful stress scenarios. Simply identifying worst-case scenarios usually contributes little to risk management because the scenarios are (hopefully) not realistic.

A simple stress test that many portfolio managers use is to apply alternative interest rate shocks to a portfolio of swaps. The swap portfolio manager may revalue the portfolio under the assumption that the entire yield curve will move up or down 50 basis points in the next year. Other simple scenarios include flattening or steepening yield curves. The main purpose of these simple scenarios is to determine whether the stress environments produce changes in portfolio value consistent with the manager's intuition. If not, it is a signal that the manager does not fully understand the risks in the portfolio and must learn more about the risk structure or trim exposure by cutting back on proprietary positions.

Because market prices for swaps, especially highly customized swaps, are not observable in the open market, these prices must be modeled using the valuation techniques studied in previous chapters. In contrast to derivatives traded on an organized exchange that are marked to market each day, privately negotiated derivatives like swaps must be marked to model. Simulation techniques used to manage the risk of swap portfolios also rely on valuation models. Because models can be specified incorrectly, managers of swap portfolios must cope with *model risk*. The term describes risk caused by using models with missing elements, inaccurately estimated parameters, faulty functional form, or unsupported underlying assumptions. Model risk can also result from computer coding errors or data input problems.

SUMMARY

Swaps can be used in several ways for managing portfolio risks. Like other financial derivatives, swaps can eliminate, decrease, or increase risk. We showed how to use interest rate swaps to manage the duration gap between a firm's assets and liabilities. We also showed how to hedge a stock portfolio with equity swaps and explored more sophisticated so-called flavored swap structures. In addition to demonstrating how to use swaps to manage risks inherent in underlying business operations, we also explained how swap dealers manage the risks resulting from their holding a portfolio of swaps.

QUESTIONS AND PROBLEMS

1. Consider the balance sheet information in Table 7.1. Suppose that the firm attempts to completely hedge its interest rate risk with a pay-fixed

swap that has a duration of four years. What should the notional principal of this swap be?

2. Suppose the firm in problem 1 has a desired duration gap of .5 years. Interpret the meaning of this gap in terms of a bond position.

3. Using the swap in problem 2, what notional principal would be required to achieve a desired duration gap of one year?

4. What is the difference between an amortizing swap and an accreting swap?

5. What is a macro swap? How does a macro swap differ from the other swaps we have considered?

6. What is a currency annuity swap?

7. What right does a payer swaption give to its holder?

8. What right does a receiver swaption give to its holder?

9. Suppose the fixed rate used in the seasonal swap example was 3.5 percent. What would the price of this swap be to the fixed-rate payer?

10. What is mismatch risk? Why is mismatch risk an important concern of swap dealers?

11. Explain how an interest rate swap can be analyzed as a strip of futures.

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CHAPTER 8

Financial Engineering and Structured Products

In this chapter, we explore how financial engineers use simple instruments as building blocks for the creation of synthetic instruments and structured products. First, we analyze the relationships among futures, forwards, options, swaps, and the financial instrument underlying them. We can simulate each of these building blocks by combining others. Therefore, we explore how to create a *synthetic instrument*—a financial structure that has the same value as another identifiable instrument. For example, we show how investment in options and a risk-free bond can create a synthetic stock position. We also show how to use swaps to change the characteristics of debt instruments.

In the second part of this chapter, we describe the techniques of the financial engineer in creating customized structured products to eliminate unique business risks. These techniques have given rise to entirely new instruments including Equity-Linked Certificates of Deposit, Preferred Equity Redemption Cumulative Stocks (PERCS), and structured notes. In addition to creating new products that can be widely disseminated, financial engineers can develop special-purpose solutions such as the real-world swap that was constructed for Northwestern Bell Telephone and the specially engineered solution to the financial problems of Sonatrach, an Algerian oil company.

Finally, we examine specific cases of corporate debacles involving financial derivatives. Although financial derivatives are powerful tools for managing risk and taking calculated risks, disasters can arise from ignorance, hubris, and outright dishonesty. We consider the bankruptcy of Orange County, California, the problems involving Bankers' Trust and its clients Gibson Greetings and Procter & Gamble. We also explore the financial disasters involving Barings Bank, Metallgesellschaft, and Long-Term Capital Management.

SYNTHETIC INSTRUMENTS

In this section, we show how to create synthetic financial instruments such as a portfolio of options that will have the same profits and losses as the underlying asset at the expiration date of the options. To understand how to create synthetic instruments, we first review the put-call parity relationship introduced in Chapter 4. We then illustrate specific synthetic instruments.

Put-Call Parity and Synthetic Instruments

In Chapter 4, we used the principle of put-call parity to find the price of a put option, given knowledge of the price of a call option on the same underlying good. To apply put-call parity, we need a call option with the same striking price and the same term to expiration as the put we are attempting to price. Subject to those conditions, we saw that the put-call parity maintains that:

$$P = C - S + \frac{E}{\left(1 + R_f\right)^T} \tag{8.1}$$

where S = stock price
P = put price
C = call price
E = common exercise price for the call and put
R_f = risk-free rate
T = common term to expiration for the call and put

This put-call relationship provides the basic blueprint for creating synthetic securities. By rearranging equation 8.1 to isolate individual instruments on the left-hand side of the equation, we see what combination of other instruments will simulate a particular instrument of interest. We now show how to create synthetic equity, synthetic puts, synthetic call options, and a synthetic T-bill.

Synthetic Equity

Rearranging equation 8.1 to isolate the stock (S), we have:

$$S = C - P + \frac{E}{\left(1 + R_f\right)^T} \tag{8.2}$$

Equation 8.2 shows that a position in the stock is equivalent to a long call plus a short put, coupled with an investment at the risk-free rate. The investment at the risk-free rate is an amount that will pay the common exercise price on the call and the put at the time of expiration. Thus, *synthetic* equity consists of a long call, short put, and an investment of the present value of the exercise price in the risk-free rate.

To illustrate this equivalence, consider the following example. Assume a call and a put have an exercise price of $80 and expire in one year. The risk-free rate of interest is 7 percent per annum. With this interest rate, an investment of $74.77 will pay the exercise price of $80 in one year. Table 8.1 presents several alternative stock prices that might arise in one year, and it shows the value of the call, put, bond, and the synthetic equity as well.

As Table 8.1 shows, the synthetic equity will have the same value as the stock in one year, no matter what the stock price might be. To see this equivalence, consider a stock price in one year of $95. With this stock price, the put will be worthless and the call will be worth $15. The risk-free bond will pay $80, so the synthetic equity position will be worth $95 as well ($15 from the call and $80 from the risk-free bond). Given the purchase of the synthetic equity, it is also possible to convert the synthetic position into the underlying equity. For example, the trader could exercise the call option and use the bond proceeds to pay the exercise price.

To complete the example, consider a terminal stock price below $80. If the stock is worth $65 at expiration, then the call expires worthless, and the risk-free bond is worth $80. However, the synthetic equity involves a short position in a put option that can be exercised against the writer. The short

TABLE 8.1 Synthetic Equity

	Elements of Synthetic Equity			
Stock Price at Expiration	**Call** $E = \$80$	**Short Put** $E = \$80$	**Risk-Free Investment**	**Synthetic Equity**
$ 60	$ 0	$–20	$80	60
65	0	–15	80	65
70	0	–10	80	70
75	0	–5	80	75
80	0	0	80	80
85	5	0	80	85
90	10	0	80	90
95	15	0	80	95
100	20	0	80	100

put is a liability of $15 for the synthetic equity holder. Considering the long call, the short put, and the bond together, the synthetic equity position is worth $65, the same as the stock itself.

Synthetic Put Options

The put-call parity relationship of equation 8.1 shows that a *synthetic put* consists of taking a long call and short stock position, while investing the present value of the exercise price in a risk-free instrument. Table 8.2 shows the values of an actual put and the synthetic put for alternative stock prices at expiration. The value of the synthetic put equals the sum of a long call, plus a short stock position, plus an investment in the risk-free bond.

Synthetic Call Options

As the put-call parity relationship indicates, a *synthetic call* consists of a long position in both the stock and the put option, and a short position in a risk-free bond that will pay the exercise price at the expiration of the option. To create the synthetic call, a trader borrows the present value of the exercise price and uses these funds to help finance the purchase of the put and the stock. Table 8.3 shows the values at expirations for the constituent elements and for a synthetic call. The table also shows that the synthetic call and the actual call have the same values at expiration for every terminal stock price.

TABLE 8.2 A Synthetic Put

		Elements of a Synthetic Put			
Stock Price at Expiration	**Put E = $80**	**Call E = $80**	**Short Stock**	**Risk-Free Investment**	**Synthetic Put**
$ 60	$20	$ 0	$–60	$80	$20
65	15	0	–65	80	15
70	10	0	–70	80	10
75	5	0	–75	80	5
80	0	0	–80	80	0
85	0	5	–85	80	0
90	0	10	–90	80	0
95	0	15	–95	80	0
100	0	20	–100	80	0

TABLE 8.3 A Synthetic Call

Stock Price at Expiration	Call E = $80	Elements of a Synthetic Call			Synthetic Call
		Put E = $80	Stock	Short Risk-Free Investment	
$ 60	$ 0	$20	$ 60	$–80	$ 0
65	0	15	65	–80	0
70	0	10	70	–80	0
75	0	5	75	–80	0
80	0	0	80	–80	0
85	5	0	85	–80	5
90	10	0	90	–80	10
95	15	0	95	–80	15
100	20	0	100	–80	20

Synthetic T-Bills

A synthetic T-bill can also be created by the proper combination of a long put, short call, and a long position in the stock. The resulting position is a synthetic T-bill, because the synthetic instrument will pay the exercise price at the expiration date of the options no matter what the stock price might be. It is ironic that "risky" instruments such as a call, put, and stock can be combined to simulate a T-bill. Table 8.4 shows the value of the constituent elements and the resulting synthetic T-bill.

TABLE 8.4 A Synthetic T-Bill

Stock Price at Expiration	Risk-Free Investment	Elements of a Synthetic T-Bill			Synthetic T-Bill
		Short Call E = $80	Put E = $80	Stock	
$ 60	$80	$ 0	$20	$ 60	$80
65	80	0	15	65	80
70	80	0	10	70	80
75	80	0	5	75	80
80	80	0	0	80	80
85	80	–5	0	85	80
90	80	–10	0	90	80
95	80	–15	0	95	80
100	80	–20	0	100	80

SYNTHETIC FUTURES AND FORWARDS AND PUT-CALL PARITY

In our discussion of forwards and futures in Chapter 2, we saw that the futures price generally conforms to the cost-carry relationship. This relationship holds almost exactly in the financial futures marketsalthough it provides a less complete understanding of markets for traditional commodities such as foodstuffs. We now consider the special case in which the cost-of-carry relationship holds exactly, and we assume that the cost-of-carry equals the risk-free rate. Most financial futures closely approximate these assumptions.

Under these assumptions, the futures price will equal the spot price times one plus the cost-of-carry:

$$F = S\left(1 + R_f\right) \tag{8.3}$$

where F = futures price
S = spot price
R_f = the risk–free rate, assumed to be the cost–of–carry for the good

We now want to integrate the cost-of-carry model with the put-call parity relationship and with the analysis of synthetic securities. Rearranging the terms of the put-call parity relationship gives:

$$C - P = S + \frac{E}{\left(1 + R_f\right)^T} \tag{8.4}$$

Combining equations 8.3 and 8.4 gives:

$$C - P = \frac{F - E}{\left(1 + R_f\right)^T} \tag{8.5}$$

Equation 8.5 says that the difference between the call and put price equals the present value of the difference between the futures price and the exercise price of the options. If the exercise price is \$100, the futures price is \$120, the risk-free rate is 10 percent, and the options expire in one year, we have:

$$C - P = \frac{F - E}{\left(1 + R_f\right)^T}$$

$$= \frac{\$120 - \$100}{1.10}$$

$$= \$18.18$$

In this example, the call price must exceed the put price by \$18.18. Although this equation gives only the relative value of the call and put, we know that the call option must be worth at least \$20 because the call is \$20 in-the-money. For its part, the put will have relatively little value because it is so far out-of-the-money.

For the special case in which the current futures price equals the exercise price, then the quantity $F - E$ equals zero. This implies that $C - P$ also equals zero, which means that the call and put must have the same price. If the futures price is less than the exercise price, the quantity $F - E$ is negative. This implies that the put will be more valuable than the call. For the same instruments of the example in the preceding paragraph, if the futures price is \$90, the quantity $F - E = -\$10$, and the value of $C - P$ must be –\$9.09.

SYNTHETIC PUT OPTIONS AND DYNAMIC HEDGING

In Chapter 5, we saw that a long stock position combined with a put option can insure the position against large losses. This type of insurance is called *portfolio insurance*. In this section, we consider another means of implementing portfolio insurance without options: We use stock index futures and cash to replicate the role of the put option in a portfolio insurance strategy. Implementing portfolio insurance strategies using futures is called *dynamic hedging*. Although the mathematics of dynamic hedging is too complex for full treatment here, we can understand the basic idea behind portfolio insurance with stock index futures.

A Dynamic Hedging Example

Consider a fully diversified stock portfolio worth \$100 million. The value of this portfolio can range from zero to infinity. Many investors would like to put a floor beneath the value of the portfolio. It is deemed highly desirable to ensure that the portfolio's value never falls below \$90 million. Portfolio insurance offers a way to control the downside risk of a portfolio. In a financial market, however, there is no free lunch, so it is only possible to limit the risk of a large price fall by sacrificing some of the potential for a gain. Like life insurance, portfolio insurance is not free, but it may be desirable for some traders.

A risk-minimizing hedge converts a stock portfolio to a synthetic T-bill. By fully hedging our example stock portfolio, we can keep the portfolio's value above $100 million. A fully hedged portfolio will increase in value at the risk-free rate, although full hedging eliminates all of the potential gain in the portfolio beyond the risk-free rate. In dynamic hedging, however, the trader holds the stock portfolio and sells some futures contracts. The more insurance the trader wants, the more futures he or she will sell.

Assume that a stock index futures contract has an underlying value of $100 million and a trader sells futures contracts to cover $50 million of the value of the portfolio. Thus, in the initial position, the trader is long $100 million in stock and short $50 million in futures, so 50 percent of the portfolio is hedged. Table 8.5 shows this initial position in the time zero row. At $t = 0$, there has been no gain or loss on either the stock or futures. In the first period, we assume that the value of the stock portfolio falls by $2 million. The 50 futures contracts cover half of that loss with a gain of $1 million. Therefore, at $t = 1$, the combined stock/futures portfolio is worth $99 million. Now the manager increases the coverage in the futures market by selling five more contracts. This gives a total of 55 short positions and coverage for 56 percent (55/99) of the total portfolio. In the second period, the stock portfolio loses another $2 million, but with 55 futures contracts, the futures gain is (55/99)$2 million = $1.11 million. This gives a total portfolio value of $98.11 million.

By $t = 4$, the stock portfolio has fallen $10 million, but the futures profits have been $6.21 million. This gives a total portfolio value of $96.21 million. Also, the manager has increased the futures position in response to each drop in stock prices. At $t = 4$, the trader is short 80 contracts, hedging

TABLE 8.5 Portfolio Insurance Transactions and Results

	Gain/Loss $ Millions		Total	Futures	Portion
Time	Stocks	Futures	Value	Position	Hedged
0	0.00	0.00	100.00	–50	.50
1	–2.00	1.00	99.00	–55	.56
2	–2.00	1.11	98.11	–60	.61
3	–2.00	1.22	97.33	–70	.72
4	–4.00	2.88	96.21	–80	.83
5	–36.86	30.65	90.00	–90	1.00
6	–10.00	10.00	90.00	–90	1.00

83 percent of the stock market portfolio. At $t = 5$, the stock price drops dramatically, losing $35.86 million. The futures profit covers $30.65 million. This leaves a total portfolio value of $90 million. However, this is the floor amount of the portfolio, so the trader must now move to a fully hedged position. If the stock portfolio is only partially hedged, the next drop in prices can take the value of the entire portfolio below the floor amount of $90 million. At $t = 6$, the price of the stocks drops $10 million, but the futures position fully covers the loss. Therefore, the combined portfolio maintains its floor value of $90 million.

Table 8.5 shows the basic strategy of portfolio insurance with dynamic hedging. Initially, the portfolio is partially hedged. If stock prices fall, the trader increases the insured portion of the portfolio. Had the stock portfolio risen in value, the futures position would have lost money. However, the loss on the futures position would have been less than the gain on the stocks, because the portfolio was only partially hedged. As the stock prices rose, the manager would have bought futures, thereby hedging less and less of the portfolio. Less hedging would be needed if the stock price rose, because there would be little chance of the portfolio's total value falling below $90 million.

Implementing Portfolio Insurance with Dynamic Hedging

By design, Table 8.5 is highly simplistic. First, it does not show how the starting futures position was determined. Second, it does not show how the adjustments in the futures position were determined. Third, it considers only large changes in the value of the stock portfolio. The smallest change in the table is 2 percent of the stock portfolio's value. The exact answer to these questions is highly mathematical. However, we can explore these issues in an intuitive way.

Choosing the initial futures position depends on several factors. First, it depends on the floor that the manager chooses relative to the initial value of the portfolio. If the lowest acceptable value of the portfolio is $100 million, then the manager must hedge 100 percent at $t = 0$. Thus, the lower the floor relative to the portfolio value, the lower the percentage of the portfolio the manager will need to hedge. Second, the purpose of the insurance strategy is to guarantee a minimum terminal portfolio value while allowing for more favorable results. As a consequence, the futures position must take into account the volatility of the stock portfolio. The higher the estimated volatility of the stock portfolio, the greater the chance of a large drop in value that will send the total portfolio value below the floor. Therefore, the portion of the portfolio that is to be hedged depends critically on the estimated volatility of the stock portfolio. This will differ both across time and for portfolios of different risk.

Adjustments in the futures position depend on the same considerations that determine the initial position. First, the value of the portfolio relative to the floor is critical. Second, new information about the volatility of the stock portfolio also affects the futures position. In Table 8.5, the volatility of the stock portfolio accelerates. Each percentage drop is larger than the previous one. Therefore, this increasing volatility will lead to a larger short futures position than would otherwise be necessary.

In Table 8.5, the drops in the stock portfolio's values are large. In actual practice, dynamic hedging works by continually monitoring the value of the portfolio. Small changes in the portfolio can trigger small adjustments in the futures position. For many portfolios, monitoring and updating can occur many times a month. This is why it is called dynamic hedging—the hedge is monitored and updated continuously, often with computerized trading programs. Table 8.5 does not show that continual monitoring. Instead, we might take the different rows in the table as snapshots of the portfolio's value at different times.

Table 8.5 abstracts from some of the cash flow issues that dynamic hedging will raise. For example, it does not explicitly consider the cash flows that come from daily settlement of the futures position. Actual dynamic hedging must face a host of such technical issues .[1]

CREATING SYNTHETIC STRUCTURES WITH SWAPS

Certain instruments can be synthesized from combinations of derivatives, such as through the put-call parity relationship. Any three of four instruments (a put, a call, the underlying good, a risk-free bond) can synthesize the fourth instrument. In this section, we explore some similar relationships using swaps. This idea is already familiar, as we have seen in the preceding section that a swap is, in effect, a portfolio of forward contracts.

Synthetic Fixed-Rate Debt

Consider a firm with an existing floating-rate debt obligation that wants to eliminate the uncertainty inherent in floating debt. This firm could create a synthetic fixed-rate debt instrument by combining its existing floating-rate obligation with an interest rate swap. Assume the firm has an outstanding issue of $50 million on which it pays a floating-rate annual coupon and that the debt matures in six years. The firm wants to transform this obligation into a fixed-rate instrument with the same maturity.

Figure 8.1 shows the firm's existing obligation in the upper time line. To transform this obligation into a fixed-rate instrument, the firm can engage in a swap agreement to receive a floating rate and pay a fixed rate, with a tenor and payment timing to match the present debt, as shown in the bottom time line of Figure 8.1. The combination of the existing debt and the pay-fixed/receive-floating interest rate swap gives the firm a synthetic fixed-rate obligation instead of its current floating-rate debt.

Synthetic Floating-Rate Debt

An existing fixed-rate obligation can be transformed into floating-rate debt by reversing the technique used to create synthetic fixed-rate debt. Assume a firm has an existing fixed-rate debt obligation with a maturity of six years that requires annual interest rate payments. The upper time line of Figure 8.2 shows the cash flows associated with this obligation. (This example parallels that of Figure 8.1, except the initial obligation has fixed-rate coupons.)

By combining this fixed-rate obligation with an interest rate swap to pay a fixed rate and receive a floating rate, the instrument can be transformed from a fixed rate to a synthetic floating-rate obligation. The lower time line of Figure 8.2 shows the cash flows on an interest rate swap to pay fixed and receive floating. By combining this swap with the existing obligation, the firm transforms its existing fixed-rate obligation into a synthetic floating-rate debt with the same maturity.

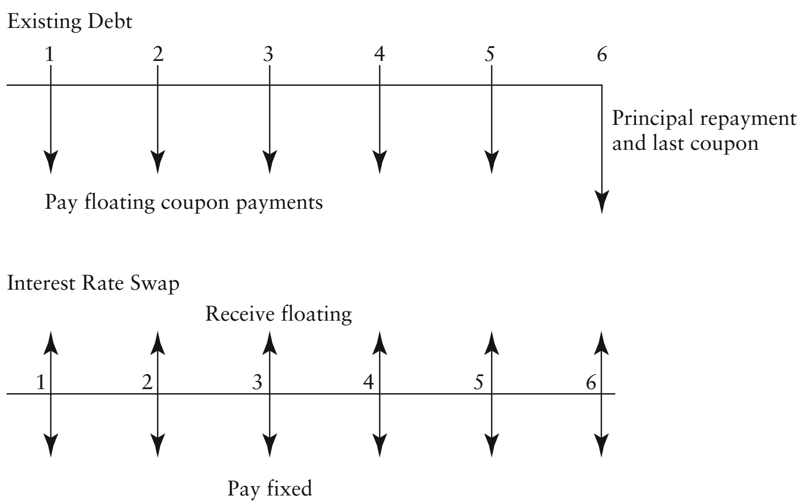

FIGURE 8.1 Elements of synthetic fixed-rate debt.

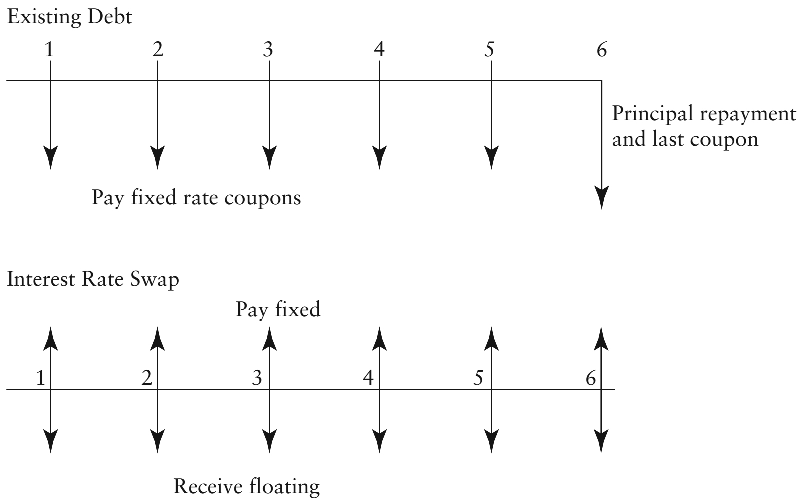

FIGURE 8.2 Elements of synthetic floating-rate debt.

Synthetic Callable Debt

Consider a firm with an outstanding fixed-rate obligation that possesses no call feature. The issuing firm would like to be able to call this debt in three years, but does not want the obligation to retire the issue. In essence, the firm wishes that the existing noncallable debt had a call provision allowing a call in three years.

When a firm calls an existing debt instrument, it repays the debt. We may view that repayment as creating a new financing need that the firm will meet from floating-rate obligations. After all, calling the debt retired the existing fixed-rate obligation. From this perspective, the decision to call an existing fixed-rate obligation is like creating a synthetic floating-rate debt obligation using a call. As we have just seen, a fixed-rate debt obligation, combined with an interest rate swap to receive fixed payments and pay floating transforms the fixed-rate instrument into a floating-rate obligation.

This issuer, however, wants to have the option, but not the obligation, to make this transformation. Therefore, the firm can create a synthetic callable bond by using a swaption—an option on a swap.[2] Because the firm wants to possess the call option, we know that the firm must purchase a swaption, because only buying an option gives that flexibility. The swap must allow the firm to receive a fixed rate and pay a floating rate; therefore, the firm needs the option to sell a swap. (Recall that purchasing a swap means to pay fixed and receive floating.) Consequently, the firm needs to purchase a put swaption with a maturity of three years. The exercise price on the put swaption will play the role of the call price on a callable bond. Thus, by combining a noncallable fixed-rate debt obligation with the purchase of a put swaption, the firm can transform its initial obligation into a callable bond.

Synthetic Non-Callable Debt

A firm with outstanding callable debt can use interest rate swaps to eliminate the call feature. When it issued the callable debt, the firm essentially purchased a call option from the bondholders. If the firm is sure that it will not call the debt, it may want to recapture the value represented by that call option.

If the issuance of callable debt involves the sale of a call option to the bondholders, this can be "unwound" by now selling a call swaption with the correct characteristics—the same characteristics possessed by the option that the firm purchased from its bondholders. Specifically, the call swaption that the firm now decides to sell should have maturity and exercise incentives that match those inherent in the original callable debt. Measured from the expiration date of the swaption, the swap should have a tenor that matches the remaining life of the bond from that date.

Once it sells the call swaption, two outcomes are possible. The option can expire worthless or it can be exercised against the issuing firm. If the option is never exercised, the firm merely continues to pay the fixed rate of interest for the full life of the bond and the original call feature on the bond never comes into play. The first line of Table 8.6 shows the firm's position if the swaption is never exercised. If it is not exercised, the firm will simply not

TABLE 8.6 Transforming Callable into Non-Callable Debt

Call Date Scenario	Swap	Issuer	Result
Interest rates higher.	Swaption not exercised.	Does not call the bond.	Issuer has fixed rate financing.
Interest rates lower.	Swaption exercised. Issuer pays fixed and receives floating for remainder of bond's life.	Calls the bond and funds floating for remainder of bond's life.	Issuer has fixed rate financing.

Source: Adapted from L. S. Goodman, "Capital Market Applications of Interest Rate Swaps," Chapter 7 of C. R. Beidleman, *Interest Rate Swaps,* Homewood, IL: Business One Irwin, 1991.

exercise the call provision in the bonds it has issued and in effect will have a straight debt obligation.

If interest rates fall sufficiently, the owners of the call swaption will exercise that option against the firm. However, the firm will then exercise the call provision of the bond it has issued. In effect, when the firm suffers the exercise, it passes that exercise through to its own bondholders and remains unaffected by the exercise of the call against it. As the second line of Table 8.6 indicates, the issuer continues to enjoy a position that is effectively like having issued noncallable straight debt initially.

Synthetic Dual-Currency Debt[3]

A dual-currency bond has principal payments denominated in one currency, with coupon payments denominated in a second currency. A firm might borrow dollars and pay coupon payments on the instrument in European Union euros. When the bond expires, the firm would repay its principal obligation in dollars. This dual-currency bond can be synthesized from a regular single currency bond with all payments in dollars (a dollar-pay bond) combined with a fixed-for-fixed currency swap.

The upper time line in Figure 8.3 shows the cash flows from owning a typical dollar-pay bond from the point of view of the bond owner. The

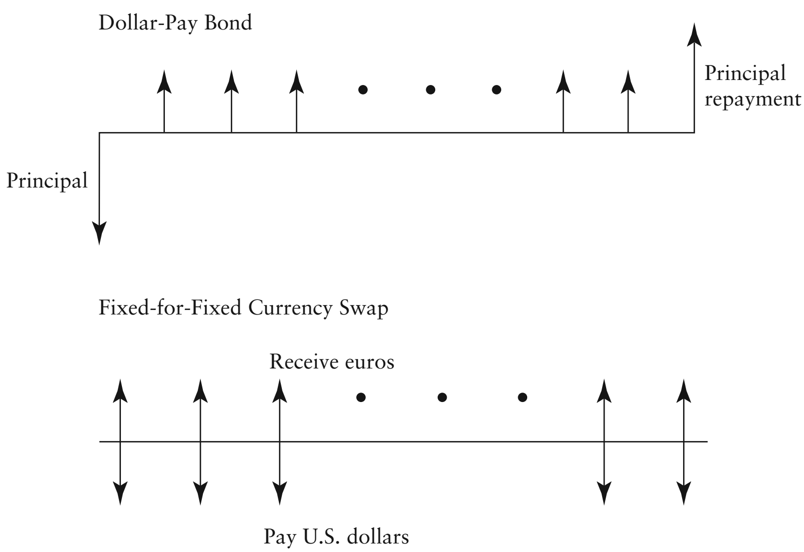

FIGURE 8.3 Elements of synthetic dual-currency debt.

purchaser of the bond invests at the outset and then receives coupon inflows and the return of principal on maturity. Thus the down arrows indicate outflows, while the up arrows represent cash inflows. The second time line in Figure 8.3 shows the cash flows for a fixed-for-fixed foreign currency swap in which the party receives fixed euro inflows and pays fixed dollar amounts. Notice that there is no exchange of borrowings in this swap. The cash flows in the currency swap are constructed to equal the coupon payments.

Figure 8.4 shows the effect of combining the dollar-pay bond with the foreign currency swap. The dollar coupon payments on the dollar-pay bond and the dollar payments on the fixed-for-fixed currency swap perfectly offset each other. This leaves euro inflows from the swap that take the place of the coupon payments. As Figure 8.4 shows, the principal payment and repayment are in dollars; all of the coupon cash flows are in euros. Thus, combining a dollar-pay bond and the appropriate fixed-for-fixed currency swap with no exchange of borrowings produces a *dual-currency bond*.

The All-In Cost

As shown in the preceding section, it is possible to use swaps to create many synthetic structures. In a financial market that functions well, synthetic structure should sell for a price that is equivalent to the structure's underlying combination of building block derivatives. Combinations of financial instruments producing the same net present value of cash flows should have the same price. However, pricing differences emerge from market imperfections and inefficiencies such as taxes, transaction costs, illiquidity in some markets, and similar factors. When applied to complex synthetic structures, these imperfections can cloud the comparison of a structure's cash flows to the cash flows of its building block equivalents. Therefore, a tool to compare financing alternatives can be useful.

The *all-in cost* can be thought of as the internal rate of return (IRR) for a given financing alternative after all financing costs have been factored in.

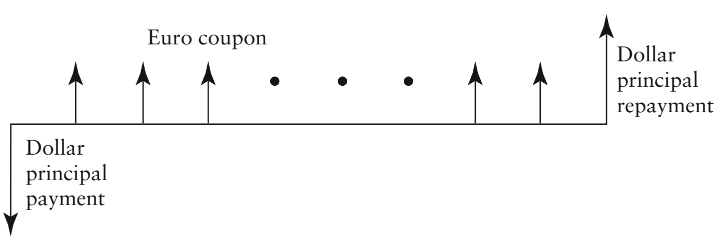

FIGURE 8.4 Cash flows on synthetic dual-currency debt.

It is called the all-in cost because it includes all costs associated with the alternative being evaluated, such as flotation costs, underwriting fees, and administrative expenses, as well as the actual cash flows for the instrument being evaluated. The all-in cost represents an effective annual percentage cost and provides a comprehensive basis for comparing different financing alternatives.

Consider two financing alternatives available to a firm that are different in structure but have similar cash flows. The first instrument is a 10-year semiannual payment bond with a principal amount of $40 million and a coupon rate of 7 percent. The bond trades at par. For simplicity, assume that no underwriting fees or administrative costs are associated with this alternative.

As a second financing alternative, the firm can use a floating-rate instrument coupled with the interest rate swap. The firm forms this structure by issuing a 10-year floating-rate bond at LIBOR plus 30 basis points, with the rate being reset every six months, and simultaneously entering into a pay-fixed/receive-floating swap. This combination of a floating-rate bond and a pay-fixed/receive-floating swap is economically equivalent to a fixed-rate bond. On inquiry, the firm learns that it can enter the swap at a par value swap rate of 6.5 percent. This means that the firm will pay a fixed rate on the swap of 6.5 percent and will receive a LIBOR. The net effect is that the firm would pay a fixed cost of 6.8 percent on $40 million. Suppose, however, that the swap dealer charges a fee for arranging the swap. This fee, combined with administrative costs, totals $400,000 in up-front costs.

In summary, the two financing alternatives have similar cash flows and both provide fixed-rate financing of about $40 million. The choice of financing, therefore, reduces to comparing the all-in costs of the two deals. For the first alternative, the IRR is equivalent to the coupon rate of 7 percent since the bond is priced at par. For the second alternative, the firm receives $39.6 million of actual financing after paying the $400,000 for the swap. The firm makes 20 semiannual payments of $1.36 million (=$40,000,000 (.068/2)), and repays the principal of $40 million at the end of 10 years. The all-in cost for the second alternative is simply the IRR that equates the present value of the initial cash inflow with the present value of all the expected cash outflows. For these cash flows, the IRR is 6.9406 in annual terms. This is slightly lower than the 7 percent IRR on the straight bond financing. Therefore, based on this all-in cost comparison, the firm prefers the floating-rate instrument coupled with the interest rate swap. However, the firm should be aware that the 6 basis point differential might simply reflect an additional credit risk on the swap. If so, the firm would be indifferent between the two financing alternatives.

ENGINEERING NEW PRODUCTS

In this section, we consider new financial products that have been created and marketed in the United States. These products are intended for trading in an active market, as opposed to custom solutions that might be designed for the particular needs of a given investor. These products are often called *hybrid instruments* because they contain features of both securities and derivatives. First, we look at Preferred Equity Redemption Cumulative Stock (PERCS). PERCS are a type of preferred stock that will necessarily be converted into the common shares of the issuing firm. The main advantage of PERCS stems from an exploitation of market imperfections. Second, we consider equity-linked certificates of deposit. For these equity-linked CDs, the investor accepts a low or zero rate of interest in return for a payoff that depends on the performance of a stock index. Third, we consider structured notes. In a structured note, the rate of return depends on the performance of some other security. These versatile instruments can be tied to the performance of various debt instruments, equities, or commodities.

PERCS

Preferred Equity Redemption Cumulative Stock (PERCS) is a hybrid security invented by Morgan Stanley; Avon issued the first PERCS in 1988. After Avon, 15 other mostly blue chip firms also issued PERCS including General Motors, Kmart, Sears, Texas Instruments, and Citicorp. Through mid-year 1994, total volume exceeded $8 billion.[4] As the name indicates, a PERCS is a type of preferred stock. Like other preferred stocks, a PERCS pays a fixed dividend, but these dividends are significantly higher than the dividend of the firm's common stock. In contrast to conventional preferred shares, PERCS must be converted into shares of common stock, typically about three years after issuance. The conversion price, or "cap," is typically about 30 to 45 percent above the price of the common share when the PERCS is issued. The PERCS must be converted at the maturity date, but it may be converted earlier. If the PERCS is converted at maturity, each PERCS will be converted into a share of stock. If the conversion occurs prior to maturity, the PERCS will be converted to a full share if the share price is less than the cap. If the share price exceeds the cap, the PERCS will be converted to a fractional share equal to the cap divided by the share price at conversion.

We noted that the dividend rate on a PERCS exceeds that of the underlying common share. If the PERCS paid the same dividend as the common stock, a PERCS would be equivalent to a covered call—a long position in the common share plus a short position in a call option on the share with an expiration date equal to the maturity of the PERCS. Because the PERCS pays a dividend above the common share, we may, following Chen, Kensinger, and Pu, analyze a PERCS as consisting of a covered call plus an annuity. The annuity is the present value of the difference between the dividend stream of the PERCS and the dividend stream of the common stock.

From the description thus far, it seems that a PERCS is not a truly new security, or at least not a unique security. If we could completely replicate a PERCS by: buying the stock, selling a call, and buying an annuity, there would be little reason for a PERCS to exist. The true importance of PERCS comes from the imperfect market factors of transaction costs and taxes. First, because a PERCS is a single security, trading it may be cheaper than trading a covered call plus annuity. Second, a PERCS effectively translates the call premium realized on a covered call into a higher dividend investment. This could be particularly valuable to an institutional investor that pays taxes on only 70 percent of dividends. Third, corporations might like to issue PERCS because they provide an alternative means of raising equity. When the PERCS is converted to a common share, a new share is issued. Compared with issuing straight equity, a PERCS may have a lower agency cost to the firm because of the built-in short position in a call option that the purchaser of the PERCS receives.

Ignoring market imperfections, a PERCS is equivalent to a long stock, short call, long annuity portfolio, and the price of the PERCS can be measured against that replicating portfolio. Of course, because of the market imperfections, such a study would not fully consider any benefits of creating PERCS. If PERCS constitute a valuable security innovation, the value of a PERCS should exceed the value of the replicating portfolio given the savings on transaction costs and agency costs.

For each of the eight PERCS examined, Chen, Kensinger, and Pu found that the value of the PERCS was worth more than the replicating portfolio. The excess value of the PERCS ranged from zero to almost 11 percent of the value of the replicating portfolio. Thus, the PERCS must provide some additional value over and above its replicating portfolio. Chen, Kensinger, and Pu also found that the price differential is not large enough to make arbitrage between the PERCS and its replicating portfolio profitable due to transaction costs. Figure 8.5 shows the difference in value between the PERCS and the equivalent portfolio for Kmart. Apparently, the excess value of the PERCS is due to the market imperfections previously discussed. The creation of the PERCS helps to economize on the costs represented by market imperfections.

Other issuers have products similar to PERCS. Salomon Brothers offers ELKS (Equity-Linked Securities) that mimic the cash flows of PERCS. Unlike a conventional PERCS deal, ELKS are redeemed for cash rather than the underlying stock. Salomon Brothers offered the first ELKS in 1993

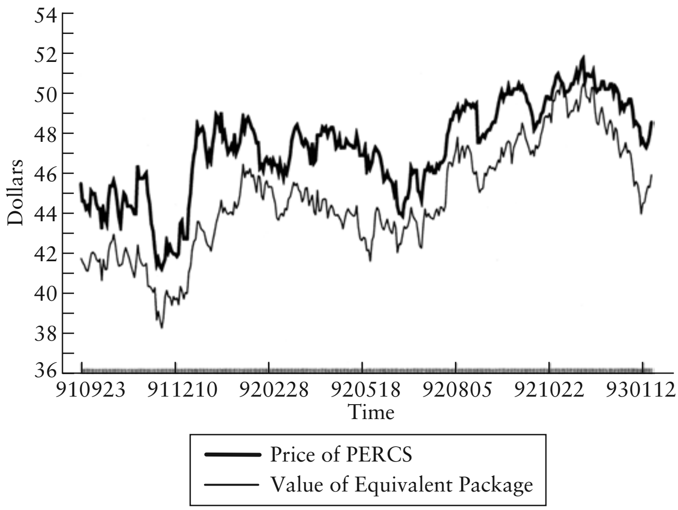

FIGURE 8.5 The value of the Kmart PERCS and its equivalent portfolio.
Source: Andrew H. Chen, John Kensinger, and Hansong Pu, "An Analysis of PERCS," *The Journal of Financial Engineering,* 3:2, June 1994, p. 100.

based on Digital Equipment's stock. Technically, the ELKS are debt obligations of Salomon Brothers and Digital Equipment was not involved in the deal. In the original deal, the note was structured so that at the end of three years ELKS holders would receive 135 percent of their original investment or the average stock price of Digital Equipment over a 10-day period. In exchange for the 35 percent cap, investors received interest payments of 6.75 percent. The Digital Equipment ELKS traded at the American Stock Exchange. This type of instrument masquerades under different names depending on who issues it. Lehman Brothers calls their version YIELDS, and Merrill Lynch has a product called EYES.

Equity-Linked Certificates of Deposits[5]

An Equity-Linked Certificate of Deposit, or an equity-linked CD, pays little or no interest. However, these CDs promise a minimum maturity value plus a return based on the performance of a stock market index. Citicorp issued a five-year CD that guarantees a return of principal, plus twice the average increase in the S&P 500 over that period. Merrill Lynch issued a 5.5-year note that guaranteed a return of 100 percent of the principal, plus 115 percent of the appreciation in the S&P 500 index.

Consider an investment instrument of $1,000,000 in an equity-linked CD for a 5-year horizon when the CD rate is 8 percent compounded annually and the S&P index stands at 500.00. The equity-linked CD guarantees the return of principal and pays 115 percent of the appreciation in the S&P 500 over the five-year period. Table 8.7 shows the payoff at maturity for various levels of the S&P index at the maturity date. The break-even level of the S&P index is 704.0557, the level of the index that leads to an 8 percent return over the horizon. By contrast, if the S&P index is 600 at maturity, the realized rate of return is only 4.23 percent, and if the index reaches 800, the annualized return is 11.07 percent.

Figure 8.6 shows the payoffs on this equity-linked CD graphically. The payoff is $1,000,000 for any S&P index value of 500.00 or less. For higher S&P values, the payoff on the CD rises dramatically. If the index value is 704.0557, the payoff is $1,469,372.08, which equals an 8 percent return on the investment. The payoffs graphed in Figure 8.6 have exactly the characteristic shape of a call option, as explored in Chapter 4. Notice, however, that the minimum value of the CD is $1,000,000. This contrasts with a regular call option in which the minimum value is zero.

Figure 8.6 suggests that the equity-linked CD of the example embodies a combination of a call option and some other security. In essence, the investor in the equity-linked CD of our example buys a call option on the S&P 500.

TABLE 8.7 Payoff at Maturity

S&P 500 Terminal Value	Payoff on CD	Annual Percentage Return on CD
400	$1,000,000	0.00
450	$1,000,000	0.00
500	$1,000,000	0.00
550	$1,115,000 = $1,000,000 + 1.15($1,000,000)(50/500)	2.20
600	$1,230,000 = $1,000,000 + 1.15($1,000,000)(100/500)	4.23
700	$1,460,000 = $1,000,000 + 1.15($1,000,000)(200/500)	7.86
704.0557	$1,469,327.08 = $1,000,000 + 1.15($1,000,000)(204.0557/500)	8.00
800	$1,690,000 = $1,000,000 + 1.15($1,000,000)(300/500)	11.07

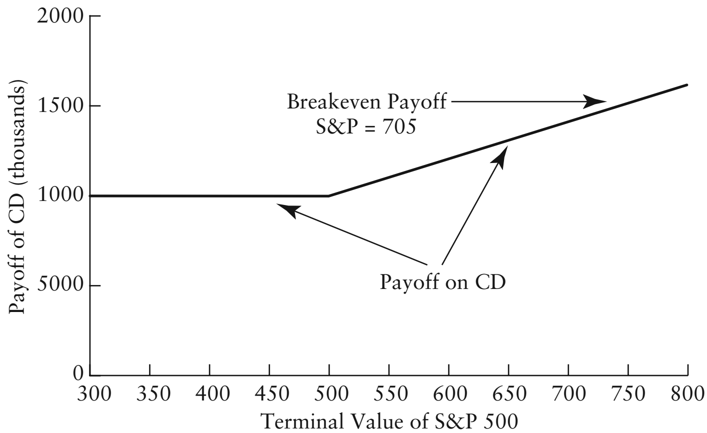

FIGURE 8.6 Payoffs on equity linked CD.

The cost of the option is the forgone interest, which is sacrificed to gain the payoff that is contingent on the index. Therefore, the cost of the option is:

$$\$1{,}000{,}000(1.08)^5 - \$1{,}000{,}000 = \$469{,}328.08$$

The exercise price of the option component is an index value of 500.00 because it is at this level that the option starts to pay. At expiration, the payoff on the option is 115 percent of the amount by which the index exceeds the exercise price of 500.00. Therefore, the payoff on the option component is:

$$\text{MAX}\{0, 1.15(\text{Terminal Index Value} - 500.00)\}$$

Thus, this equity-linked CD may be viewed as the combination of a zero interest CD plus a call option on the S&P 500. Equivalently, we may view this equity-linked CD as a combination of a CD plus the purchase of a call option with a premium equal to the interest on the CD.

If this equity-linked CD is merely a combination of a normal CD with the purchase of a call option on the S&P index, we might well wonder why this instrument is of any interest. After all, CDs exist, as do options, on the S&P 500. An important feature of this type of equity-linked CD is the deposit insurance provided by the Federal Deposit Insurance Corporation.

This feature can make these instruments more attractive than otherwise. In essence, the ability to capture deposit insurance represents a market imperfection that the creation of an equity-linked CD can capture to the mutual benefit of the issuing bank and the investor. These instruments are fairly new, and their future is uncertain. However, they have attracted considerable investor interest, and some observers believe that they may be positioned for considerable growth.

STRUCTURED NOTES

A *structured note* is a debt instrument combined with some type of derivative instrument. The payoffs on a structured note depend on interest earnings and the performance of the derivative embodied in the structure of the note. The equity-linked CDs of the previous section qualify as a structured note because their payoffs depend on the performance of the S&P 500. According to some estimates, the annual issuance of structured notes is approaching $100 billion, so these new instruments represent a potent force in today's capital markets.[6]

Structured notes embrace a rich variety of instruments because tying the payoff on the note to a new index or instrument can create new types of structured debt. The notes can be valued by decomposing them into the basic building blocks of debt or equity, forwards or options. Consider the following types of structured notes:

- *Floating Rate Note.* The rate of return on a structured note might depend on the floating rate of interest on another instrument. For example, a multiyear debt obligation might pay LIBOR plus a spread.
- *Inverse Floating Rate Note.* The rate of return on a structured note might depend on the inverse of a floating rate of instrument. A multiyear debt obligation might pay 15 percent per year minus LIBOR. This type of instrument is called an *inverse floater.*
- *Yield Curve Notes.* The rate of return on a structured note might depend on the spread between two-year and thirty-year Treasury yields. The yield on the structured note might depend on the difference between a thirty-year and two-year constant maturity Treasury (CMT) index. Such a note would be advantageous if the yield curve became steeper.
- *Relative Rate Differential Notes.* The rate of return on a structured note might depend on the spread between yields on two kinds of debt instruments. The yield on a structured note might depend on the yield differential between the prime rate and LIBOR.

- *Foreign Rate Structured Notes.* The rate of return on a structured note might depend on the yield differential between LIBOR for European Union euros and U.S. dollars.
- *Equity Performance Structured Notes.* The rate of return on a structured note might depend on the performance of a stock market index. The equity-linked CDs considered earlier are a type of equity performance structured note.
- *Commodity Performance Structured Notes.* The rate of return on a structured note might depend on the price change in a physical commodity, such as the price of a barrel of crude oil. Consider an oil-indexed zero-coupon note issued by the Standard Oil Company in 1986. At maturity, the holder of the note received $1,000 plus the excess of the crude oil price over $25 per barrel multiplied by 170 barrels. The payout in excess of $25 per barrel was capped at $40 per barrel. The market value of the notes at the date of issue was contingent on the price of oil in the future. The notes could be decomposed into a regular note plus a long call option exercisable at $25 per barrel plus a short call exercisable at $40 per barrel.

We now consider some detailed examples of structured notes including an inverse floating rate note, a constant maturity indexed note, a yield curve note, and a credit-linked note.

INVERSE FLOATING RATE NOTE[7]

Consider an investor who believes that six-month LIBOR will fall over the next three years from its present level of 4.75 percent. The investor buys a note paying 13 percent minus six-month LIBOR, which yields 8.25 percent at the outset. If LIBOR rises, the return on the inverse floater falls. If LIBOR reaches 13 percent in a given period, the note pays zero interest for that period; if LIBOR falls to 3 percent, the note pays 10 percent. (This kind of note might have a floor of zero percent interest, specifying that if LIBOR exceeds 13 percent, the interest rate remains at zero, rather than becoming negative.)

An investment bank is the typical issuer for such a structured note. The investment bank might prefer a fixed-rate obligation even though it issues the inverse floater to accommodate the investor. To avoid the floating obligation, the investment bank could enter a swap agreement to change its floating obligation to a fixed obligation. In this example, the issuing investment bank might enter a swap in which it pays the lesser of LIBOR or 13 percent and receives a fixed rate of 6.6 percent paid semiannually.

Figure 8.7 shows the transactions involved, both for the investor and the issuing investment bank. As the figure shows, the investor buys a note that pays 13.00 percent minus LIBOR. For its part, the investment bank pays 13.00 percent minus LIBOR on the structured note, pays LIBOR in the swap agreement, and receives a fixed swap rate of 6.6 percent. For the investment bank, the net obligation is a fixed-rate obligation of 6.4 percent:

$$-(13.00 \text{ percent} - \text{LIBOR}) - \text{LIBOR} + 6.6 \text{ percent} = 6.4 \text{ percent}$$

The investor who purchases the inverse floater essentially is making a bet about the future course of interest rates, effectively betting that LIBOR will be less than the forward rates predicted by the Eurodollar yield curve. To see the bet implicit in this inverse floater, we assume that the instruments are fairly priced, so that the price of the structured note equals the present value of the anticipated payments discounted at the forward rates implicit in the Eurodollar yield curve. Assume that the forward rate for two years in the future is 7 percent. If this forward rate correctly predicts the future rate, the structured note will pay 6 percent for that period (13 percent minus LIBOR for that period of 7 percent). If LIBOR in that period is less than the forward rate of 7 percent, say 5 percent, then the investor in the inverse floater will receive 8 percent for that period (13 percent minus the LIBOR of 5 percent).

Constant Maturity Treasury (CMT) Indexed Note In a CMT indexed note, the payoff on the note is tied to changes in the yield on a constant maturity Treasury rate. For example, an investor might buy a two-year note that pays

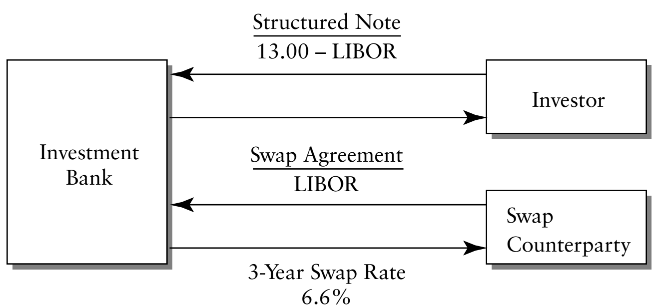

FIGURE 8.7 Cash flows for an inverse floater. *Source:* Adapted from "Anatomy of the Structured Note Market," *Journal of Applied Corporate Finance,* 7:3, Fall 1994, pp. 85–98.

the ten-year CMT rate minus 3 percent. At each period, the yield on the structured note will vary with the CMT rate. This yield must reflect the prevailing rate on a two-year floating-rate note. The investor will profit if the CMT rises enough so that the realized yield exceeds the available two-year floating-rate yield. The issuer of the structured note undertakes a floating-rate obligation, which it can translate into a fixed-rate obligation through a swap transaction.

A Yield Curve Note An investor can use a yield curve note to express a view about future changes in the yield curve. An investor who believes that the yield curve will steepen necessarily expects the difference between ten-year and two-year CMT to increase. Such an investor might buy a floating-rate note that pays a rate of interest depending on the difference between the ten-year and two-year CMT index rates.

Credit-Linked Notes A credit-linked note is a debt instrument bundled with an embedded credit derivative. In exchange for a higher yield on the note, investors accept exposure to a specified credit event. Such a note might provide for principal repayment to be reduced below par if a reference asset defaults prior to the maturity of the note.

In August 2000 and in May 2001, Citigroup sought to hedge its credit exposure to Enron by issuing $1.4 billion in five-year credit-linked notes.[8] These notes became newsworthy after Enron's December 2001 bankruptcy. Investors received a steady stream of fixed payments on the notes. Citigroup invested the investors' money in a combination of highly rated corporate and government securities.

Citigroup promised to return the investors' principal if the notes' five-year terms elapsed without incident. But the contract stated that if Enron ever went bankrupt, Citigroup would take possession of the highly rated securities and give the investors unsecured Enron debt instead. The investors would then settle with Enron's other creditors in bankruptcy court.

The notes worked like an insurance policy: Citigroup paid a premium in the form of interest payments, and if Enron collapsed, the bank would receive significant compensation in the form of high-quality securities. The bulk of the hedge was created in May 2001 and was a record issue of credit-linked notes: $855 million worth, in three currencies.

The notes paid 7.37 percent if denominated in dollars, and less if denominated in euros or British pounds. The rates were similar to the average interest then being paid by Aaa-rated bonds. The average for companies with Enron's rating, Baa, was 8.07 percent. Enron itself was among the notes' first buyers.

ENGINEERING TAILORED SOLUTIONS

In contrast with the new products discussed in the previous section, this section considers two examples of custom solutions to the needs of a particular customer. Both the creation of new products and the development of custom-tailored solutions illustrate the techniques of the financial engineer. We first examine a swap agreement developed for Northwestern Bell Telephone to economize on taxes. We then consider an inverse oil-indexed bond developed for an Algerian government agency. This oil-indexed bond is a type of structured note.

Northwestern Bell Telephone Debt-for-Debt Swap[9]

Northwestern Bell Telephone (NWB) is a subsidiary of U.S. West, one of the Baby Bell telephone companies. On May 2, 1990, it extinguished $42.7 million of several debt issues with coupons ranging from 4.875 to 9.5 percent having maturities of 8 to 26 years. To extinguish this debt, NWB issued a new security paying 7.75 percent maturing in May 2030. Morgan Stanley & Company assisted in this transaction.

Morgan Stanley purchased $42.675 million of NWB's debt for about $34.5 million through open market and privately negotiated transactions. NWB gave to Morgan Stanley a cash payment of $408,000 and $42.883 million face value of its new issued 7.75 bonds due in 2030 in exchange for the $42.675 million of NWB bonds that Morgan Stanley had acquired in the open market. Figure 8.8 shows the transactions involved. In these transactions, Morgan did not act as the agent of NWB. If Morgan had acted as NWB's agent, buying the old bonds for NWB's account, the difference between the basis and the market value of each bond would have been treated as taxable income to NWB. At the time of the transaction, the tax law allowed for a tax-free exchange if the principal amount of the new issue was about the same as the principal amount retired. Because the transaction was structured as an exchange, NWB avoided any unfortunate tax implications.

The advantage of this transaction stemmed from the difference between the corporate tax rate and the marginal investor's tax rate. By creating a transaction that avoided taxation on the book gain from extinguishing the old debt, NWB was able to profit by repurchasing the old discount bonds and simultaneously issuing par bonds. Constructing this swap as an exchange was necessary to make the deal profitable. Table 8.8 shows the profits from the NWB transaction. The total present value gain was $1.125 million; the cost of the exchange was $248,000, leaving a profit of $877,000.

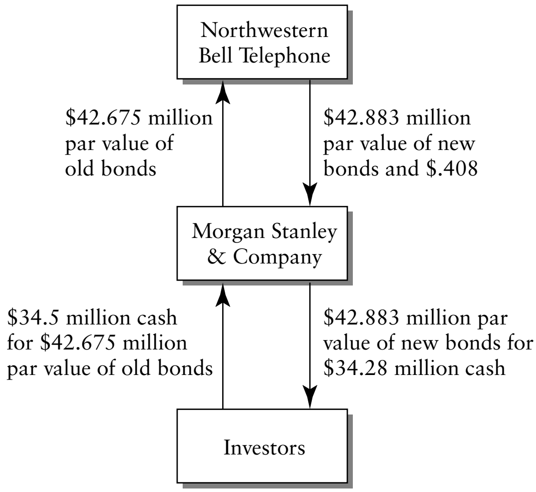

FIGURE 8.8 The Northwestern Bell Telephone swap transaction. *Source:* Andrew Kalotay and Bruce Tuckman, "A Tale of Two Bond Swaps," *The Journal of Financial Engineering,* 1:3, December 1992, pp. 235–343.

SONATRACH'S INVERSE OIL-INDEXED BONDS[10]

Sonatrach is the state-owned hydrocarbon producer of Algeria. In late 1989, Sonatrach was faced with financial difficulty in meeting payments to a banking syndicate on its borrowings through a floating-rate note (FRN). Sonatrach was paying LIBOR plus a large spread of several full percentage points. To resolve this financial embarrassment, Chase Manhattan Bank helped Sonatrach retire its existing FRNs by issuing a series of inverse oil-indexed bonds. The new debt structure substantially reduced the cash flows due from Sonatrach in each period, thereby reducing the likelihood of financial difficulty.

The Sonatrach transaction was organized as follows. First, Sonatrach issued new FRNs paying LIBOR plus 100 basis points to a group of syndicate banks. Second, Sonatrach wrote two-year call options on oil with a striking price of $23 to Chase. Third, Chase wrote seven-year calls on oil with a striking price of $22 to the syndicate banks. Fourth, Chase wrote seven-year puts on oil with a striking price of $16 to the syndicate banks. Fifth, the banks extinguished the previously existing FRNs, accepting the new FRN from Sonatrach and the calls and puts from Chase as a substitute. Figure 8.9 summarizes the transaction.

After the transaction, Sonatrach was obligated to pay a lower annual sum on the FRN. Before the transaction, Sonatrach was paying LIBOR plus

TABLE 8.8 Profits from the Northwestern Bell Telephone Swap Transaction

Coupon	Maturity	Market Yield	Market Price	Corporate Value	Principal Retired	PV Gain
4.875	6/01/98	9.30	75.23	81.60	9.590	.611
7.50	4/01/05	9.42	84.77	87.79	4.000	.121
6.25	1/01/07	9.45	73.39	78.29	10.300	.506
7.00	101/09	9.54	78.03	81.73	7.185	.266
7.875	1/01/11	9.53	85.14	87.45	4.600	.106
9.5	8/15/96	9.78	97.34	97.66	7.000	.023
7.75	5/01/30	9.80	79.54	80.72	42.833	–.508

Total present value gain:	$1.125 million
Costs of doing the exchange:	.248
Net gain:	$ .877 million

Source: Andrew Kalotay and Bruce Tuckman, "A Tale of Two Bond Swaps," *The Journal of Financial Engineering,* 1:3, December 1992, pp. 235–343.

approximately 300 basis points; after the transaction Sonatrach's interest payment was LIBOR plus 100 basis points. This change greatly reduced the financial pressure on Sonatrach. However, Sonatrach was also obligated to make additional payments if the price of oil rose above $23. If this were to occur, Sonatrach would be enjoying substantially larger income from the sale of its oil production and would be able to afford the additional payments without incurring financial distress. The banks in the syndicate accepted a lower interest payment in exchange for the chance to profit from oil price volatility. In the new arrangement, the syndicate banks held a strangle on oil with exercise prices of $16 and $22. In effect, Sonatrach relinquished a portion of its upside potential from rising oil prices to escape high interest payments that were generating financial distress. Chase received a two-year call option from Sonatrach and wrote a strangle on oil prices to the banks in the syndicate. The difference in value between the call option Chase received and the strangle Chase wrote was the compensation to Chase for arranging the transaction. By this restructuring of debt and the issuance of the inverse oil-indexed structured note, all three parties received a benefit.

DERIVATIVES DEBACLES

To this point, we have concentrated on the techniques for using derivatives. This is appropriate, as the proper use of derivatives provides substantial benefits to transactors and to the economy as a whole. We must also recognize

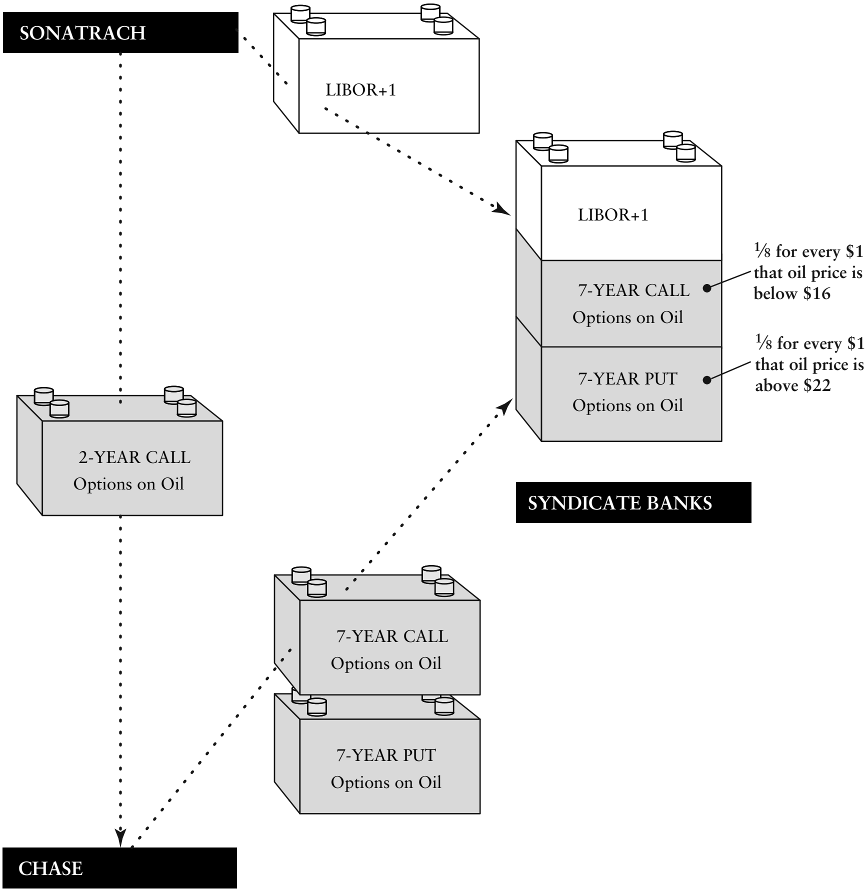

FIGURE 8.9 Sonatrach's inverse oil-indexed bond. *Source:* Christopher L. Culp, Dean Furbush, and Barbara T. Kavanagh, "Structured Debt and Corporate Risk Management," *The Journal of Applied Corporate Finance,* 7:3, Fall 1994, pp. 73–84.

that derivatives are powerful instruments that ignorant or malicious economic agents can misuse. In this section, we consider some notorious financial debacles in which the abuse of financial derivatives has played a role. First, we consider the bankruptcy of Orange County, California, in which a mistaken bet on interest rates lost about $1.6 billion of public funds. Second, we consider misfortunes that have beset Bankers Trust, an investment banking firm that has been a leader in financial engineering and in the marketing of derivative instruments to Gibson Greetings and Procter & Gamble. Third, we analyze the demise of Barings Bank, a firm that lost more than its entire net worth in a few months, apparently through the actions of a single trader. Fourth, we explore Metallgesellschaft, where an apparently sensible futures hedging strategy broke down in its implementation. Finally, we examine the collapse of Long-Term Capital Management (LTCM), a hedge fund run by some of the best and brightest minds on Wall Street. Although financial derivatives did not trigger the collapse, the case highlights many potential dangers of derivatives usage.

The Bankruptcy of Orange County, California

In December 1994, the government of Orange County, California, shocked the financial world as well as its citizens by declaring bankruptcy. The sole cause of this disaster was losses approaching $2 billion in the investment portfolio of the county managed by the treasurer of the county, Robert Citron.

The investment pool consisted of about $7 billion, with funds being contributed from many governmental agencies in Orange County, such as school districts and water boards. These funds were effectively on deposit with the county for investment management, with the idea that they would be readily available to meet the financial needs of the governmental entities, such as payrolls. In most similar situations, the funds would be invested in very safe, very short maturity, money market instruments.

In the sharply upward sloping yield curve environment of the early to mid-1990s, Citron followed a strategy of investing the funds of the county in longer maturity instruments. This decision alone could not account for the stupendous losses incurred in Orange County, however. Orange County used its $7 billion of capital as a foundation on which it borrowed additional funds through reverse repurchase agreements, or reverse repos. By using the principal of the fund as collateral, Citron borrowed about $13 billion in additional funds on a short-term basis. Having assembled a war chest of about $20 billion, Citron invested the vast bulk of these funds in long maturity instruments. Thus, Orange County was attempting to capture the large yield differential between short-term and long-term securities. As long as the yield curve maintained a steep upward slope, Orange County was effectively borrowing at a low rate of interest and investing at a substantially higher rate. However, this strategy involved considerable risk. The value of longer maturity debt instruments can change dramatically if interest rates change, so a rise in the long-term rates could create a substantial capital loss on the invested funds.

This yield curve play, combined with substantial leverage, was the essence of the Orange County strategy. It was a high-risk strategy that worked well for several years, but its inherent dangers finally gained dominance. As such, derivatives did not play a key role in the decision to use leverage and make the yield curve bet. However, derivatives did play an important role in allowing Citron to increase leverage and effectively lengthen the average maturity of the investments in the portfolio.

The principal derivative strategy used by Orange County focused on structured notes; the portfolio included as much as \$8 billion in such notes.[11] One of the Orange County structured notes was an inverse floater that paid interest according to the following formula:[12]

$$\text{Principal} \times \left(10.5 \text{ percent} - \text{six-month LIBOR}\right)$$

The principal for this particular note was \$50 million with a maturity of September 24, 1997. At the time of the bankruptcy, this note had a market value of \$43.3 million. Compared with a normal floating-rate note, this structured note had approximately twice the sensitivity to changes in interest rates. For example, a 1 percent increase in rates on this note would result in a loss of about \$2 million, or about 4.67 percent of the value of the note. This greater sensitivity in the investment, combined with 300 percent leverage is the kind of transaction that created the dramatic risk exposure that destroyed Orange County.

In 1994, not only did interest rates rise in general, but the yield curve also flattened. This meant that the differential between long and short rates that Orange County was trying to capture narrowed at the same time that the rising short-term rates increased the county's borrowing cost. The rise in long-term rates also generated considerable capital losses on the long-term instruments in the portfolio.

In the aftermath, Orange County's bankruptcy led to a major layoff of government workers, reduced county salaries and services, and required higher taxes. Work on new roads was postponed indefinitely. The county canceled or postponed projects such as school renovations. The county's school districts, which had \$1 billion invested in the Citron pool, worried about paying monthly bills for items such as school supplies and lighting. Robert Citron pleaded guilty to lying to bond investors and misappropriating public funds. He was sentenced to a year in jail, a \$100,000 fine, 1,000 hours of community service, and psychological counseling. Citron also was a witness in Orange County's \$2.4 billion lawsuit against Merrill Lynch. Merrill Lynch sold the bonds to the county and also helped fund Citron's strategy through repurchase agreements and underwriting securities for the county, the proceeds of which were invested in the yield curve bet. In the end, Merrill Lynch agreed to pay Orange County \$400 million to settle the case and to pay a $2 million civil monetary penalty to the Securities and Exchange Commission (SEC). The dismissal of Citron and the rupture with Merrill Lynch left the county with an enormously complex portfolio that still involved considerable interest rate risk. Therefore, the county moved swiftly to dispose of the more complex instruments and to deleverage the portfolio. As this sad story shows, derivatives are powerful instruments that can greatly increase risk as well as manage risk.

Bankers Trust-Gibson Greetings

On April 19, 1994, Gibson Greetings, Inc., a manufacturer of seasonal greeting cards, wrapping paper, and related products, with headquarters in Cincinnati, Ohio, filed its quarterly financial statement with the SEC. The company stated that it had taken a $16.7 million charge resulting from losses on two swap transactions with BT Securities Corporation, a subsidiary of Bankers Trust New York Corporation (BT), which is now part of Deutsche Bank.[13] The announcement stated that this loss was in addition to a $3 million charge announced earlier related to the same swap transactions. The announcement also stated that cumulative losses from the two swap transactions could potentially reach $27.5 million.

Gibson's announcement and the resulting legal and regulatory events involving Gibson's swap counterparty, BT, attracted considerable attention in the financial press and among participants in the financial markets. These newsworthy events included a lawsuit filed by Gibson against BT that was settled out of court in November 1994, with Gibson paying BT only $6.2 million out of $20.7 million owed under the terms of its swap agreements with BT.[14] In addition to this private lawsuit, three government regulatory actions resulted from the Gibson-BT dispute. In December 1994, the Federal Reserve Bank of New York (NY Fed) entered into a written agreement with BT regarding the future conduct of BT's "leveraged derivatives transactions." That same month, the Commodity Futures Trading Commission (CFTC) and the Securities and Exchange Commission entered and settled simultaneous actions against BT in which BT paid a civil penalty of $10 million to the United States Treasury.

Although the BT-Gibson dispute revolved around two swap contracts, BT and Gibson had engaged in 27 previous transactions. The two contracts in dispute represented the cumulative position resulting from the earlier transactions. The relationship between BT and Gibson began innocently enough with simple plain vanilla fixed-for-floating interest rate swaps. Over time, the transactions evolved to more complex, customized structures. The dispute between Gibson and BT centered around the duties of the two parties in determining the value of these complex structures. Gibson claimed that BT breached its fiduciary duty as Gibson's financial adviser by dispensing advice that advantaged BT at Gibson's expense. BT claimed that their relationship with Gibson was purely arm's length, without any fiduciary or advisory role. Characterizing the nature of the relationship between BT and Gibson was an important element of the dispute.

Gibson entered into its first interest rate swaps with BT on November 12, 1991. These plain vanilla interest rate swaps were, according to Gibson, intended to "reduce its interest costs" related to a $50 million fixed-rate (9.33%) borrowing completed in May 1991. Attracted to the low short-term rates at the time, Gibson transformed its fixed obligation into a floating obligation by entering into two fixed-for-floating interest rate swaps on this date, each with a notional amount of $30 million.

By transmuting its fixed-rate expense into a floating-rate expense, the swaps allowed Gibson to offset a portion of its fixed interest expense on its borrowing during 1992 and 1993. In each of those years, Gibson would have received a net payment equal to 1.21% of $30 million from BT. Based on expectations about LIBOR at the initiation of the swap agreements, Gibson's overall interest rate expense during 1994 through 1996 was expected to rise above its initial fixed-rate obligation. Of course, this is reasonable, since when all the swap payments were taken together, the swaps initially had zero value. *Ex ante,* Gibson had only succeeded in shifting its anticipated interest expense from one period to another.

However, if future interest rates were to be below the rates initially expected to prevail, then Gibson would have profited from the swap (Gibson would have received a value of fixed payments higher than the value of the floating payments made). Further, if future rates were to be low enough relative to expectations, then Gibson's interest-rate expense could be lower during this period as well. Achieving this reduction in interest rate expense was dependent on winning what was, in effect, a bet on the future direction of rates.

Gibson did not hold these swaps to term, but, after amending the contracts in January 1992, terminated them on July 7, 1992, receiving a payment from BT of $260,000, representing the value of the swaps at that time. This payment reflected that Gibson had profited under its swaps because of falling interest rates during the first half of 1992.

On October 1, 1992, Gibson entered into another swap with BT called a "ratio swap." In this swap, the payment to Gibson from BT was to be as follows:

$$\text{Net payment from BT} = 5.5\% - \frac{\left(\text{LIBOR}^2\right)}{6.0\%}$$

This swap increased Gibson's exposure to increases in the level of short-term interest rates. Under the new swap, the future net payments to Gibson would become negative more rapidly than under the original swaps, and the difference would become exponentially greater, the greater the rate.

This swap was amended three times to shorten the agreement before being terminated on April 21, 1993, with BT making a payment to Gibson of $978,000. In one of the amendments, the termination date of the swap was shortened by a year in exchange for Gibson entering into another swap. Through March 4, 1994, Gibson entered into several additional swaps with BT. According to Gibson's complaint, the swaps "ultimately involved complex structures highly sensitive to even small movements" in rates. When interest rates spiked upward sharply beginning in February 1994, Gibson had two outstanding swaps with BT, and it was still exposed to interest rate increases. According to Gibson, between February 25 and March 3, 1994, the present value of Gibson's two outstanding swaps fell $9.5 million for a cumulative loss of $17.5 million. On March 4, Gibson rolled its existing swaps into two final swaps that became the subject of Gibson's suit.

Although the plain vanilla and ratio swap transactions described earlier were not at the center of the Gibson-BT dispute, it is important to understand how these transactions became intertwined with more complex future transactions. It is this process of terminating a swap, or a portion of a swap (a so-called *tear-up agreement*), in consideration for entering into a new or amended swap agreement that ultimately triggered the dispute. Of the 29 transactions between Gibson and BT, many involved the termination of one position in exchange for entering into another position. This process of rolling from one position to another is called *morphing* by some practitioners. It requires agreement between the parties as to the terms that will equate the tear-up value of the existing swap (or swap portion) to the value of the new position (or amendment) received in exchange for the tear-up. The dispute between Gibson and BT centered on the duties of the two parties in determining the value of the positions involved in tear-ups and rollovers. Because of the complexity of the customized deals, valuation relied on model prices—comparable market quotes were not observable as they would be with a plain vanilla transaction. Gibson, and the government, alleged that BT knew that Gibson relied on BT's information to value swap positions. In addition, Gibson, and the government, alleged that BT misled Gibson about the value of those swap positions. Gibson alleged that BT provided it with valuations that significantly understated the magnitude of Gibson's losses, leaving the company unaware of the actual extent of its losses from the swap transactions. Moreover, Gibson, and the government, alleged that an advisory relationship existed between BT and Gibson. Under this legal theory, BT owed Gibson a duty not to misrepresent valuation information. BT argued that their transactions with Gibson were strictly arm's length deals and that the master swap agreement did not establish any advisory or fiduciary relationship. BT argued that the tear-up values they quoted were simply that—quotations at which BT stood ready to transact a tear-up.

BT was certainly aware of Gibson's reliance on BT's models. A taped conversation of a BT managing director and his supervisor includes the passage: "From the very beginning, [Gibson] just, you know, really put themselves in our hands like 96 percent. . . . And we have known that from day one . . . these guys have done some pretty wild stuff. And you know, they probably did not understand it quite as well as they should. I think that they have a pretty good understanding of it, but not perfect. And that's like perfect for us."[15] The SEC alleged that on two occasions BT provided Gibson with valuations that differed by more than 50 percent from the value generated by BT's models and recorded on BT's books.

We can characterize the factual background to the dispute between Gibson and BT with the following four points:

1. The swap transactions between Gibson and BT exposed Gibson to increases in short-term interest rates.
2. Gibson and BT frequently amended existing swap positions or tore up existing swap positions in exchange for rolling into new swap positions.
3. Market prices about the tear-up values and the value of new customized swap positions were not observable. Instead, valuation models were used and positions were "marked to model."
4. The dispute between Gibson and BT centered on the extent of Gibson's reliance on BT to determine the value of the positions involved in tear-ups and rollovers.

Bankers Trust-Procter & Gamble Procter & Gamble (P&G) sued over its similar experience with BT. P&G accused BT of misleading statements about the terms and consequences of two interest rate swaps in which P&G alleged damages of $195 million. In one of the swaps, BT was the fixed-rate payer and P&G paid a floating rate equal to the commercial paper rate minus 75 basis points plus a spread.[16] The spread depended on the yield of the five-year and thirty-year U.S. Treasury issues. In this swap, P&G effectively borrowed at 75 basis points below the commercial paper rate, but to capture this very attractive rate, P&G accepted some risk. The notional amount of the swap was $200 million with a five-year tenor, and the agreement was consummated on November 2, 1993. The swap agreement also included some very complicated option features designed to allow P&G to lock in a favorable interest rate, even if interest rates rose.

By February 22, 1994, interest rates had risen significantly, and BT informed P&G that it would have to pay an additional $44 million in interest over the remaining 4.5 years of the swap. In the next few weeks, rates continued to rise. By March 29, 1994, P&G found it was obligated to pay interest to BT at the commercial paper rate plus 14.12 percent, giving a total loss of more than $150 million.

The pricing of the deal was from a proprietary model of BT. P&G had placed itself in a position in which it had to rely on the computations of BT, without understanding how the results were reached. In large part, this was because of the swap's complex option provisions. P&G claimed that it relied on BT to correctly assess and convey the exact terms of the provision for locking in a favorable rate. Instead, P&G claimed that it was the victim of a financial fraud, a charge that BT strongly contested. BT argued that P&G was fully aware of the risks when they agreed to the swaps.

In May 1996, BT and P&G announced a settlement to the dispute. Under the terms of the agreement, P&G paid BT $35 million and assigned to BT the value of its swaps, worth at the time about $14 million.

The Demise of Barings Bank

In previous chapters, we have considered arbitrage strategies using futures contracts. Theoretically, arbitrage is risk-free, and properly executed arbitrage transactions involve very low levels of actual risk. Thus, arbitrage is a low-risk strategy that seeks to capture small and temporary pricing discrepancies between markets.

Nicholas Leeson, a trader for Barings Bank stationed in Singapore, was supposed to be conducting arbitrage between Japanese stock index futures contracts traded in Japan and similar futures contracts traded on the Singapore exchange (SIMEX). Such trading involves buying the cheaper contract and simultaneously selling the more expensive one, then reversing the trade when the price difference has narrowed or disappeared.

Government studies conducted after the collapse revealed that Leeson also had written options on the Nikkei 225 stock index. Leeson's overall position in December 1994 resembled a short straddle position similar to the position described in Chapter 5 and illustrated in Figure 5.4. The strike prices for most of Leeson's straddle positions ranged from 18,500 to 20,000 index points. To make money, Leeson needed the Nikkei 225 to trade in this range. The Kobe earthquake on January 17, 1995, however, rocked the entire Japanese economy and led to a dramatic drop in the Japanese stock market. This shattered Leeson's short straddle strategy and exposed his position to large losses. It appears that Leeson tried to make up this loss by establishing large long positions in Nikkei 225 futures. Instead of conducting risk-free arbitrage, Leeson made very large one-sided bets that Japanese stocks would rise. When Barings filed for bankruptcy in February 1995, it was discovered that Leeson, in the name of Barings, had established (and concealed in an error account) outstanding notional futures positions on Japanese equities of $7 billion. In addition, Leeson had outstanding notional futures positions on Japanese bonds and Euroyen totaling $20 billion.[17] Leeson had also sold Nikkei put and options with a nominal value of about $7 billion. The reported capital of Barings at the time was $615 million. The highly leveraged bets on a rising Japanese market turned out to be giant losers. In a short period, Leeson's trades lost about $1.4 billion. These losses, completely exhausted the capital of Barings, which declared bankruptcy and was acquired by the Dutch investment bank, ING for £1.00.

After the losses became public, Leeson was arrested, convicted, and sentenced to a 6½-year prison term in Singapore. By 1999, Leeson was out of prison, giving speeches on the dangers posed by rogue traders (at $100,000 per appearance), and receiving numerous job offers in risk management.

Leeson's rogue trading, while spectacular, is hardly an isolated incident. The 1990s witnessed a steady stream of staggering losses caused by rogue traders. In the mid-1990s, Daiwa and Sumitomo Corporation each lost over $1 billion from rogue traders in their employ. In 1997, Codelco lost $200 million, allegedly caused by a rogue trader. In February 2002, Allied Irish Banks (AIB) announced a $750 million loss attributed to rogue trading. The threat of rogue trading highlights the need for corporations to adopt strict internal control procedures to monitor the derivatives positions taken by traders on the corporation's behalf.

METALLGESELLSCHAFT

In December 1993, MG Refining and Marketing Inc. (MGRM), a subsidiary of Metallgesellschaft AG, a German conglomerate, revealed that it was responsible for losses of approximately $1.5 billion. MGRM committed to sell, at prices fixed in 1992, certain amounts of petroleum every month for up to 10 years. These contracts initially proved to be very successful since they guaranteed a price over the current spot petroleum prices. By September 1993, MGRM had sold forward contracts amounting to the equivalent of 160 million barrels. Embedded in MGRM's contracts was an option clause allowing counterparties to terminate the contracts early if the front-month New York Mercantile Exchange (NYMEX) futures contract was greater than the fixed price at which MGRM was selling the oil product. If the buyer exercised this option, MGRM would be required to pay in cash one-half of the difference between the futures price and the fixed prices times the total volume remaining to be delivered on the contract. This option would be attractive to a customer if they were in financial distress or simply no longer needed the oil.

MGRM thought their risk management expertise, combined with the financial backing of their parent company, allowed them to offer their customers guaranteed contracts with sell-back options. To manage the risk from offering these contracts, MGRM employed a stack hedge using the front-end month futures contracts on the New York Mercantile Exchange (NYMEX). Recall from Chapter 3 that a stack hedge involves establishing a large "stacked" futures position in the front month and then rolling the position forward (less the portion of the hedge that is no longer needed) into the next front month contract. This strategy involves risk.

The futures contracts MGRM used to hedge customer contracts were written on unleaded gasoline and No. 2 heating oil. MGRM also held some West Texas Intermediate sweet crude contracts. MGRM went long in the futures and entered into privately negotiated swap agreements to receive floating and pay fixed energy prices. At one point, MGRM held a notional futures position of 55 million barrels of gasoline and heating oil. Their swap positions accounted for nearly 100 million additional barrels.

It is not clear that there was anything conceptually wrong with MGRM's hedging strategy. But it is clear that MGRM had not adequately communicated its intentions to its parent company and financial backers. In late 1993, MGRM's futures positions lost money as spot energy prices fell, requiring additional funds to meet margin calls. Presumably, the value to MGRM of its long-term, fixed-rate customer contracts increased as the value of the energy prices fell, leaving the net position hedged as planned. But the gains on the long-term customer contracts were not realized, while the futures losses were marked to market daily. By the end of 1993, the heavy cash outflows required to maintain the stack hedge, combined with concern about the credit risk taken on with the large swap position, caused MGRM's parent to change its assessment of the potential risks involved in its forward delivery contracts with customers. After reviewing the program, MGRM's parent decided to end MGRM's participation in the hedge program. In December 1993, MGRM's futures positions were unwound and customer contracts were canceled. Given that many of these contracts were in-the-money to MGRM, this cancellation was costly.

Liquidating such huge futures positions on short notice was also extremely costly. The average trading volume in NYMEX's heating oil and unleaded gasoline futures contracts averaged around 25,000 contracts per day. With MGRM needing to liquidate 55,000 long futures contracts, other traders were able to extract a large premium from MGRM for providing liquidity. The net result of MGRM's actions was to lose on both legs of its hedge. By unwinding the hedge, MGRM lost on the futures leg of its hedge while forgoing the unrealized gains in the forward customer contracts. In total, MGRM lost $1.5 billion in the disaster.

MGRM's parent blamed the management of MGRM for the massive buildup of MGRM's forward and hedge positions. But MGRM's parent was clearly not blameless. If they truly were ignorant of MGRM's positions, then they were not doing their job. On the other hand, if they knew of MGRM's positions and did not understand them, then they were not doing their job. Wherever the truth lies, MGRM's parent shares the blame for this situation.

LONG-TERM CAPITAL MANAGEMENT

In September 1998, Long-Term Capital Management (LTCM), a private hedge fund, sat on the brink of bankruptcy. Because of LTCM's huge derivatives positions, and its large debts to many lenders, the Federal Reserve System, particularly the Federal Reserve Bank of New York, feared that LTCM's bankruptcy could trigger a chain of events that would threaten the world's financial system. To avert the pending bankruptcy, the New York Fed took the unprecedented step of arranging a bailout of the fund. Although a derivatives trading strategy was not the initial cause of LTCM's downfall, the fund did hold extremely large derivatives positions, including many privately negotiated swap contracts with some of Wall Street's largest financial institutions. The suddenness of LTCM's collapse left its swap counterparties with credit risk exposures greatly in excess of their initial expectations. A bankruptcy by LTCM would have created large losses for its swap counterparties, lenders, and other creditors. In the judgment of the Federal Reserve's authorities, these losses were potentially large enough to impair not only these direct market participants, but also the economies of many nations.

John Meriwether founded LTCM in 1993. He had previously headed up the highly profitable bond-arbitrage group at Salomon Brothers. At LTCM, Meriwether assembled a partnership consisting of former Salomon traders, two Nobel laureates (Robert Merton and Myron Scholes), and a former Vice-Chairman of the Board of Governors of the Federal Reserve System (David Mullins). They set up LTCM as a hedge fund, employing the same trading strategies that Meriwether had used at Salomon Brothers.

A hedge fund is a private investment partnership, accessible only to large investors, and is unregulated by the rules governing other investment companies. Hedge funds can employ any trading strategy they choose, including highly risky strategies, hence the term "hedge" is misleading in describing the risk appetite of the fund's investors.

In the early days of LTCM, the fund took little outright risk. The firm's goal was to limit the fund's risk to the same level of risk as the overall stock market. The core strategy involved trades designed to take advantage of small differences in prices of nearly identical bonds. In essence, the fund placed bets on small price differences that were likely to narrow as the nearly identical bonds converged to the same value. Based on an analysis of historical correlations, LTCM judged that convergence was likely to happen, barring default or market disruptions. The strategy worked well for the firm in 1995 and 1996 returning over 40 percent annually to investors after fees and expenses. By 1997, the firm had amassed investment capital of $7 billion and controlled $125 billion in assets.

During 1997, convergence trades had become less profitable, and LTCM produced a return of only 17 percent compared with U.S. stocks, which gained 33 percent. Since the fund claimed to have the same overall level of risk as the stock market, producing lower returns than the stock market was embarrassing. The fund had to find a way to produce higher returns for its investors.

Part of LTCM's strategy to produce higher returns involved returning capital to investors. By shrinking the capital base to $4.7 billion while keeping assets at $125 billion, investors who remained in the fund, the thinking went, would see higher returns. Of course, a consequence of this action is that the leverage ratio of the fund went up.

On Wall Street, LTCM enjoyed huge respect from financial firms, many of which were clamoring to invest in the fund. But LTCM was not seeking new investors especially when it was trying to shrink its capital base and increase leverage. Nevertheless, some investors were allowed to participate in the fund under the condition, allegedly, that they would also loan funds to LTCM. Union Bank of Switzerland invested $266 million and also loaned the fund $800 million. Credit Suisse Financial Products invested $33 million and loaned the fund $100 million.

In addition to increasing leverage, LTCM also looked for new opportunities to exploit its trading strategy, adding new risks to the equation. The fund applied its convergence arbitrage strategy to the swaps market, betting that the spread between swap rates and the most liquid Treasury bonds would narrow. It also applied the strategy to spreads between callable bonds and interest rate swaptions as well as to mortgage-backed securities and double-A corporate bonds. Then LTCM ventured into equity trades, selling equity index options and taking speculative positions in takeover stocks with total return swaps. It became one of the largest players on the world's futures exchanges in debt and equity products. The fund had positions in futures, options, swaps, and other OTC derivatives totaling more than $1 trillion in notional principal.[18] Many of these trades offset each other, so the notional principal did not reflect the fund's level of risk.

In May and June 1998, LTCM's troubles began: A downturn in the mortgage-backed securities market caused the value of the fund to drop about 16 percent. In August, Russia's announcement that it was defaulting on its bond payments sent shock waves through the world's financial markets. The Russian bond default led the market to reassess credit risks in general, but credit risks on sovereign (i.e., government-issued) debt in particular. The Russian bond default caused credit spreads to increase sharply and stock markets to plunge. LTCM now found that it was on the losing side of the bets it had placed on the direction of swap spreads and stock market volatility.

By the end of August, LTCM's year-to-date loss exceeded 50 percent. In September, the portfolio's losses accelerated, with the investors losing a cumulative 92 percent of their year-to-date investment. The losses triggered huge margin calls on LTCM's losing T-bond futures positions. Meeting the margin calls depleted the fund's liquid resources. It was now on the brink of bankruptcy, unable to meet further margin calls.

At the same time that it was responding to margin calls on its futures positions, LTCM's 36 swap counterparties began to call for more collateral to cover the large credit exposure that LTCM now posed. They feared that the posted collateral would be insufficient to cover large losses. In addition, the counterparties feared that there was potential for losses to accrue before the collateral could be liquidated. Although each counterparty monitored the credit risk of their individual positions with LTCM, it is clear in retrospect that they were unaware of the extent of the fund's swap positions with other counterparties. Swap counterparties also failed to gauge the full extent of LTCM's leverage, resulting in a serious underestimate of the counterparty credit exposure posed by the fund.

By late September, lenders, swap counterparties, and other creditors began to sense the magnitude of the problem. Bankruptcy, once thought to be a remote possibility, was now a distinct possibility. Because the fund was organized in the Cayman Islands, it was believed that the LTCM would seek bankruptcy protection under Cayman, as opposed to U.S., law. This fact added to the uncertainty faced by swap dealers, lenders, and other creditors.

In late September, the Federal Reserve Bank of New York organized a bailout for LTCM by encouraging 14 banks to invest $3.6 billion in return for a 90 percent stake in the firm. The New York Fed took action out of fear that an LTCM bankruptcy would trigger a contagion effect—a domino-like series of sequential defaults—spreading out across the globe. In addition, an unknown number of other players in the market had positions similar to LTCM's and any market disruptions caused by a LTCM bankruptcy would create a liquidity crisis as these other players rushed for the exits.

The bailout averted near-certain bankruptcy. But many commentators have questioned the wisdom of the New York Fed's actions. By intervening, it is argued, the New York Fed undermined the market discipline that is enforced by pain of failure. Allowing LTCM to fail, it is argued, would have provided powerful incentives for other hedge funds to more carefully examine their risk management practices. It also would have forced swap dealers and lenders to do a better job of accessing the true extent of their credit exposure to funds like LTCM.

By the end of 1999, all money had been paid back to investors and John Meriwether had started a new hedge fund. Because the possibility of bankruptcy was averted by the bailout, we will never know if the New York Fed's fears of a contagion effect were justified. Although derivatives were not the initial cause of the LTCM fiasco, the case offers important lessons about potential dangers of using financial derivatives. It highlights the importance of managing counterparty credit risk as well as the need for stress testing day-to-day risk management. It also illustrates the potential problems of model risk. The trading models used by LTCM contained assumptions about the historical correlation of assets that failed to account for the possibility of market disruptions.

SUMMARY

In this chapter, we considered techniques for managing risk and launched a preliminary exploration of financial engineering. We began our analysis of risk management by considering how financial futures can be used to hedge foreign exchange, equity, and interest rate risk. We turned to a real-world example of program trading in the support of index arbitrage. We then considered more complex swap transactions, such as equity swaps, forward swaps, and extension swaps. Options can be combined with swaps to create swaptions, which are options on swaps. As we move from simple risk management techniques, such as hedging with futures, to index arbitrage, to more complicated swap structures, we enter the world of the financial engineer.

The chapter provided an analysis of several products that financial engineers have created in recent years. First, we considered PERCS—Preferred Equity Redemption Cumulative Stock. A PERCS can be analyzed as a share of stock, plus a long call, plus an annuity. Compared with this portfolio, a PERCS gives added value because of its ability to exploit market imperfections. An Equity-Linked Certificate of Deposit is a financial product engineered to exploit market imperfections as well. Structured notes, a third general class of engineered products, are instruments that embrace a wide range of specialized debt structures with complicated payoffs.

In addition to engineered products, we considered examples of custom-engineered products designed to meet the needs of a particular customer. We explored the terms of Northwestern Bell Telephone's debt-for-debt swap, which was designed to realize tax savings. Sonatrach's inverse oil-indexed bond, a quite different example, helped maintain Sonatrach's solvency.

The chapter concluded with a brief overview of some recent derivatives debacles. Massive leverage with a huge term structure bet helped bring Orange County, California, to bankruptcy, and structured notes helped Orange County get enough leverage to ruin itself. We reviewed the dubious business transactions of Bankers Trust that brought the firm into litigation with its customers, Gibson Greetings and Procter & Gamble. This example highlighted the duties of customers and derivatives dealers. Barings Bank went bankrupt through massive losses incurred in a supposedly safe arbitrage program. For Barings, the problem was not only a flawed trading strategy, but also a serious lack of internal controls and managerial supervision. These poor controls created a situation in which trading directed by a single individual bankrupted a merchant bank that had been in business for more than 200 years. The case of Metallgesellschaft and its subsidiary MG Refining and Marketing showed how a conceptually sensible futures hedging program could break down in its implementation. Finally, we examined the collapse of Long-Term Capital Management. Although financial derivatives did not initially trigger the collapse, the case highlighted many potential dangers of derivatives usage.

QUESTIONS AND PROBLEMS

1. Is a call option a synthetic instrument? Explain what makes a financial instrument synthetic.

2. You hold a portfolio consisting of only T-bills. Explain how to trade futures to create a portfolio that behaves like the S&P 500 stock index.

3. A stock trades at $100 per share. A call option on the stock has an exercise price of $100, costs $16, and expires in six months. A put on the stock also has an exercise price of $100, costs $10, and expires in six months as well. The six-month risk-free rate of interest is 10 percent. State exactly what instruments to buy and sell to create a synthetic equity position in the stock.

4. For the preceding question, make a table showing the profits and losses from the option portion of the synthetic equity as a function of the stock price in six months.

5. A stock trades for $40 and a call on the stock with an expiration date in three months and a $40 strike price sells for $5. The risk-free rate of interest is 12 percent. State exactly how you would trade to create a synthetic put with a strike price of $40. Make a table showing the terminal values of your synthetic put and the actual put at expiration as a function of the stock price.

6. A stock sells for $75 and a call with an exercise price of $75 sells for $7 and expires in six months. The risk-free rate of interest is 10 percent. What is the price of a put with a striking price of $75 and the same term to expiration?

7. How much would you pay for a portfolio consisting of a short stock, short put, and a long call? Assume that the options have a striking price equal to the current price of the stock and that both options expire in one year.

8. For the same underlying stock, a call and put both expire in one year, and their exercise price equals the current market price of the stock. Assume that you sell the stock short and can use 100 percent of the proceeds. You buy the call option and sell the put. You invest all remaining funds in the risk-free asset for one year. How much will this entire portfolio (stock, put, call, and bond) be worth in one year? Explain.

9. How much should the put cost? Explain.

10. The manager asks you to devise a strategy to keep the value of the portfolio no less than $92. How would you transact? Explain.

11. The manager asks you to devise a strategy that will provide a terminal portfolio value of $112. How would you transact? Explain.

12. The manager asks you to devise a strategy that will dramatically increase the expected return on the portfolio. Give a qualitative description of how you would transact to achieve this goal.

13. The manager is determined not to trade any stocks to avoid transaction costs. Nonetheless, she desires a risk-free portfolio. How would you transact to meet her wishes? Explain.

14. Consider two interest rate swaps to pay fixed and receive floating. The two swaps require the same payments each semiannual period, but one swap has a tenor of 5 years, while the second has a tenor of 10 years. Assume that you buy the 10-year swap and sell the 5-year swap. What kind of instrument do these transactions create? Explain.

15. Assume you can borrow at a fixed rate for 10 years for 11 percent or that you can borrow at a floating rate of LIBOR plus 40 basis points for ten years. Assume also that LIBOR stands at 10.60 percent. Under these circumstances, your financial advisor states: "The all-in cost is the same on both deals—11 percent. Therefore, the two are equivalent and one should be indifferent between these two financing alternatives." How would you react? Explain.

16. How is the floating rate payment structured in an inverse floating-rate note?

17. What role did derivatives play (if any) in the Orange County debacle?

18. What were the key issues in the BT-Gibson Greetings case?

19. How did the conceptually sensible futures hedging program of Metallgesellschaft's subsidiary, MG Refining and Marketing break down in its implementation?

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appendix

Cumulative Distribution Function for the Standard Normal Random Variable

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

questions and problems with answers

CHAPTER 1 INTRODUCTION

1. What are the two major cash flow differences between futures and forward contracts?

   A futures contract requires an initial margin deposit and daily resettlement cash flows.

2. What is the essential difference between a forward contract and a futures contract?

   A futures contract requires an initial margin deposit and daily resettlement.

3. What problems with forward contracts are resolved by futures contracts?

   With a forward contract, one must be assured of the financial probity of one's trading partner. A forward contract can only be settled by fulfilling the contract terms or getting the opposite trading party to agree to release one from the forward contract obligation.

4. Futures and options trade on a variety of agricultural commodities, minerals, and petroleum products. Are these derivative instruments? Could they be considered financial derivatives?

   No, because the underlying goods are not financial obligations.

5. Why does owning an option only give rights and no obligations?

   The owner of an option has paid to possess the option, so he or she has no obligations, only privileges.

6. Explain the differences in rights and obligations as they apply to owning a call option and selling a put option.

The owner of a call option has paid for the right to demand that a seller of the option surrender the underlying instrument. At the time of exercise, the call owner must be prepared to pay the exercise price stipulated in the option contract. The seller of a put option has received a payment and has agreed to stand ready to receive the underlying good at the discretion of the owner of the put. If the owner of the put decides to exercise, the seller of the put must pay the exercise price and accept the underlying good.

7. Are swaps ever traded on an organized exchange? Explain.

   No. By their very nature, swaps are custom-tailored financial agreements. In principle, each swap is unique, so they are not suited to exchange-trading in which the exchange relies on the fact that the goods it trades are homogeneous.

8. Would all uses of financial derivatives to manage risk normally be considered an application of financial engineering? Explain what makes an application a financial engineering application.

   In broadest terms, financial engineering is the use of financial derivatives to manage risk. In normal usage, "financial engineering" generally refers to a custom-tailored application of financial derivatives to manage a particular risk.

9. List three advantages of exchange trading of financial derivatives relative to over-the-counter trading.

   Exchange-traded financial derivatives are homogeneous, so there is little need to investigate contract terms each time one wishes to trade. It is easy to close positions on exchange-traded goods compared to goods traded over-the-counter. Exchange-trading solves the problem of credit risk—the need to trust your trading party to fulfill his or her trading obligation.

10. Consider again the pension fund manager example in this chapter. If another trader were in a similar position, except the trader anticipated selling stocks in three months, how might such a trader transact to limit risk?

   The trader that anticipates selling holds a mirror-image position relative to the pension fund manager of the chapter. Thus, the trader could sell the same stock index futures position that the pension fund manager of the chapter example bought.

CHAPTER 2 FUTURES

1. What are the two most important functions of the clearinghouse of a futures exchange?

First, by acting as buyer to every seller and as seller to every buyer, the futures clearinghouse makes it possible for each trader to close a futures position at will. This result is achieved because the trader has obligations to the clearinghouse, rather than to another specific trader. Second, the clearinghouse guarantees that each trader's contract will be fulfilled, so the trader does not need to worry about the creditworthiness of his or her trading partner.

2. What is the investment for a trader who purchases a futures contract? Justify your answer.

There is no investment. Some regard the margin deposit as the investment, but that is really only a good faith deposit or a performance bond.

3. What is open interest? What happens to open interest over the life of a futures contract?

Open interest is the number of contracts obligated for delivery. When the contract is first listed for trading, open interest is necessarily zero. As traders take positions, open interest builds. As the contract approaches expiration, many traders will offset their positions to avoid delivery. This reduces open interest. At expiration, open interest will again be zero as each contract will have been fulfilled by offset, delivery, or an EFP.

4. What are the two ways to fulfill a futures contract commitment? Which is used more frequently? Why?

A futures contract can be fulfilled by delivery in accordance with the contract terms or through a reversing trade. Most contracts are settled by a reversing trade because it is usually more economical to settle the contract in this way.

5. Explain why the futures price might reasonably be thought to equal the expected future spot price.

If the futures price did not equal the expected future spot price, speculators would trade to capture the difference. Speculative activity ensures that the futures price and expected future spot price are reasonably close.

6. What is the implied repo rate? What information does the implied repo provide about the relationship between cash and futures prices?

Most participants in the futures markets face a financing charge on a short-term basis that is equivalent to the repo rate, that is, the interest rate on repurchase agreements. In a repurchase agreement, a person sells securities at one time, with the understanding that they will be repurchased at a certain price at a later time. Most repurchase agreements are for one day only and are known, accordingly, as overnight repos. The repo rate is relatively low, exceeding the rate on Treasury bills by only a small amount.

In trading vernacular, the theoretical rate of return on a cost-of-carry strategy is the implied repo rate. An arbitrageur calculates the implied repo rate and compare it to his own financing cost (proxied by the actual repo rate) to determine whether or not an arbitrage opportunity exists. In a well-functioning market without arbitrage opportunities, the implied repo rate is equivalent to the actual repo rate. Deviations from this relationship lead to arbitrage opportunities in a perfect market.

7. What kinds of transaction costs do traders face in conducting cost-of-carry arbitrage?

In actual markets, traders face a variety of direct transaction costs. First, the trader must pay a fee to have an order executed. For a trader off the floor of the exchange, these fees include brokerage commissions and various exchange fees. Even members of the exchange must pay a fee to the exchange for each trade. Second, in every market there is a bid-asked spread. A market maker on the floor of the exchange must try to sell at a higher price (the asked price) than the price at which he or she is willing to buy (the bid price). The difference between the asked price and the bid price is the bid-asked spread.

8. As part of a cash-and-carry arbitrage strategy, you are obligated to make delivery of a T-bill against the nearby T-bill futures contract when it expires in 30 days. Which T-bill should you purchase today?

The T-bill futures contract at the Chicago Mercantile Exchange calls for the delivery of a 90- or 91-day T-bill. The fact that the futures contract has only 30 days until maturity means that the trader would need to purchase a 120- or 121-day T-bill today in order for it to meet the contract requirements at delivery.

9. How can the Cost-of-Carry Model be modified to account for dividends?

For many financial assets, cash flows such as stock dividends and bond coupons will be important considerations. These cash flows can be easily included in the Cost-of-Carry Model as part of a "net" financing cost. The Cost-of-Carry Model must be adjusted to include the dividends that will receive between the present and the expiration of the futures. In essence, the chance to receive dividends lowers the cost of carrying the stocks. Carrying stocks requires that a trader finance the purchase price of the stock from the present until the futures expiration. For stocks, the cost-of-carry is the financing cost for the stock, less the dividends received while the stock is carried forward.

10. What is the fair value futures price? What market forces drive the futures price toward fair value?

The futures price that conforms with the Cost-of-Carry Model is called the fair value futures price. The fair value price reflects the proper economic relation of the futures price to its underlying instrument. The difference between the actual futures price and the fair value futures price is called basis error. If the actual futures price differs from the fair value futures price, this is an indication that risk-free arbitrage profits can be gained. If the futures price is too low relative to the fair value price, arbitragers will buy the futures and simultaneously sell the cash instrument underlying the futures. If the futures price is too high relative to the fair value futures price, arbitragers will sell the futures and simultaneously buy the cash market instrument. In the absence of transaction costs, arbitrage will be possible whenever the futures price differs from the fair value futures price. In reality, transactions costs are a significant concern. Arbitrage will only be possible when the difference between the futures price and the fair value futures price is big enough to overcome transaction costs.

11. What real-world complications can frustrate a stock index arbitrage strategy?

(1) Dividend payments may be suspended or altered; (2) the composition of underlying index continually changes through time; (3) transaction costs; (4) stock exchange restrictions on short sales of stock; (5) the prices observed on trading screens may not reflect the actual state of the market at the time arbitrageurs make their decisions; (6) trade execution

risk, that is, the arbitraguer's order may move the price; and (7) stock exchange "collars" on index arbitrage on days of large price moves. This is not an exhaustive list.

12. How is the spot FX rate related to the futures FX rate? What market forces keep spot and futures FX rates in proper alignment?

The spot FX rate and the futures FX rate are related through cost-of-carry arbitrage. Cost of carry arbitrage keeps spot and futures prices in proper alignment, just as with other markets. For example, the cost-of-carry relationship between US dollars and euro can be expressed as: $F_{0,t} = S_0 (1 + C)$, where C is the percentage cost of carrying foreign currency (in our example, euros) forward from $t = 0$ to time t, that is, $C = (r_{US} - r_{euro})$, where r_{US} is the interest rate in the United States and r_{euro} is the interest rate in Europe.

13. According to widely held belief, an upward sloping yield curve generally implied that spot interest rates were expected to rise. If this is so, does it also imply that futures prices are expected to rise? Does this suggest a trading strategy? Explain.

No, it does not imply that futures prices should rise. Instead, in an efficient market, the futures price impounds the information contained in the yield curve. Aside from small discrepancies due mainly to institutional differences in futures and cash markets, the yield implied on the futures contract will equal the forward yield implied by the term structure of interest rates. Therefore, reflection upon the term structure does not give rise to any kind of special (arbitrage) trading strategy in futures.

14. Assume that the spot corn price is $3.50, that it costs $.017 cents to store a bushel of corn for 1 month, and that the relevant cost of financing is 1 percent per month. If a corn futures contract matures in 6 months and the current futures price for this contract is $3.95 per bushel, explain how you would respond. Explain your transactions for one contract, assuming 5,000 bushels per contract and assuming that all storage costs must be paid at the outset of the transaction.

Transactions at $t = 0$:

Sell 5,000 bushels of corn in the spot market at $3.50 per bushel, or a $17,500 savings:

5,000(6).017 = $510 in storage costs, for a total of $18,010.

Invest $18,010 for six months at 1 percent per month.

Buy one corn contract for delivery in six months, at a futures price of $3.75 per bushel.

Transactions at $t = 6$ months:

Liquidate investment, receiving

$$\$18{,}010(1.01)^6 = \$19{,}117.98$$

Buy corn under the futures contract, paying:

$$5{,}000(\$3.75) = \$18{,}750$$

This gives a profit of:

$$\$19{,}117.98 - \$18{,}750 = \$367.98$$

This is an arbitrage profit because there was no investment and there was a certain profit. There was no investment, because the cash flow at time = 0 was zero. There was only a cash inflow at $t = 6$, and this was certain once the transactions at time = 0 were initiated.

15. Explain the risks inherent in a reverse cash-and-carry strategy in the T-bond futures market

The reverse cash-and-carry strategy requires waiting to receive delivery. However, the delivery options all rest with the short trader. The short trader will initiate delivery at his or her convenience. In the T-bond market, this exposes the reverse cash-and-carry trader to receiving delivery at some time other than the date planned. Also, with so many different deliverable bonds, the reverse cash-and-carry trader is unlikely to receive the bond he or she desires. (These factors are fairly common for other commodities as well.) In the T-bond futures market, the short trader holds some special options such as the wildcard and end-of-month options. The reverse cash-and-carry trader suffers the risk that the short trader will find it advantageous to exploit the wildcard play or exercise the end-of-month option.

CHAPTER 3 RISK MANAGEMENT WITH FUTURES CONTRACTS

1. Assume that you believe the futures prices for corn are too low relative to wheat prices. Explain how you could take advantage of this belief.

One could buy corn and sell wheat futures. When the two prices move closer together, this spread position will be profitable.

2. Assume that you are a bond portfolio manager and that you anticipate an infusion of investable funds in three months. How could you use the futures market to hedge against unexpected changes in interest rates?

   If you expect new funds in three months, you will be anticipating the purchase of bonds, and you will fear a rise in the price of bonds. You can protect against that possibility by buying bond futures now to match your expected date for new funds. Notice that you are locking in current futures prices, not spot prices, for the bonds you buy.

3. You believe that the yield difference between T-bills and Eurodollar CDs will widen. How would you use the futures market to take advantage of this belief?

   First, recognize that because of credit risk, Eurodollar CD yields will always exceed risk-free T-bill yields. To simplify the analysis, assume that T-bill rates are constant and that any widening in yield spread occurs because of increasing Eurodollar yields. Under this assumption, a belief in a widening yield spread translates into a fall in the Eurodollar futures price relative to the T-bill futures price because of the inverse relationship between yields and prices. To take advantage of this belief, a trader would simultaneously take a long position in T-bill futures and a short position in Eurodollar futures.

4. Which is likely to have the greater variance—the basis between cash and futures, or the cash price of the good? Why?

   The cash price is almost certainly more variable than the basis, because the cash and futures prices, which determine the basis, will be strongly correlated.

5. Describe the difference between a stack hedge and strip hedge. What are the advantages and disadvantages of each?

   A hedge implemented by establishing futures positions in a series of futures contracts of successively longer expirations is called a strip hedge. A hedge implemented by stacking the entire futures position in the front month and then rolling the position forward (less the portion of the hedge that is no longer needed) into the next front month contrac is called a stack hedge.

   Each strategy involves trade-offs. The strip hedge has a higher correlation with the underlying risks than the stack hedge (i.e., has lower tracking error), but may have higher liquidity costs because the more distant contracts may be very thinly traded and may have high bid/ask spreads accompanied by high trade-execution risk. The stack hedge has lower liquidity costs but has higher tracking error.

6. Why might it be inappropriate for a corporation to hedge?

First, reducing risk also means reducing expected return. Whether hedging improves the trade-off between risk and expected return depends on the risk preferences of individual traders. Second, when applied to publicly held corporations, we find that hedging may not add to shareholder value. These companies are organized using the corporate form specifically for the purpose of spreading the risk of corporate investments across many shareholders who further spread the risk through their individual ownership of diversified portfolios of stocks from many corporations. In a sense, a publicly held corporation is hedged naturally through its ownership structure. Shareholders are therefore likely to be at best indifferent to hedges constructed at the corporate level since such hedges can be replicated or undone by the portfolio composition of individual shareholders. The shareholders' indifference means that they are unwilling to pay a premium for shares of stock where earnings are hedged at the corporate level. Yet in spite of this indifference, many publicly held corporations are observed to hedge. We must assume that since capital market discipline creates powerful incentives for corporations to make value-maximizing decisions, that not all observed hedging is being done over the objections of shareholders.

7. What risks are associated with using a cross hedge?

A hedge in which the characteristics of the instrument being hedged and the instrument underlying the futures contract do not perfectly match is called a cross hedge. The danger is that the futures price will not be perfectly correlated with the price of the instrument being hedged. This exposes the hedger to basis risk.

8. Assume you hold a T-bill that matures in 90 days, when the T-bill futures expires. Explain how you could transact to effectively lengthen the maturity of the bill.

Buy the T-bill futures that expires in 90 days. After this transaction, you will be long a spot 90-day bill, and you will hold (effectively) a spot position in a 90-day bill to begin in 90 days. The combination replicates a 180-day bill.

9. Assume that you will borrow on a short-term loan in six months, but you do not know whether you will be offered a fixed rate or a floating-rate

loan. Explain how you can use futures to convert a fixed rate to a floating-rate loan and to convert a floating rate to a fixed-rate loan.

For convenience, we assume that the loan will be a 90-day loan. If the loan is to be structured as a floating-rate loan, you can convert it to a fixed-rate loan by selling a short-term interest rate futures contract (Eurodollar or T-bill) that expires at the time the loan is to begin. The rate you must pay will depend on rates prevailing at the time of the loan. If rates have risen you must pay more than anticipated. However, if rates have risen, your short position in the futures will have generated a profit that will offset the higher interest you must pay on the loan.

Now assume that you contract today for a fixed interest rate on the loan. If rates fall, you will be stuck paying a higher rate than the market rate that will prevail at the time the loan begins. To convert this fixed-rate loan to a floating-rate loan, buy an interest rate futures that expires at the time the loan is to begin. Then, if rates fall, you will profit on the futures position, and these profits will offset the higher than market rates you are forced to pay on your fixed-rate loan.

10. Assume you hold a well-diversified portfolio with a beta of 0.85. How would you trade futures to raise the beta of the portfolio?

Buy a stock index futures. In effect, this action levers up the initial investment in stocks, effectively raising the beta of the stock investment. In principle, this levering up can continue to give any level of beta a trader desires.

11. An index fund is a mutual fund that attempts to replicate the returns on a stock index, such as the S&P 500. Assume you are the manager of such a fund and are fully invested in stocks. Measured against the S&P 500 index, your portfolio has a beta of 1.0. How could you transform this portfolio into one with a zero beta without trading stocks?

Sell S&P 500 Index futures in an amount equal to the value of your stock portfolio. After this transaction you are effectively long the index (your stock holdings) and short the index by the same amount (your short position in the futures). As a result, you are effectively out of the stock market, and the beta of such a position must be zero.

12. You expect a steepening yield curve over the next few months, but you are not sure whether the level of rates will increase or decrease. Explain two ways you can trade to profit if you are correct.

If the yield curve is to steepen, distant rates must rise relative to nearby rates. If this happens we can exploit the event by trading just short-term instruments. The yield on distant expiration short-term instruments must rise relative to the yield on nearby expiration short-term instruments. Therefore, one should sell the distant expiration and buy the nearby expiration. This strategy could be implemented by trading Eurodollar or T-bill futures.

As a second basic technique, one could trade longer term T-bonds against shorter maturity T-notes. Here the trader expects yields on T-bonds to rise relative to yields on T-notes. Therefore, the trader should sell T-bond futures and buy T-note futures. Here the two different contracts can have the same expiration month.

13. The Iraqi invasion of Alaska has financial markets in turmoil. You expect the crisis to worsen more than other traders suspect. How could you trade short-term interest rate futures to profit if you are correct? Explain.

Greater than expected turmoil might be expected to result in rising yields on interest rate futures. To exploit this event, a trader could sell futures outright. A second result might be an increasing risk premium on short-term instruments. In this case, the yield differential between Eurodollar and T-bill futures might increase. To exploit this event, the trader could sell Eurodollar futures and buy T-bill futures of the same maturity.

14. You believe that the yield curve is strongly upward sloping and that yields are at very high levels. How would you use interest rate futures to hedge a prospective investment of funds that you will receive in nine months? If you faced a major borrowing in nine months, how would you use futures?

If you think yields are near their peak, you will want to lock in these favorable rates for the investment of funds that you will receive. Therefore, you should buy futures that will expire at about the time you will receive your funds. The question does not suggest whether you will be investing long term or short term. However, if the yield curve is strongly upward sloping, it might favor longer term investment. Consequently, you might buy T-bond futures expiring in about nine months.

If you expect to borrow funds in nine months you may not want to use the futures market at all. In the question, we assume that you believe rates are unsustainably high. Trading to lock in these rates only ensures that your borrowing takes place at the currently very high effective rates. Given your beliefs, it might be better to speculate on falling rates.

15. For the most part, the price of oil is denominated in dollars. Assume that you are a French firm that expects to import 420,000 barrels of crude oil in six months. What risks do you face in this transaction? Explain how you could transact to hedge the currency portion of those risks.

Here we assume that the price of oil is denominated in dollars. Further, contracts traded on the NYMEX in oil are also denominated in dollars. Therefore, hedging on the NYMEX will not deal with the currency risk the French firm faces. However, the French firm can hedge the currency risk it faces by trading forwards for the euro. To see how the French firm can control both its risk with respect to oil prices and foreign exchange consider the following data. We assume a futures delivery date in six months for the oil and for foreign exchange forward contracts. The futures price of oil is $30 barrel, and the six-month forward price of a euro is $.20. With these prices, the French firm must expect a total outlay of $12.6 million for the oil, and a total euro outlay of 63 million euros. By trading oil futures and euro forwards, it can lock in this euro cost. Because the crude oil contract is for 1,000 barrels, the French firm should buy 420 contracts. This commits it to a total outlay of $12.6 million. The French firm then sells 63 million euros in the forward market for $12.6 dollars. These two transactions lock in a price of 63 million euros for the oil.

CHAPTER 4 OPTIONS

1. Respond to the following claim: "Buying a call option is very dangerous because it commits the owner to purchasing a stock at a later date. At that time, the stock may be undesirable. Therefore, owning a call option is a risky position."

Owning a call option does not involve any commitment to purchase a stock. It gives the owner the option to buy a stock if he or she wishes, but it is completely discretionary. The line of reasoning is completely incorrect. Owning a call option is a risky position, however, because the value of the option is uncertain.

2. "I bought a call option with an exercise price of $110 on IBM when IBM was at $108 and I paid $6 per share for the option. Now the option is about to expire and IBM is trading at $112. There's no point in exercising the option because I will wind up paying a total of $116 for the shares—$6 I already spent for the option plus the $110 exercise price." Is this line of reasoning correct? Explain.

Bad reasoning. The $6 option price that is already paid is a sunk cost. The only question is what to do now. With the price of IBM at $112, the option to buy it at $110 is worth at least $2, so the option should be exercised or sold before expiration.

3. What is the value of a call option on a share of stock if the exercise price of the call is $0 and its expiration date is infinite? Explain.

The call price would equal the stock price, because the option could be transformed into the stock without cost by exercise at a zero price at any time.

4. Why is the value of a call option at expiration equal to the maximum of zero or the stock price minus the exercise price?

The price cannot be less than zero because the option involves no obligations. If the stock price exceeds the exercise price and the call price is less than the stock price minus the exercise price, there will be an arbitrage opportunity. One could buy the call, exercise it, and sell the stock for a profit. Similarly, if the call price exceeded the stock price, one could sell the call, buy the stock and profit by the difference between the call price and the stock price.

5. Two call options on the same stock have the following features: The first has an exercise price of $60, a time to expriation of three months, and a premium of $5. The second has an exercise price of $60, a time to expiration of six months, and a premium of $4. What should you do in this situation? Explain exactly, assuming that you transact for just one option. What is your profit of loss at the expiration of the nearby option if the stock is at $55, $60, or $65?

Buy a 6 month option at $4, and sell a 3 month option at $5: Net cash flow = $1.

If the stock price in three months is:

$55

The 3 month option cannot be exercised and you can keep the $1, plus your long position in the call.

$60

The 3 month option cannot be exercised and you can keep the $1, plus your long position in the call.

$65

The 3 month option will be exercised against you. You must surrender the stock and receive \$60. You exercise your call, and pay \$60 to acquire stock. This generates a zero net cash flow, but leaves you with the \$1 from the original transaction.

In summary, no matter what the stock price in 3 months, you will have at least a \$1 profit. If the option you sold cannot be exercised against you, your profit will be \$1 plus the value of your remaining call option.

6. Two call options are identical except that they are written on two different stocks with different risk levels. Which will be worth more? Why?

The call on the riskier stock should be worth more. A call option has inherent in it an insurance policy against extremely adverse outcomes because a call option is a leveraged instrument and the value of the call cannot fall below zero. Insurance against an adverse outcome from a greater risk must be worth more, so the option on the riskier stock must be worth more.

7. Explain why owning a bond is like taking a short position in a put option.

If you own a bond, the stockholders of the firm may refuse to pay on the bond and put the company to you, the bondholder. In this case, the stockholders sell the company to the bondholder and avoid paying the bond payments.

8. Why does ownership of a convertible bond have features of a call option?

For assets with both futures contracts and option contracts, the difference between the value of the call option and the value of the put option is equal to the present value of the difference between the futures price and the option exercise price, discounted at the risk-free rate from expiration until today.

9. Assume tht following: A stock is selling for \$100, a call option with an exercise price of \$90 is trading for \$6 and matures in one month, and the interest rate is 1 percent per month. What should you do? Explain your transactions.

Buy the call for \$6, exercise the call, paying \$90, and sell the stock, receiving \$100.

$$\text{Total profit} = \$100 - \$90 - \$6 = \$4$$

10. Consider a euro futures contract with a current price of \$.35 per euro. There are also put and call options on the euro with the same expiration date in three months that happen to have a striking price of \$.35. You buy a call and sell a put. How much should your combined option position cost? Explain. What if the interest rate were 1 percent per month and the striking prices were \$.40? How much should the option position be worth then?

If the futures is at \$.35, a long call/short put portfolio with a striking price of \$.35 should cost zero, assuming the futures and options have the same expiration. A long call/short put portfolio exactly duplicates the payoffs on a futures position with a contract price equal to the common exercise price of the options. As it costs nothing to enter the futures contract, the same must be true of the option portfolio.

For the same circumstances, except considering options with a striking price of \$.40, we see that the long call/short put replicates a long futures with a contract price of \$.40. If the current futures price is \$.35, the option portfolio will pay \$.05 less than the futures at expiration. (For example, if the terminal futures price is \$.40, the long futures will be worth \$.05, but the option portfolio will be worth zero.) Therefore, the option portfolio has a certain payoff \$.05 less than the long futures, which is itself costless. To induce a trader to accept the portfolio of options, with their inherent loss of \$.05, the market must pay the acquirer of the option portfolio the present value of the \$.05 loss that will be incurred at expiration. With three months to expiration and a discount rate of 1 percent per month, the value of the option portfolio will be $-\$.05/(1.01)^3 = -\$.04853$.

11. Two call options on the same stock expire in two months. One has an exercise price of \$55 and a price of \$5. The other has an exercise price of \$50 and a price of \$4. What transactions would you make to exploit this situation?

Buy the call with an exercise price of \$4 and sell the call with an exercise price of \$5.

This gives a net cash flow of \$1. If the stock price at expiration is above \$55, the option you sold will be exercised against you, and your total profit will be \$1. If the stock price at expiration is \$50 or below, neither option can be exercised and your total profit will be \$1.

If the stock price is greater than \$50 but less than or equal to \$55, you will make more. For example, assume that the stock price is \$53 at expiration. The option you sold cannot be exercised against you, but your option will be worth \$3. In this case your total profit will be \$4, the $3 current value of your option plus the $1 cash flow you received originally.

CHAPTER 5 RISK MANAGEMENT WITH OPTIONS CONTRACTS

1. Explain the difference between a straddle and a strangle.

   A straddle is a long position in a call and a long position in a put, with the two options having the same exercise price and the same expiration. A strangle is the same, except the exercise price of the put is less than the exercise price of the call.

2. Consider the following information about a stock and two call options, C_1 and C_2 written on the stock. The current stock price: $100. C_1 current price: 6.0581, delta = .4365, gamma = .0187. C_2 current price: 16.3328, delta = .7860, gamma = .0138. What combination of stock and the two options will produce a simultaneously delta-neutral and gamma-neutral portfolio? Assume you are long the stock.

   Assuming that we are holding one share of stock long (i.e. N_s=1), we must now determine the positions we must make in options C_1 and C_2 in order to satisfy the conditions expressed above. We know before we start that the delta on a share of stock, delta_s is 1 and that gamma_s is 0. The resulting 2-equation system looks like:

$$1(1) + N_1(.4365) + N_2(.7860) = 0$$
$$1(0) + N_1(.0187) + N_2(.0138) = 0$$

   By premultiplying the bottom expression by (.4365/.0187) the resulting system looks like:

$$1 + N_1(.4365) + N_2(.7860) = 0$$
$$N_1(.4365) + N_2(.322123) = 0$$

   Since both expressions are equal to zero then they must be equal to each other. We can now write our system of two equations with two unknowns as one equation with one unknown:

$$1 + N_2(.7860) = N_2(.322123)$$

which means that $N_2 = (1/-.46388) = -2.15574$. In other words, sell 2.15574 calls with an exercise price of \$110. Now that we have solved for N_2 we can easily solve for N_1 by going back to the original system of equations and plugging in N_2 as −2.15574 and doing the algebra to determine N_1, that is,

$$1 + N_1(.4365) + (-2.11574)(.7860) = 0$$
$$N_1(.0187) + (-2.11574)(.0138) = 0$$

Using either the top or bottom expression (or both) we find that N_1 = 1.590867. The positve sign denotes a long position, indicating that we should buy 1.590867 of C_1.

3. Your largest and most important client's portfolio includes option positions. After several conversations, it becomes clear that your client is willing to accept the risk associated with exposure to changes in volatility and stock price. However, your client is not willing to accept a change in the value of her portfolio resulting from the passage of time. Explain how you can protect her portfolio against changes in value due to the passage of time.

Your client wants to avoid changes in the value of her portfolio due to the passage of time. Theta measures the impact of changes in the time until expiration on the value of the option. Your client should create a theta neutral portfolio to protect the value of her options positions against changes in the time until expiration. To protect her portfolio from the wasting away effect associated with option contracts, she must first determine the theta for her current portfolio. Given the theta value of her portfolio, she should construct a position in option contracts that has a theta value that is of equal magnitude and opposite sign of the theta of her portfolio. Thus the theta for the hedge portfolio, the original portfolio plus the additional options contracts used to create the hedge, is zero. The value of this portfolio should not change with the passage of time. However, the portfolio will have exposure to the changes in other market variables, that is, interest rates, volatility, and stock price changes.

4. Your newest client believes that the Asian currency crisis is going to increase the volatility of earnings for firms involved in exporting, and that this earnings volatility will be translated into large stock price changes for the affected firms. Your client wants to create speculative positions using options to increase his exposure to the expected changes in the riskiness of exporting firms. That is, your client wants to prosper from changes in the volatility of the firm's stock returns. Discuss which Greek your client should focus on when developing his option positions.

Your client wants to create exposure to changes in the volatility of stock returns. Vega measures the change in the value of an option contract resulting from changes in the volatility of the underlying stock. Once you have identified stocks with traded options that have significant Asian exposures, you want to construct positions based on the vega of the option. Because your client wants exposures to volatility risk, you would construct a portfolio with a large vega.

5. Your brother-in-law has invested heavily in stocks with a strong Asian exposure, and he tells you that his portfolio has a positive delta. Give an intuitive explanation of what this means. Suppose the value of the stocks that your brother-in-law holds increases significantly. Explain what will happen to the value of his portfolio.

   Delta measures the change in value of an option due to a change in the price of the underlying asset, which is usually a stock. If an investor holds a portfolio consisting of a single stock, the delta of the portfolio is one, because a one dollar increase in the stock price will produce a one dollar per share increase in the value of the portfolio. If the asset in question is an option, then the delta of the option measures the change in the value of the option contract because of a change in the value of the underlying stock price. If your brother-in-law's portfolio has a positive delta, the value of his portfolio will move in the same direction as the value of the underlying asset. If the value of the stocks he holds increases, then the value of his portfolio will increase at a rate of delta times the dollar change in the asset price.

6. Your mother-in-law has invested heavily in the stocks of financial firms, and she tells you that her portfolio has a negative rho. Give an intuitive explanation of what this means.

   Rho measure the change in the value of an asset due to changes in interest rates. If the investor holds an option, then the rho of the option measures the change in the value of the option contract because of a change in the risk-free rate. If your mother-in-law's portfolio has a negative rho, that implies that the value of the portfolio moves in the opposite direction as changes in the interest rate. If the short-term interest rate is increased by the Federal Reserve, then the value of her portfolio will decrease.

7. Your brother, Daryl, has retired. With the free time necessary to follow the market closely, Daryl has established large option positions as a stock investor. He tells you that his portfolio has a positive theta. Give an intuitive explanation of what this means.

   Theta measures the change in the value of an option because of changes in the time until expiration for the option contract. That is, with the passage of time, the value of an option contract will change. This is known as time decay. Formally, theta is the negative of the first derivative of the option pricing model with respect to changes in the time to expiration. Since your brother has constructed the portfolio with a positive theta, the passage of time should increase the value of his portfolio. Thus, he should, all things being equal, return from his vacation to find that the value of his portfolio has increased.

CHAPTER 6 THE SWAPS MARKET

1. Explain the differences between a plain vanilla interest rate swap and a plain vanilla currency swap.

   In a plain vanilla interest rate swap, one party agrees to make a series of payments based on a fixed rate of interest, while the second party promises to make payments based on a floating rate of interest. Both payment streams are based on the same notional amount and are in the same currency. No principal changes hands; the parties just make payments based on the notional principal. In a plain vanilla currency swap, each party conveys to the other a principal amount in its home currency. This occurs at the initiation of the swap. During the life of the swap, the parties pay interest to each other based on the contract terms. At the end of the swap, the parties re-exchange the principal amount. The key difference between the plain vanilla interest rate swap and the plain vanilla currency swap is two-fold: the currency swap involves an exchange of principal, while the interest rate swap does not; the currency swap involves cash flows in two currencies, while the interest rate swap has cash flows in only one currency.

2. What role does the swap dealer play?

   Swap dealer facilitate swap transactions by standing ready to accept either side of a transaction (e.g., pay-fixed or receive-fixed) depending on the customer's demand at the time. Swap dealers act as financial intermediaries in swap transactions. Swap dealers generally run a matched book, in which the cash flows on numerous transactions on both sides of a market net to a relatively small risk exposure on one side of the market. Many of these matched transactions are termed customer facilitations, meaning that the dealer serves as a facilitating agent, simultaneously providing a swap to a customer and hedging the associated risk with an offsetting swap position or a futures position. The dealer collects a fee for the service and, if done properly, incurs little risk. Dealers may also choose to bear some amount of residual risk as part of a proprietary trading position. Swap dealers must have a strong credit standing, large relative capitalization, good access to information about a variety of end users, and relatively low costs of managing the residual risks of an unmatched portfolio of customer transactions. Most dealers are commercial banks, investment banks, and other financial enterprises such as insurance company affiliates.

3. Assume that you are a financial manager for a large commercial bank and that you expect short-term interest rates to rise more than the yield curve would suggest. Would you rather pay a fixed long-term rate and receive a floating short-term rate, or the other way around? Explain your reasoning.

   You would prefer to pay a fixed long-term rate and receive a floating short-term rate. The initial short-term rate that you receive will merely be the spot rate that prevails today. However, if your hunch is correct, the short-term rate will rise more than the market expects, and you will then receive that higher rate. Because your payments are fixed, you will reap a profit from your insight.

4. Explain the role that the notional principal plays in understanding swap transactions. Why is this principal amount regarded as only notional? (Hint: What is the dictionary definition of "notional"?)

   In interest rate swaps, all of the cash flows are based on a notional or fictional amount. This is essentially a matter of convenience in helping to conceptualize the transaction. The entire contract could be stated without regard to the principal amount. One definition of "notional" is "existing in idea only."

5. Consider a plain vanilla interest rate swap. Explain how the practice of net payments works.

   In a typical interest rate swap, each party is scheduled to make payments to the other at certain dates. For the fixed payor, these amounts are certain, but the payments that the floating payor will have to make are unknown at the outset of the transaction. In each period, one party will owe a large amount to the other, depending on how interest rates have changed. Rather than make two payments, the party owing the greater amount simply pays the difference between the two obligations.

6. Assume that the yield curve is flat, that the swap market is efficient, and that two equally creditworthy counterparties engage in an interest rate swap. Who should pay the higher rate, the party that pays a floating short-term rate or the party that pays a fixed long-term rate? Explain.

   They should pay the same. If the yield curve is flat, short-term rates equal long-term rates. Barring a change in rates, the two parties should pay the same amounts to each other in each period. If interest rates change, however, the payments will no longer be the same.

7. In a currency swap, counterparties exchange the same sums at the beginning and the end of the swap period. Explain how this practice relates to the custom of making interest payments during the life of the swap agreement.

   At the outset, the two parties exchange cash denominated in two currencies. Each party pays interest on the currency it receives from the other. Thus, the exchange of currencies is the basis for computing all of the interest payments that will be made over the life of the agreement.

8. Explain why a currency swap is also called an "exchange of borrowings."

   In a currency swap, both parties pay and both parties receive actual cash. Each has borrowed from the other, so they have exchanged borrowings.

9. Assume that LIBOR stands today at 9 percent and the seven-year T-note rate is 10 percent. Establish an indication pricing schedule for a seven-year interest rate swap, assuming that the swap dealer must make a gross spread of 40 basis points.

   In this problem, the T-note rate is above LIBOR, indicating a strongly upward sloping yield curve. A customer may elect to either pay or receive LIBOR, which now stands at 9 percent. For the swap dealer to make a gross spread of 40 basis points, and assuming that the spread is set to be even around LIBOR, the swap dealer would be prepared to receive LIBOR and pay 8.80 percent. Alternatively, if the swap dealer must pay LIBOR, he or she must receive 9.20. With the T-note rate at 10.00

percent, the swap dealer must be prepared to pay the T-note rate minus 120 basis points or to receive the T-note rate minus 80 basis points.

10. Why are only positive swap values used to measure counterparty credit risk?

Only positive swap values are of interest in determining swap credit exposure. This is because with negative- or zero-value swaps (i.e., out-of-the-money or at-the-money swaps), the counterparty owes nothing in the event of a default.

11. Suppose that the fixed-rate payer in the interest rate swap used in our pricing example agreed to pay a fixed coupon of 4 percent per year (as opposed to the 3.298 percent for the par value swap). What would the price of the swap be now?

The present value of the expected floating would remain unchanged:

$$\frac{\$150{,}000}{1.0075}+\frac{\$160{,}000}{1.01556}+\frac{\$170{,}000}{1.024192}+\frac{\$180{,}000}{1.03341}=\$646{,}597.01$$

For the fixed side we now have:

$$\frac{(.04\times\$5{,}000{,}000)}{1.0075}+\frac{(.04\times\$5{,}000{,}000)}{1.01556}+\frac{(.04\times\$5{,}000{,}000)}{1.024192}+\frac{(.04\times\$5{,}000{,}000)}{1.03341}=\$784{,}256.80$$

The difference in the present values represents the value of the swap:

\$784,256.80 – \$646,597.01 = \$137,659.80 to the floating-rate payer. The value is –\$137,659.80 to the fixed-rate payer.

12. What is "cherry picking"? Why does cherry picking complicate the scenario analysis of swap credit risk?

It is possible that in the event of bankruptcy, a bankruptcy court may allow each swap to run until its settlement, maturity, or expiration date, and then close out only those swaps that have positive replacement cost. Selectively closing out only those swaps with positive value is called cherry picking. The possibility of cherry picking is a scenario that must be accounted for in measuring potential counterparty credit exposure.

13. What is replacement cost? Why is potential replacement cost an important consideration for measuring swap counterparty credit risk?

Current replacement cost is the amount required to replace the swap in the event of counterparty default today. Current replacement cost alone does not accurately portray the potential credit risk over the life of the swap. A counterparty might default at some future date with swap values significantly different than current swap values. The potential loss is larger because the replacement cost can potentially become larger over the life of the swap.

14. What is the key difference between a credit default swap and a total return swap?

The key difference between a credit default swap and a total return swap is the fact that the credit default swap provides protection against specific credit events. The total return swap provides protection against loss of value irrespective of cause. Finally, either credit default swaps or total return swaps entail two sources of credit exposure: one from the underlying reference asset and another from possible default by the counterparty to the transaction.

15. What features of swaps can be customized?

Swaps can be custom-tailored to the needs of the counterparties. The counterparties can select the dollar amount that they wish to swap and the exact maturity that they need (i.e., the swap's "tenor"). In addition, swaps have numerous other terms that can be customized, including: (a) whether or not the notional amount is subject to an amortization schedule, and if so what that schedule is; (b) the index to which the floating rate resets (e.g., six-month LIBOR); (c) the spread (if any) to be added to the floating-rate index, reflecting considerations such as credit risk; (d) the frequency and timing of the floating-rate reset; and (e) any terms affecting the credit risk of the settlements.

CHAPTER 7 RISK MANAGEMENT WITH SWAPS

1. Consider the balance sheet information in Table 7.1. Suppose that the firm attempts to completely hedge its interest rate risk with a pay-fixed swap that has a duration of four years. What should the notional principal of this swap be?

$$D_G^* = 4 = 2.396201 + 5.151369\left(\frac{\text{Notonal principal of swap}}{\$155,467.133}\right)$$

This equation yields a notional principal of $48,402,285.38.

2. Suppose the firm in problem 1 has a desired duration gap of .5 years. Interpret the meaning of this gap in terms of a bond position.

   A gap of .5 years means that the firm wants the company to have the same interest rate sensitivity as a six-month T-bill.

3. Using the swap in problem 2, what notional principal would be required to achieve a desired duration gap of one year?

$$D_G^* = .5 = 2.396201 + 5.151369\left(\frac{\text{Notonal principal of swap}}{\$155,467.133}\right)$$

   This equation yields a notional principal of $57,226,910.57.

4. What is the difference between an amortizing swap and an accreting swap?

   With an amortizing swap, the notional principal is reduced over time. With an accreting swap, the notional principal becomes larger during the life of the swap.

5. What is a macro swap? How does a macro swap differ from the other swaps we have considered?

   A macro swap ties the floating rate of the swap to a macroeconomic variable or reference index such as the growth rate in GNP or the wholesale price index. Macro swaps differ from the other types of swaps we have considerd that exist primarily to manage interest rate or exchange rate risk. For companies whose sales and profits are highly correlated with the business cycle, macro swaps can be used to manage business cycle risk.

6. What is a currency annuity swap?

   A currency annuity swap is similar to a plain vanilla currency swap without the exchange of principal at the initiation or the termination of the swap. In a currency annuity swap, one party might make a sequence of payments based on 3-month European Union Euribor while the other makes a sequence of payments based on 3-month US dollar LIBOR. The currency annuity swap generally requires one party to pay an additional spread to the other or to make an up-front payment at the time of the swap. Variations on this structure can be created by allowing one, or both, parties to pay at a fixed. In pricing these swaps, the key is to specify a spread or up-front payment that equates the present value of the cash flows incurred by each party.

7. What right does a payer swaption give to its holder?

A payer swaption gives the holder the right to enter into a swap as the fixed-rate payer.

8. What right does a receiver swaption give to its holder?

A receiver swaption gives the holder the right to enter into a swap as the fixed-rate receiver.

9. Suppose the fixed rate used in the seasonal swap example was 3.5 percent. What would the price of this swap be to the fixed-rate payer?

The fixed-rate payments are:

First quarter	$.035 \times .25 \times \$10{,}000{,}000 = \$87{,}500$
Second quarter	$.035 \times .25 \times \$10{,}000{,}000 = \$87{,}500$
Third quarter	$.035 \times .25 \times \$10{,}000{,}000 = \$87{,}500$
Fourth quarter	$.035 \times .25 \times \$50{,}000{,}000 = \$437{,}500$

The floating-rate payments are:

First quarter	$.030 \times .25 \times \$10{,}000{,}000 = \$70{,}500$
Second quarter	$.032 \times .25 \times \$10{,}000{,}000 = \$80{,}000$
Third quarter	$.034 \times .25 \times \$10{,}000{,}000 = \$80{,}500$
Fourth quarter	$.036 \times .25 \times \$50{,}000{,}000 = \$450{,}000$

The floating-rate side is valued at:

$$\frac{\$70{,}500}{1.0075} + \frac{\$80{,}000}{1.01556} + \frac{\$80{,}500}{1.024192} + \frac{\$450{,}000}{1.03341} = \$667{,}193.30$$

The fixed-rate side is valued at:

$$\frac{\$87{,}500}{1.0075} + \frac{\$87{,}500}{1.01556} + \frac{\$87{,}500}{1.024192} + \frac{\$437{,}500}{1.03341} = \$681{,}796.90$$

For the party paying fixed, this swap is out of the money. It is worth \$−14603.61 to the fixed-rate payer and \$14,603.62 to the fixed rate receiver.

10. What is mismatch risk? Why is mismatch risk an important concern of swap dealers?

    Mismatch risk refers to the risk that the swap dealer will be left with a position that he cannot offset easily through another swap. This arises if there is a mismatch in the needs between the swap dealer and other participants. The swap dealer's transactions with multiple counterparties can leave the swap dealer with a residual risk position due to the mismatch between the needs of the counterparties.

11. Explain how an interest rate swap can be analyzed as a strip of futures.

    We have already noted that a swap may be regarded as a portfolio of forward contracts. A swap may also be thought of as a portfolio of forward contracts. For example, a swap agreement with quarterly payments based on Eurodollar deposit rates is essentially similar to a strip of Eurodollar futures contracts in which the futures maturities match the payment dates on the swap.

CHAPTER 8 FINANCIAL ENGINEERING AND STRUCTURED PRODUCTS

1. Is a call option a synthetic instrument? Explain what makes a financial instrument synthetic.

    A call option is not a synthetic instrument, although it may be possible to create a synthetic call from other instruments. Essentially, a synthetic instrument is a combination of instruments that has a terminal value and profit profile that closely replicates an identifiable security. For example, it is possible to create a synthetic call by trading a put, the underlying good, and a risk-free bond.

2. You hold a portfolio consisting of only T-bills. Explain how to trade futures to create a portfolio that behaves like the S&P 500 stock index.

    Buy S&P 500 Index futures. You should buy an amount of futures that equals the value of funds invested in T-bills. The resulting portfolio will replicate a portfolio that is fully invested in the S&P 500.

3. A stock trades at \$100 per share. A call option on the stock has an exercise price of \$100, costs \$16, and expires in six months. A put on the

stock also has an exercise price of $100, costs $10, and expires in six months as well. The six-month risk-free rate of interest is 10 percent. State exactly what instruments to buy and sell to create a synthetic equity position in the stock.

From put-call parity we know:

$$S - C - P + \frac{E}{(1+r)^1}$$

assuming that the call and put have the same expiration and exercise price, and the bond pays the exercise price at the expiration of the options. Thus, we can create a synthetic position in the stock by buying the call, selling the put, and investing $100/(1.05) = $95.24 in the risk-free bond for maturity in six months.

4. For the preceding question, make a table showing the profits and losses from the option portion of the synthetic equity as a function of the stock price in six months.

Terminal Stock Price	Long Call Profit Max{0, S – E} – C	Short Put Profit Max{0, E – S} – P
80	$–16	$10
85	–16	5
90	–16	0
95	–16	–5
100	–16	–10
105	–11	–10
110	–6	–10
115	–1	–10
120	+4	–10

5. A stock trades for $40 and a call on the stock with an expiration date in three months and a $40 strike price sells for $5. The risk-free rate of interest is 12 percent. State exactly how you would trade to create a synthetic put with a strike price of $40. Make a table showing the terminal values of your synthetic put and the actual put at expiration as a function of the stock price.

$$P = C - S + \frac{E}{(1+r)^1}$$

assuming that the call and put have the same expiration and exercise price, and the bond pays the exercise price at the expiration of the options. We create the synthetic put by buying a call, selling the stock, and buying a risk-free bond that pays $40 in six months.

Stock Price at Expiration	Value of a Synthetic Put at Expiration	Value of a Put at Expiration
30	$10	$10
35	5	5
40	0	0
45	0	0
50	0	0

6. A stock sells for $75 and a call with an exercise price of $75 sells for $7 and expires in six months. The risk-free rate of interest is 10 percent. What is the price of a put with a striking price of $75 and the same term to expiration?

From put-call parity we know:

$$P = C - S + \frac{E}{(1+r)^1}$$

assuming that the call and put have the same expiration and exercise price, and the bond pays the exercise price at the expiration of the options. Applying this equation to our data, we have:

$$P = \$7 - \$75 + \frac{\$75}{(1.05)} - \$7 - \frac{\$75}{\$71.43} = \$3.43$$

7. How much would you pay for a portfolio consisting of a short stock, short put, and a long call? Assume that the options have a striking price equal to the current price of the stock and that both options expire in one year.

Together the short put and long call have the same profits and losses as a long stock position. These gains or losses will exactly offset the gains or losses on the short stock position. At expiration, with the option portfolio and the exercise price you can acquire the funds necessary to acquire the stock and close the short sale. That means that the short stock, long call, short put portfolio should be worth the same as the present value of the exercise price, and that is the minimum you should demand for accepting the portfolio.

8. For the same underlying stock, a call and put both expire in one year, and their exercise price equals the current market price of the stock. Assume that you sell the stock short and can use 100 percent of the proceeds. You buy the call option and sell the put. You invest all remaining funds in the risk-free asset for one year. How much will this entire portfolio (stock, put, call, and bond) be worth in one year? Explain.

The long call/short put position will have profits and losses that exactly offset the profits or losses on the stock. You must also repay the short sale on the stock, so the bond must pay the exercise price of the options. Thus, the entire portfolio is worth the exercise price in one year. Use this information for the remaining questions. Assume a stock portfolio manager believes that her portfolio has an expected return of 12 percent and a standard deviation of 20 percent. Also, assume that the portfolio mimics an index on which call and put options trade. Assume that the index value is now 100 and the portfolio is worth \$100. The risk-free rate of interest is 8 percent. Calls and puts on the index trade with a \$100 strike price and are one year away from expiration. The call costs \$11.40. The focus is on the value of the portfolio in one year.

9. How much should the put cost? Explain.

Applying put-call parity we have:

$$P = C \times S + \frac{E}{(1+r)^1} = \$11.40 - \$100 + \frac{\$100}{1.08} = \$4.00$$

10. The manager asks you to devise a strategy to keep the value of the portfolio no less than \$92. How would you transact? Explain.

You could use \$4 of the portfolio's value to buy the put to create an insured portfolio. Then the value of the entire portfolio could never fall below \$92.

11. The manager asks you to devise a strategy that will provide a terminal portfolio value of \$112. How would you transact? Explain.

The original portfolio is worth \$100 and it has an expected value of \$112 in one year. There is no way to guarantee that you will achieve the expected value. The best that you can guarantee is that you will earn the risk-free rate of 8 percent for a terminal value of $108.

12. The manager asks you to devise a strategy that will dramatically increase the expected return on the portfolio. Give a qualitative description of how you would transact to achieve this goal.

To dramatically increase the expected return, you must increase the risk. This can be achieved by increasing leverage. For example, you could sell the stock portfolio and invest the entire proceeds in a long call/short put position. This will dramatically increase both the expected return and the risk.

13. The manager is determined not to trade any stocks to avoid transaction costs. Nonetheless, she desires a risk-free portfolio. How would you transact to meet her wishes? Explain.

You can create a synthetic risk-free bond. Because you already have a long position in the stock, merely sell the call and buy the put. This yields a cash inflow of $11.40 – $4.00 = $7.40, which can be invested at 8 percent to pay $8.00 in one year. The long stock/short call/long put will have no profits or losses and you will have achieved the riskless return of 8 percent.

14. Consider two interest rate swaps to pay fixed and receive floating. The two swaps require the same payments each semiannual period, but one swap has a tenor of 5 years, while the second has a tenor of 10 years. Assume that you buy the 10-year swap and sell the 5-year swap. What kind of instrument do these transactions create? Explain.

The net payment from these swaps during the first five years is zero, because the two exactly cancel each other. This leaves a long position in a swap that begins in five years with a tenor of five years. Thus, the resulting position is essentially a forward swap.

15. Assume you can borrow at a fixed rate for 10 years for 11 percent or that you can borrow at a floating rate of LIBOR plus 40 basis points for 10 years. Assume also that LIBOR stands at 10.60 percent. Under these circumstances, your financial advisor states: "The all-in cost is the same on both deals—11 percent. Therefore, the two are equivalent and one should be indifferent between these two financing alternatives." How would you react? Explain.

Your financial advisor is naive, because she is neglecting the shape of the term-structure. Her advice would have some foundation if the yield curve were flat at 11 percent. In this situation, the expected cost on the two alternatives would be the same. However, the short-term strategy still involves risks (and opportunities) that the fixed-rate strategy does not possess.

16. How is the floating rate payment structured in an inverse floating-rate note?

The floating rate on an inverse floater is determined by an algorithm that causes the floating-rate payments to fall as interest rates rise and floating-rate payments to rise as interest rates fall. For example, suppose an investor buys a note paying 13 percent minus six-month LIBOR, which yields 8.25 percent at the outset. If LIBOR rises, the return on the inverse floater falls.

17. What role did derivatives play (if any) in the Orange County debacle?

Derivatives did not play a key role in the Orange County debacle, which was caused primarily by the decision to use leverage and make yield curve bets. However, derivatives did play an important role in allowing the county's treasurer to increase leverage and effectively lengthening the average maturity of the investments in the portfolio. The principal derivative strategy used by Orange County focused on structured notes, with the portfolio including as much as $8 billion in structured notes, in particular inverse floaters.

18. What were the key issues in the BT-Gibson Greetings case?

The dispute between Gibson and BT centered around the duties of the two parties in determining the value of their swap transactions. Gibson claimed that BT owed it a fiduciary duty in objectively valuing the swaps. BT claimed that their relationship with Gibson was purely arm's length, without any fiduciary role. Whether to characterize the relationship between Gibson and BT as arm's length or advisory was an important, but not the only, issue in the dispute. Another issue was the allegation that BT's representatives misled Gibson about the value of the company's swap positions by providing Gibson with values that significantly understated the magnitude of Gibson's losses. As a result, Gibson remained unaware of the actual extent of its losses from its swap transactions and caused Gibson to understate its unrealized losses used to prepare financial statements filed with the SEC. The SEC alleged that on two occasions BT provided Gibson with valuations that differed by more that 50 percent from the value generated by BT's models and recorded on BT's books.

19. How did the conceptually sensible futures hedging program of Metallgesellschaft's subsidiary, MG Refining and Marketing break down in its implementation?

It is not clear that there was anything conceptually wrong with MGRM's hedging strategy. But it is clear that MGRM had not adequately communicated its intentions to its parent company and financial backers. In late 1993, MGRM's futures positions lost money as spot energy prices fell, requiring additional funds to meet margin calls. Presumably, the value to MGRM of its long-term, fixed-rate customer contracts increased as the value of the futures fell, leaving the net position hedged as planned. But the gains on the long-term customer contracts were not realized, while the futures losses were marked to market daily. By the end of 1993, the heavy cash outflows required to maintain the stack hedge, combined with concern about the credit risk taken on with the large swap position, caused MGRM's parent to change its assessment of the potential risks involved in its forward delivery contracts with customers. After reviewing the program, MGRM's parent decided to end MGRM's participation in the hedge program. In December 1993, MGRM's futures positions were unwound and customer contracts were cancelled. Given that many of these contracts were in-the-money to MGRM, this cancellation was costly.

notes

Chapter 1

1. We say "essentially" because in some legal proceedings over-the-counter swap transactions have been alleged to be futures for purposes of invoking the antifraud provisions of the Commodity Exchange Act. See, for example, CFTC docket no. 95-3 concerning disputed swaps transactions between Gibson Greetings, Inc., and BT Securities Corporation.
2. There are other futures exchanges that do not trade financial futures.
3. There is a notable exception in the forward market for foreign currency, where the forward market is extremely large and overshadows the futures market.
4. This estimate is derived from examining statistics compiled by the Bank for International Settlements (BIS). The BIS regularly publishes a semiannual statistical review of worldwide OTC markets.
5. See C. W. Smithson, "A LEGO Approach to Financial Engineering: An Introduction to Forwards, Futures, Swaps, and Options," *Midland Corporate Finance Journal,* 4:4, Winter 1987, pp. 16–28. Smithson refers to these building blocks as the LEGOS® with which all derivatives contracts are built.
6. Although the identity of traders on futures and options exchanges is not public information, large traders often cannot preserve their anonymity. Traders at the exchange know each other, and generally know which traders execute orders for large firms. Thus, large transactions executed by employees of a large firm signal the trading intentions of the firm to other traders present at the exchange.

Chapter 2

1. For more information on the sale of price and quote information, see J. Harold Mulherin, Jeffry Netter, and James Overdahl, "Who Owns the Quotes: A Case Study into the Definition and Enforcement of Property Rights at The Chicago Board of Trade," *The Review of Futures Markets,* 1991.
2. By contrast, more than 90 percent of foreign exchange forward contracts are completed by actual delivery.
3. In many cases, the owner of these goods will choose to insure them for him- or herself. Nonetheless, there is an implicit cost of insurance even when the owner self-insures.

4. For an informative and readable account of repurchase agreements, see Bowsher, "Repurchase Agreements," *Instruments of the Money Market,* Richmond: Federal Reserve Bank of Richmond, 1981.
5. For studies of this approach to pricing T-bill futures, see I. Kawaller and T. Koch, "Cash-and-Carry Trading and the Pricing of Treasury Bill Futures," *Journal of Futures Markets,* 4:2, Fall 1984, pp. 115–123.
6. The T-bond futures contract is complex, and our discussion abstracts from many of its features. For a comprehensive discussion of the features of the contract, see R. W. Kolb, *Understanding Futures Markets,* 5th ed., Malden, MA: Blackwell Publishers, 1997.
7. Obtaining expected dividend information on the 500 individual stocks in the underlying index can be a tedious task. Standard & Poor's Index Services Division tracks this information and makes it available to its customers. A rough estimate of the dividend stream, based on the annualized dividend yield can be found on www.spglobal.com, the S&P Web site.
8. Trading for the S&P 500 futures contracts ends on one day, and the final settlement price is set at the next day's opening cash price.
9. For a discussion of stock index futures and stock market volatility with references to many specific studies, see R. W. Kolb, *Understanding Futures Markets,* 5th ed., Malden, MA: Blackwell Publishers, 1997.
10. For a discussion of the impact of restrictions on stock index arbitrage trading, see James Overdahl and Henry McMillan, "Another Day, Another Collar: An Evaluation of the Effects of NYSE's Rule 80A on Trading Costs and Intermarket Arbitrage" (with Henry McMillan), *Journal of Business,* January 1998.

Chapter 3

1. For an excellent analysis of the Metallgesellschaft case, see C. Culp and M. Miller, "Metallgesellschaft and the Economics of Synthetic Storage," *Journal of Applied Corporate Finance,* Winter 1995, pp. 62–76.
2. See R. Kolb and R. Chiang, "Improving Hedging Performance Using Interest Rate Futures," *Financial Management,* 10:4, 1981, pp. 72–79; and "Duration, Immunization, and Hedging with Interest Rate Futures," *Journal of Financial Research,* 10:4, Autumn 1982, pp. 161–170.
3. Duration is a key concept in bond portfolio management. In essence, it is a single index number reflecting the sensitivity of the price of a bond to changes in interest rates. It depends most critically on the maturity and the coupon rate of the bond. For more on duration, see Robert W. Kolb, *Investments 4e* (Miami, FL: Kolb Publishing Company, 1995), particularly Chapters 7 and 8.
4. For a derivation of this relationship and a more extended discussion, see Robert W. Kolb, *Understanding Futures Markets, 5e* (Malden, MA: Blackwell Publishers, 1997).
5. Strictly speaking, there is no return on a futures contract because a position in a futures contract requires no investment. By the futures return, we mean the percentage change in the futures price.

6. For more on the subject of when corporations should hedge (and when they should not), see Smith and Stulz, 1985, "The Determinants of Firms' Hedging Policies," *Journal of Financial and Quantitative Analysis,* 20, 391–405.

Chapter 4

1. In the place of some prices, the letters *r* and *s* appear. An *r* indicates that a particular option was not traded on the day being reported. An *s* indicates that no option with those characteristics is being made available for trading by the exchange.
2. Because expiration is imminent, we are assuming that it is too late to sell the option.
3. Most of the principles indicated here were originally proven rigorously by Robert C. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science,* 1973, pp. 141–183.
4. Throughout this chapter, solid lines are used to indicate long positions and dashed lines are used to indicate short positions.
5. There is still an interesting result to be noted here. If the prices are equal, a trader could buy the option with the lower striking price and sell the one with the higher striking price. This strategy would not guarantee a profit, but it could not lose. Further, there would be some situations in which it would pay off. For this reason, options with lower exercise prices almost always sell for higher, not just equal, prices, as the quotations from the *Wall Street Journal* make clear.
6. Strictly speaking, this argument holds for *American options*. An American option allows exercise at any time until maturity. By contrast, a *European option* allows exercise only at maturity. Thus, an American option gives all the advantages of the European option, plus it allows early exercise. For this reason, an American option must always have a value at least as great as a European option, other factors being equal.
7. This result requires that the six-month option be an American option so it may be exercised at will prior to expiration.
8. In the case of a dividend-paying stock, this will not always be true.
9. The arbitrage transactions would involve buying Portfolio B and selling Portfolio A short. Assume that the price of the call option is $1,000 and try to work out the transactions and the arbitrage profit that must result.
10. Strictly speaking, this condition will hold necessarily if at least one of two conditions is met. First, the equation holds if the stock underlying the option pays no dividends. Second, the equation holds if the option is a European option. A European option can be exercised only at the expiration of the option. As these two restrictions imply, the equation might fail depending on the cash flows associated with the stock before expiration. The complications these interim cash flows present are beyond the scope of this text.
11. See Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 1973, pp. 637–654.

12. We consider some simple dividend adjustments within the Black-Scholes Model in this chapter. However, more complicated extensions to the model are beyond the scope of this text. The interested student should see Robert A. Jarrow and Andrew Rudd, *Option Pricing,* Homewood, IL: Richard D. Irwin, 1983, for a complete exposition of extensions to the Black-Scholes Model.
13. Actually, part of the difference is due to the discounting method. Had our example used continuous discounting at 12 percent, we would have found that the value of the option had to be at least as great as $100 – $100 (.8869) = $11.31. This is much closer to the OPM value of $11.84.
14. The put-call parity relationship was first derived by Hans Stoll, "The Relationship between Put and Call Option Prices," *The Journal of Finance,* December 1969, pp. 802–824.
15. For more on the role of dividends in triggering option exercise, see James A. Overdahl and Peter Martin, "The Exercise of Equity Options: Theory and Empirical Evidence," *Journal of Derivatives,* Fall, 1994.
16. The acronym SPDR stands for Standard & Poor's Depository Receipts.
17. Two books discuss options on futures in great detail: John W. Labuszewski and Jeanne Cairns Sinquefield, *Inside the Commodity Options Market,* New York: Wiley, 1985; and John W. Labuszewski and John E. Nyhoff, *Trading Options on Futures,* New York: Wiley, 1988.
18. If we define the net cost-of-carry to include all benefits that come from holding the commodity, we can specify a revised Cost-of-Carry Model that fits more commodities. These benefits of holding the physical good are known as the *convenience yield*. However, this strategy merely saves the model by stipulating that apparent discrepancies in the model equal the unobservable convenience yield. For applying the option model, we would then need to measure the convenience yield, which appears difficult at best.
19. Fischer Black, "The Pricing of Commodity Contracts," *Journal of Financial Economics,* 3, 1976, pp. 167–179, was the first to develop a pricing model for options on futures.
20. K. Shastri and K. Tandon, "An Empirical Test of a Valuation Model for American Options on Futures Contracts," *Journal of Financial and Quantitative Analysis,* 21:4, December 1986, pp. 377–392.
21. In Figure 4.9, we indicate the special number "e" by the notation "exp."
22. R. Whaley, "Valuation of American Futures Options: Theory and Empirical Tests," *Journal of Finance,* March 1986, pp. 127–150. Figure 4.9 is based on a figure in Whaley's article. See also Robert E. Whaley, "On Valuing American Futures Options," *Financial Analysts Journal,* 42:3, May–June 1986, pp. 49–59.
23. Giovanni Barone-Adesi and Robert E. Whaley, "Efficient Analytical Approximation of American Option Values," *Journal of Finance,* 42:2, June 1987, pp. 301–320, show how to find the critical futures price efficiently. Gerald D. Gay, Robert W. Kolb, and Kenneth Yung, "Trader Rationality in the Exercise of Futures Options," *Journal of Financial Economics,* 23:2, August 1989, pp. 339–362, examine the correspondence between actual exercise behavior and the guidance provided by the Barone-Adesi and Whaley model. In general,

Gay, Kolb, and Yung find very few instances in which traders exercise when they should not. In many instances, however, traders should have exercised but did not. There are rare occasions of traders exercising when they should not have, with these exercises resulting in large losses.

24. For more on the relationships between options and futures on foreign currencies, see I. Giddy, "Foreign Exchange Options," *The Journal of Futures Markets,* 1983, pp. 143–166.

Chapter 5

1. The use of terms such as *bear spread* and *bull spread* is not standardized. Although this book uses these terms in familiar ways, other traders may use them differently.

Chapter 6

1. This does not mean to imply that exchange trading sacrifices all anonymity. However, traders watch the activities of major institutions. When these institutions initiate major transactions, it is impossible to maintain complete privacy. It is ironic that individual traders can trade on futures and options markets with a discretion that is not available to multi-billion-dollar financial institutions.
2. The US Dollar London InterBank Offered Rate, or LIBOR, represents the rate of return on large, negotiable, Eurodollar certificates of deposit maturing at some date within the next year. LIBOR can be quoted for a variety of currencies although US Dollar LIBOR is the most commonly quoted. The world's center for cash Eurodollar trading is London, but there are active Eurodollar markets in other parts of the world. London-based Eurodollar interest rates are quoted as LIBOR; quotes from other market centers use a similar format (e.g., Paris and Tokyo are quoted as PIBOR and TIBOR, respectively).

   As there are Eurodollars, there are euro versions of other currencies such as the Swiss franc, the Japanese yen, the British pound sterling, and the euro Two competing sets of banks are presently struggling to become the benchmark for quoting interest rates on the euro. One group is concentrated in London and consists of the member banks of the British Bankers Association. This group quotes rates for other currencies and is the reason for the dominance of the London interbank rates. Its quotations for the euro are known as Eurolibor. As Great Britain is not a full participant in the European Monetary Union, a competing group of banks on the European continent has sprung up to challenge the primacy of the British bankers. This quotation by continental banks is known as Euribor. The actual rates of the two quoting associations are scarcely distinguishable.
3. The practice of making net payments and not actually exchanging principal also protects each counterparty from default by the other. It would be very unpleasant

for Party A if it paid the principal amount of $1 million in our example and Party B failed to make its payment to Party A. Making only net payments greatly reduces the potential impact of default.

4. *Source: Risk,* December 2001, page 17.
5. Since September 2000, controversy has erupted in two instances over just what constitutes a "triggering event" in a standard credit default swap. In one instance, a large debt restructuring by Indianapolis-based life insurer Conseco caused some lenders, who had bought protection with credit default swaps written with Conseco as the reference asset, to exercise the default triggers in the swap agreements. Some protection sellers cried foul, alleging that certain protection buyers had a conflict of interest resulting from their dealings with Conseco. These protection buyers were the same lenders whose actions allegedly forced the triggering event in the first place. In the second instance, AT&T's announcement that it planned to split into four separate companies generated controversy because the standard master agreements for credit default swaps had not contemplated a split into more than two separate companies. The International Association of Swaps and Derivatives Dealers (ISDA), which writes the standard master agreements, has tried to clarify the language in standard agreements. At issue is whether the aforementioned types of credit event should be included as standard triggering events in credit default swaps, or whether they should be contracted for separately by swap counterparties. For more information on this subject, see "Splitting Headaches," *Risk,* July 2001, pages 36–37.
6. For more information on credit swaps and other credit derivatives, see Chapter 14 (Credit Derivatives) in *Credit Risk Measurement,* by Anthony Saunders, New York: John Wiley & Sons, 1999; and Chapter 56 (Credit Derivatives) in *Paul Wilmott on Quantitative Finance,* Volume 2, by Paul Wilmott, New York: John Wiley & Sons, 2000.
7. This discussion of comparative advantage draws on the excellent analysis by K. Kapner and J. Marshall in *The Swaps Handbook,* New York: New York Institute of Finance, 1990.
8. The swap dealer will also consider other issues in setting final pricing terms. If the swap is very complicated, the swap dealer may charge a higher price than otherwise. Similarly, if the swap is to involve cross-border currency flows, the dealer may be concerned with regulatory constraints that might impede the flow of funds.
9. In actual market practice, the participants must carefully consider the actual way in which yields are calculated on Treasury securities versus the money market computations that govern LIBOR. We abstract from these technicalities.
10. See J. A. Overdahl, B. Schachter, and I. Lang, "The Mechanics of Zero-Coupon Yield Curve Construction," in *Controlling and Managing Interest Rate Risk,* Klein, Cornyn, and Lederman, editors, New York Institute of Finance, 1997.
11. For more complex examples of swap pricing, see Robert W. Kolb, *Futures, Options, and Swaps,* 4th edition, Malden, MA: Blackwell Publishers, 2002.

Chapter 7

1. The Standard & Poor's Corporation licenses the use of the S&P 500 index for use as a reference index in swaps and other privately negotiated derivatives. This is common practice of other index providers as well. For more information on the use of proprietary indexes in the construction of swap contracts, see James A. Overdahl, "The Licensing of Financial Indexes: Implications for the Development of New Index-Linked Products," in *Indexing for Maximum Investment Management Results,* Albert S. Neubert, editor, Glenlake Publishing Co., 1997.
2. This section draws on an article by Peter A. Abken, "Beyond Plain Vanilla: A Taxonomy of Swaps," Federal Reserve Bank of Atlanta, *Economic Review,* March/April 1991. Reprinted in R. Kolb, *The Financial Derivatives Reader,* Miami: Kolb Publishing, 1993.
3. See I. G. Kawaller, "Interest Rate Swaps versus Eurodollar Strips," *Financial Analysts Journal,* September/October 1989, for a discussion of the relationship between swaps and strips.
4. For more information on value at risk, see Philippe Jorion, *Value at Risk,* Chicago: Irwin Professional Publishing, 1997.

Chapter 8

1. The following articles introduce various aspects of portfolio insurance. For the most part, they are not highly mathematical. M. Rubinstein, "Alternative Paths to Portfolio Insurance," *Financial Analysts Journal*, 41:4, July/August 1985, pp. 42–52; F. Black and R. Jones, "Simplifying Portfolio Insurance," *Journal of Portfolio Management*, 14:1, Fall 1987, pp. 48–51; T. O'Brien, "The Mechanics of Portfolio Insurance," *Journal of Portfolio Management*, 14:3, Spring 1988, pp. 40–47; P. Abken, "An Introduction to Portfolio Insurance," *Economic Review*, Federal Reserve Bank of Atlanta, 72:6, November/December 1987, pp. 2–25; H. Bierman, "Defining and Evaluating Portfolio Insurance Strategies," *Financial Analysts Journal*, 44:3, May/June 1988, pp. 84–87. The following articles are somewhat more technical or specialized, but of considerable interest. F. Gould, "Stock Index Futures: The Arbitrage Cycle and Portfolio Insurance," *Financial Analysts Journal*, 44:1, January/February 1988, pp. 48–62; J. Merrick, "Portfolio Insurance with Stock Index Futures," *Journal of Futures Markets*, 8:4, 1988, pp. 441–455; R. Bookstaber and J. Langsam, "Portfolio Insurance Trading Rules," *Journal of Futures Markets*, 8:1, 1988, pp. 15–31; Y. Zhu and R. Kavee, "Performance of Portfolio Insurance Strategies," *Journal of Portfolio Management*, 14:3, Spring 1988, pp. 48–54; J. Singleton and R. Grieves, "Synthetic Puts and Portfolio Insurance Strategies," *Journal of Portfolio Management*, 10:3, Spring 1984, pp. 63–69; M. Kritzman, "What's Wrong with Portfolio Insurance?" *Journal of Portfolio Management*, 13:1, Fall 1986, pp. 13–17. Finally, Donald L. Luskin, *Portfolio Insurance: A Guide to Dynamic Hedging*, New York:

John Wiley & Sons, 1988, contains a collection of interesting articles on portfolio insurance.

2. Chapter 6 discusses swaptions in more detail.
3. This example is adapted from John F. Marshall and Kenneth R. Kapner, *The Swaps Market,* 2e, Miami: Kolb Publishing, 1993, pp. 143–146.
4. This account of PERCS relies on Andrew H. Chen, John Kensinger, and Hansong Pu, "An Analysis of PERCS," *Journal of Financial Engineering,* 3:2, June 1994, pp. 85–108. Merrill Lynch has created a similar security known as Mandatory Conversion Premium Dividend Preferred Stock.
5. This analysis of equity-linked CDs relies on Charles Baubonis, Gary Gastineau, and David Purcell, "The Banker's Guide to Equity-Linked Certificates of Deposit," *Journal of Derivatives,* 1:2, Winter 1993, pp. 87–95. See also E. H. Cantor and B. Schachter, "Indexed Certificates of Deposit," in *Derivatives, Regulation, and Banking,* B. Schachter editor, Elsevier, 1997.
6. See Leland E. Crabbe and Joseph D. Argilagos, "Anatomy of the Structured Note Market," *Journal of Applied Corporate Finance,* 7:3, Fall 1994, pp. 85–98.
7. These examples are drawn from Leland E. Crabbe and Joseph D. Argilagos, "Anatomy of the Structured Note Market," *Journal of Applied Corporate Finance,* 7:3, Fall 1994, pp. 85–98.
8. *Source: New York Times,* "How Citigroup Hedged Bets against Enron," February 8, 2002, page C1.
9. This example is drawn from Andrew Kalotay and Bruce Tuckman, "A Tale of Two Bond Swaps," *Journal of Financial Engineering,* 1:3, December 1992, pp. 235–343.
10. This example is drawn from Christopher L. Culp, Dean Furbush, and Barbara T. Kavanagh, "Structured Debt and Corporate Risk Management," *Journal of Applied Corporate Finance,* 7:3, Fall 1994, pp. 73–84.
11. See "Orange on a Banana Skin," *Risk,* 8:1, January 1995, p. 6.
12. The analysis of this particular inverse floater appeared in Zahid Ullah, "Valuing the Orange County Notes," *Derivatives Strategy,* December 12, 1994, p. 3.
13. U.S. Securities and Exchange Commission, form 8-K filing, April 19, 1994.
14. *New York Times,* "Gibson Suit on Trades Is Settled" (November 24, 1994), D1.
15. *Derivatives Strategy,* 4:1, January 2, 1995, p. 1.
16. This account relies on "P&G vs. BT," *Derivatives Strategy,* 3:17, October 31, 1994, p. 1.
17. *Source: Report of the Banking Supervision Inquiry into the Circumstances of the Collapse of Barings, Ordered by the House of Commons,* July 1995, Her Majesty's Stationary Office, London.
18. Total futures positions amounted to over $500 billion, swaps over $750 billion, and options and other OTC derivatives over $150 billion. *Source:* Philippe Jorion, "Risk Management Lessons from Long-Term Capital Management," *European Financial Management,* 6 (September 2000): 277–300.

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